IERS TECHNICAL NOTE 3

IERS Standards (1989)

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IERS Standards

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IERS Technical Notes

This series of publications gives technical information related to the IERS activities, e.g. reference frames, excitation of the Earth rotation, computational or analysis aspects, models, etc. It also contains the description and results of the analyses performed by the IERS Analysis Centres for the Annual Report global analysis.

Issues available

No 1 : C. Boucher and Z.Altamimi. The initial IERS Terrestrial Reference Frame.

No 2 : Earth orientation and reference frame determinations, atmospheric excitation functions, up to 1988, (Annex to the IERS Annual Report for 1988).

No 3 : IERS Standards (1989)

INTRODUCTION

This document is intended to define the standard reference system to be used by the International Earth Rotation Service (IERS). It is based largely on the Project MERIT Standards (1983) with revisions being made to reflect improvements in models or constants since the Project MERIT Standards were published. If contributors to IERS do not fully comply with these guidelines, they will carefully identify the exceptions. In these cases, the institution is obliged to provide an assessment of the effects of the departures from the standards so that its results can be referred to the IERS Reference System. In the case of models, contributors may use models equivalent to those specified herein. Different observing methods have varying sensitivity to the adopted standards and reference systems. No attempt has been made in this document to assess the sensitivity of each technique to the adopted reference systems and standards.

The recommended system of astronomical constants corresponds closely to those of the Merit Standards (Melbourne, et al., 1983) with the exception of the following changes: the radius of the Earth; the dynamical form factor of the Earth; the gravitational constants G and $GM_\oplus$; the obliquity of the ecliptic ϵ_0; and the masses of the Earth-Moon system and of Saturn. The units of length, mass, and time are in the International System of Units (SI) as expressed by the meter (m), kilogram (kg) and second (s). The astronomical unit of time is the day containing 86400 SI seconds. The Julian century contains 36525 days of atomic time. The Gaussian constant, k = 0.01720209895, is the defining constant relating the heliocentric gravitational constant ($GM_\odot$) to the astronomical unit of length (A) and to the unit of time through the relationship

$$GM_\odot = A^3 k'^2$$

where $GM_\odot$ is expressed in $m^3\ s^{-2}$, A is the astronomical unit in meters (derived from the measured value of the astronomical unit in light-seconds and the defined value of the velocity of light in $m\ s^{-1}$), and k' is k/86400.

In general, each observational technique uses different realizations of both the terrestrial and celestial frames. In addition, the techniques use different transformations between these frames. The J2000.0 epoch is recommended for use in reference system algorithms. Further, the mean equator and equinox at J2000.0 is recommended as the space-fixed coordinate frame. The transformation from the 1950.0 frame to J2000.0 should use the IAU 1976 value of the precession constant. The value of the correction to the FK4 equinox is (Fricke, 1982)

$$E(T) = 0^s.035 + 0^s.085T,$$

where T is measured in Julian centuries from 1950.0. This expression for E(T) is adopted and is applied at the epoch J2000.0. A corresponding relationship between Greenwich Mean Sidereal Time (GMST) and Universal Time is given by (Aoki, et al., 1982)

$$\text{GMST at } 0^h\text{UT1} = 6^h\ 41^m\ 50^s.54841 + 8640184^s.812866T_u$$
$$+ 0^s.093104T_u^2 - 6^s.2 \times 10^{-6}T_u^3,$$

where T_u is measured in Julian centuries from J2000.0. This expression does lead, however, to a discontinuity and a change in rate of UT1 as measured by space techniques that are fixed to an inertial celestial frame. (See Williams and Melbourne, 1982; also Zhu and Mueller, 1983.) Although optical observations are not used in the IERS system, Chapter 5 "Procedure for Computing Apparent Places" is included for the purpose of clarifying coordinate transformations.

REFERENCES

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Fricke, W., 1982, "Determination of the Equinox and Equator of the FK5," Astron. Astrophys., **107**, pp. L13-16.

Melbourne, W., Anderle, R., Feissel, M., King, R., McCarthy, D., Smith, D., Tapley, B., Vicente, R., 1983, Project MERIT Standards, U.S. Naval Observatory Circular No. 167.

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Zhu, S. Y. and Mueller, I. I., 1983, "Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame," Bull. Geod., **57**, pp. 29-42.

CHAPTER 1 NUMERICAL STANDARDS

The tables herein and the succeeding chapters list the IERS Standards for constants, models, and reference frames. The format for the constants parallels that of the Project MERIT Standards. The tables are organized into three columns: the item, the standard value, and comments. The comments note departures from the IAU values and direct the reader to the appropriate chapter for an expanded discussion or listing of values. In some cases, the succeeding chapters contain tutorial material that might prove helpful. Algorithms are, in some cases, provided to clarify a formulation.

REFERENCES

Astronomical Almanac for 1984, U. S. Government Printing Office, Washington D. C.

Chovitz, B., 1987, Report of the IAG Special Study Group 5:100, Parameters of Common Relevance of Astronomy, Geodesy, and Geodynamics, submitted to XIX General Assembly of IUGG.

Cohen, E. R., and Taylor, B. N., 1986, The 1986 Adjustment of the Fundamental Physical Constants, CODATA Bulletin No. 63, Pergamon Press.

Jacchia, L. G., 1971, "Revised Static Models of the Thermosphere and Exoshpere with Empirical Temperature Profiles," Smithson. Astrophys. Observ. Spec. Rep., **332**, Cambridge, Mass.

Lieske, J. H., Lederle, T., Fricke, W., and Morando, B., 1977, "Expression for the Precession Quantities Based upon the IAU (1976) System of Astronomical Constants," Astron. Astrophys., **58**, pp. 1-16.

Marini, J. W. and Murray, C. W., 1973, Correction of Laser Range Tracking Data for Atmospheric Refraction at Elevations Above 10 Degrees, NASA GSFC X-591-73-351.

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NUMERICAL STANDARDS

ITEM	RECOMMENDED VALUE	COMMENTS
ASTRONOMICAL CONSTANTS		
Defining Constants		
- Gaussian Gravitational Constant	$k = 0.01720209895$	
- Velocity of Light	$c = 2.99792458 \times 10^8 ms^{-1}$	
Primary Constants		
- Astronomical Unit in Light-Seconds	$\tau_A = 499.00478370$ s	IAU (1976) Value = 499.004782 s.
- Equatorial Radius of the Earth	$a_e = 6378136$ m	IUGG Value; IAU Value = 6378140.
- Dynamical Form Factor for Earth	$J_2 = 0.001082626$ $\dot{J}_2 = -2.8 \times 10^{-11}\ yr^{-1}$	Use value in adopted gravity model adjusted to include zero frequency tide.
- Geocentric Constant of Gravitation	$GM_\oplus = 3.98600440 \times 10^{14} m^3 s^{-2}$	IAU (1976) Value = $3.986005 \times 10^{14} m^3 s^{-2}$. Barycentric value given by $GM_{\oplus TDB} = (1-L)GM_\oplus$ where $L = 1.4808 \times 10^{-8}$.
- Constant of Gravitation	$G = 6.67259 \times 10^{-11} m^3 kg^{-1} s^{-2}$	
- Earth-Moon Mass Ratio	$\mu = 0.012300034$	IAU (1976) Value = 0.0123002.
- General Precession in Longitude Per Century for J2000.0	$p = 5029''.0966$	
- Obliquity of the Ecliptic for J2000.0	$\epsilon_0 = 23°\ 26'\ 21''.4119$	IAU (1976) Value = 23°26'21".448. Recommended values use rotating definition (see Chapter 4).
Derived Constants		
- Nutation Constant for J2000.0	This constant is superseded by the adoption of the IAU 1980 nutation based on the Wahr Nutation Model. See Ch. 4 for the recommended Nutation Series in longitude and obliquity.	
- Astronomical Unit	$A = c\tau_A = 1.4959787066 \times 10^{11}$ m	IAU (1976) Value = $1.49597870 \times 10^{11}$ m.
- Solar Parallax	$\pi_\odot = \sin^{-1}(a_e/A) = 8''.794144$	IAU (1976) Value = 8".794148.
- Aberration Constant for J2000.0	The aberration corrections are calculated directly from ephemeris files.	IAU (1976) Value = 20".49552.
- Earth Flattening	$f^{-1} = 298.257$	Uses the IAG (1975) value of $\omega = 7.292115 \times 10^{-5}$ rad s^{-1}.
- Heliocentric Constant for Gravitation	$GM_\odot = 1.32712440 \times 10^{20} m^3 s^{-2}$	IAU (1976) Value = $1.32712438 \times 10^{20} m^3 s^{-2}$.

NUMERICAL STANDARDS
(continued)

ITEM	RECOMMENDED VALUE	COMMENTS
Derived Constants (continued)		
- Ratio of the solar Mass to the Mass of the Earth	$GM_\odot/GM_{TDB} = 332,946.038$	IAU (1976) Value = 332,946.0.
- Ratio of the Solar Mass to the Mass of the Earth-Moon System	$GM_\odot/GM_{TDB}(1+\mu) = 328,900.55$	IAU (1976) Value = 328,900.5.
- Solar Mass	$M_\odot = 1.9891 \times 10^{30}$ kg	
System of Masses (Expressed in Reciprocal Solar Masses)		
- Mercury	6,023,600	
- Venus	408,523.5	
- Earth-Moon System	328,900.55	IAU (1976) Value = 328,900.5. (adjustable in LLR)
- Mars	3,098,710	
- Jupiter	1,047.350	IAU (1976) Value = 1,047.355.
- Saturn	3,498.0	IAU (1976) Value = 3,498.5.
- Uranus	22,960	IAU (1976) Value = 22,869.
- Neptune	19,314	
- Pluto-Charon	130,000,000	IAU (1976) Value = 3,000,000.
- Ceres	1.695×10^9	IAU (1976) Value = 1.7×10^9.
- Pallas	9.247×10^9	IAU (1976) Value = 9.1×10^9.
- Vesta	7.253×10^9	IAU (1976) Value = 8.3×10^9.
Lunar Gravitational Parameters for LLR		The values of these parameters are consistent with the DE200/LE200 ephemerides but they are adjustable in LLR.
$\gamma = (B-A)/C$	2.280043×10^{-4}	IAU (1976) Value = 2.278×10^{-4}.
$\beta = (C-A)/B$	6.316769×10^{-4}	IAU (1976) Value = 6.313×10^{-4}.
C/MR^{2*}	0.39053	IAU (1976) Value = 0.392.
I^*	5553″.5	IAU (1976) Value = 5552″.7 = 1° 32′ 32″.7.
GM^*	4902.7993 km^3/sec^2	

ITEM	RECOMMENDED VALUE	COMMENTS
Lunar Gravitational Parameters for LLR (continued)		
Love Number (k_2)	0.0222	
Rotational Dissipation (k_2T)	4.643×10^{-3} days	
C_{20}	-2.02151×10^{-4}	IAU (1976) Value = -2.027×10^{-4}.
C_{22}*	2.2302×10^{-5}	IAU (1976) Value = $+2.23 \times 10^{-5}$.
C_{30}	-8.626×10^{-6}	IAU (1976) Value = -6×10^{-6}.
C_{31}	3.071×10^{-5}	IAU (1976) Value = $+2.9 \times 10^{-5}$.
S_{31}	5.6107×10^{-6}	IAU (1976) Value = $+4 \times 10^{-6}$.
C_{32}	4.8348×10^{-6}	IAU (1976) Value = $+4.8 \times 10^{-6}$.
S_{32}	1.684×10^{-6}	IAU (1976) Value = $+1.7 \times 10^{-6}$.
C_{33}	1.436×10^{-6}	IAU (1976) Value = $+1.8 \times 10^{-6}$.
S_{33}	-3.3435×10^{-7}	IAU (1976) Value = -1×10^{-6}.
C_{40}	1.5×10^{-7}	
C_{41}	-7.18×10^{-6}	
S_{41}	2.95×10^{-6}	
C_{42}	-1.440×10^{-6}	
S_{42}	-2.884×10^{-6}	
C_{43}	-8.5×10^{-8}	
S_{43}	-7.89×10^{-7}	
C_{44}	-1.549×10^{-7}	
S_{44}	5.64×10^{-8}	

*Derived Constants

DYNAMICAL MODELS

Geopotential

- Laser Satellites	GEM-T1	See Chapter 13.
- LLR	IAU (1976) zonals through degree 4 for DE200/LE200.	

NUMERICAL STANDARDS
(continued)

ITEM	RECOMMENDED VALUE	COMMENT
Drag		
- Ballistic Coefficient $\Psi = C_D A/m$	Lageos $A = 0.283\ m^2$ $m = 407.0$ kg	C_D is an adjusted parameter.
Radiation Pressure		
- Reflectance Coefficient, C_R		See Chapter 14.
- Reflectance Model		See Chapter 14.
- Earth Radiation Pressure	Ignored	
- Penumbra Model	6402000 m 1738000 m	Radius of Earth for shadow model. Radius of Moon for shadow model.
Solid Earth Tides		See Chapter 6.
Ocean Tides	Schwiderski Ocean Tide Model	See Chapter 7.
Relativistic Corrections		
- Propagation		
- LLR	Retardation due to Sun and Earth	See Chapter 16.
- VLBI	Retardation and bending due to Sun, Earth, and Moon	See Chapter 16.
- SLR	Retardation due to Earth	See Chapter 16.
- Time Epoch and Interval		
- LLR, VLBI	Annual, diurnal, and other periodic terms	See Chapter 16.
- SLR	none	See Chapter 16.
- Dynamics		
- LLR	Barycentric (n-body) formulation (β=γ=1)	See Chapter 15.
- SLR	Geocentric (1-body) formulation (β=γ=1)	See Chapter 15.

NUMERICAL STANDARDS
(continued)

ITEM	RECOMMENDED VALUE	COMMENTS
Empirical Force		
-LAGEOS	$C_T \times 10^{-12}$ m s^{-2} $\bar{V}/V$ per unit mass	C_T is an adjusted parameter that ranges from 0 to -6 over time. Mean value over LAGEOS lifetime is around -3.4.
Secular Acceleration of the Moon, $\dot{n}$		$\dot{n}$ is an adjusted parameter in LLR. Value in DE200/LE200 is near $-23.9/cy^2$.
MEASUREMENT MODEL		
Troposphere		
- SLR and LLR	Surface meteorology measurement plus Marini and Murray Model (1973).	See Chapter 10.
- VLBI		See Chapter 10. Water vapor radiometry if available-otherwise use model plus possible adjustment of vertical delay.
Satellite Center of Mass Correction		
- Lageos	0.24 m	
Solid Earth Tides Displacement	Wahr Solid Tide Model	See Chapter 6.
Ocean Loading Site Displacement	Schwiderski tides	See Chapter 8.
REFERENCE SYSTEMS	1984 Conventions except as noted.	
Conventional Inertial System	Mean equinox and equator of J2000.0	See Chapter 2.
Time Synchronization	UTC as given by BIPM	If using UTC(USNO), then use UTC(USNO)-UTC(BIPM) as published by BIPM to correct to UTC(BIPM).
Precession	IAU 1976	See Lieske, et al. (1977) for application.

ITEM	RECOMMENDED VALUE	COMMENT
Nutation	IAU 1980	Based on Wahr Theory. Reference pole is the Celestial Ephemeris Pole (CEP). See Chapter 4.
Terrestrial Reference Frame		See Chapter 3.
Tidal Variations in UT1		See Chapter 11.
VLBI Radio Source Positions and Designations		See Chapter 2.
Tectonic Motion	AM0-2	See Chapter 9.
Ephemeris System	Astronomical Almanac, 1984 (DE200/LE200).	Uses the Equinox and Equator of J2000.0. Origin in right ascension is set equal to the dynamical equinox of J2000.0. See Chapter 4.
Lunar Reference Frame		
- Retro-Reflector Coordinates (meters)		These coordinates are consistent with the DE200/LE200 ephemeris system but they are adjustable in LLR.

Apollo 11

	X1	X2	X3
PA	1592012.174	690605.998	21006.310
ME	1591752.786	691221.955	20394.850
	R	LONG	LAT
PA	1735477.073	23.45093088	.69352820
ME	1735477.073	23.47299617	.67333975

NUMERICAL STANDARDS
(continued)

ITEM		RECOMMENDED VALUE		COMMENT
Apollo 14				
		X1	X2	X3
	PA	1652662.237	-521095.647	-109727.640
	ME	1652821.419	-520455.963	-110364.156
		R	LONG	LAT
	PA	1736339.050	-17.50041767	-3.62321101
	ME	1736339.050	-17.47866283	-3.64425710
Apollo 15				
		X1	X2	X3
	PA	1554686.268	98004.046	765010.082
	ME	1554942.413	98604.650	764412.078
		R	LONG	LAT
	PA	1735481.089	3.60702873	26.15530389
	ME	1735481.089	3.62847880	26.13331104
Lunakhod 2				
		X1	X2	X3
	PA	1339413.779	801793.356	756361.607
	ME	1339394.295	802310.618	755847.426
		R	LONG	LAT
	PA	1734642.539	30.90537743	25.85105146
	ME	1734642.539	30.92203167	25.83218088

PA = Principal Axis Coordinates
ME = Mean Earth Coordinates

Rotation Angles between mean Earth and principal axis coordinates are tau = 79.815, P1 = -79.350, P2 = 0.295 arcseconds.

CHAPTER 2 CELESTIAL REFERENCE FRAME

The directions of the axes of the Celestial Reference Frame (ICRF) are defined by the coordinates of 23 primary radio sources published in the Annual Report of the BIH for 1987. The frame is densified by 205 secondary and supplementary sources, the positions of which are known with lower precision. The origin of the ICRF is at the barycenter of the solar system. The direction of the axis of rotation of the Earth is the one given for epoch J2000.0 by the IAU 1976 Precession and the IAU 1980 Theory of Nutation. The origin of right ascensions is in agreement with that of the FK5 within $\pm 0\overset{''}{.}04$. The most recent realization is given in the Annual Report of the IERS and future information will be found in the most recent Annual Report of the IERS.

REFERENCES

Arias, E. F., Feissel, M., and Lestrade, J.-F., 1988, Annual Report of the BIH for 1987, p. D-113.

Arias, E. F., Feissel, M., 1989, "The Celestial System of the International Earth Rotation Service," Proceedings of IAU Symposium 141, in press.

CHAPTER 3 CONVENTIONAL TERRESTRIAL REFERENCE FRAME

The IERS Terrestrial Reference Frame (ITRF) adopted for either the analysis of individual data sets by techniques (VLBI, SLR, LLR) or the combination of individual solutions into a unified set of data (station coordinates, Earth orientation parameters, etc.) follows these characteristics (Boucher, 1987):

a) It is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere.

b) Its scale is that of a local Earth frame, in the meaning of a relativistic theory of gravitation.

c) Its orientation is given by the BIH orientation at 1984.0.

d) Its time evolution follows a no-global-net-rotation-or-translation condition.

The unit of length is the meter (SI). The IERS Reference Pole (IRP) and Reference Meridian (IRM) are consistent with the corresponding directions in the BIH Terrestrial System (BTS) within $\pm 0''.005$. The BIH reference pole was adjusted to the Conventional International Origin (CIO) in 1967; it was then kept stable independently until 1987. The uncertainty of the tie of the IRP with the CIO is $\pm 0''.03$.

When one wants to realize such a conventional terrestrial reference system through a reference frame; i.e. a network of station's coordinates, it will:

a) Include the permanent solid Earth tidal deformation so that the adopted coordinates will differ from the instantaneous coordinates by only periodic terms (see Chapter 6).

b) Be specified by Cartesian equatorial coordinates X, Y, and Z, by preference. If geographical coordinates are needed, the GRS ellipsoid is recommended (a_e = 6378136 m, f^{-1} = 298.257; see IERS recommended values).

The way followed by various analysis centers depends on their own views on modelling, but also on the techniques themselves. For the origin, only data which can be modelled by dynamical techniques (presently SLR and LLR for IERS) can determine the center of mass. The VLBI system can be referred to a geocentric system by adopting for a station its geocentric position at a reference epoch as provided from external information. It is recommended to use a value coming from the initial IERS Terrestrial Reference frame

(ITRF-0) - see below.

The scale is obtained by appropriate relativistic modelling. This is particularly true for VLBI and LLR which are usually modelled in a barycentric frame.

The orientation is defined by adopting IERS (or BIH) Earth orientation parameters at a reference epoch. In the case of SLR, an additional constraint in longitude is necessary. The use of ITRF-0 values is recommended for this purpose.

The time evolution of the orientation will be ensured by using a no-net-rotation condition either directly, or by adopting a plate motion model which fulfills this condition (for information on AMO-2, see Chapter 9).

Coordinates of sites may be found in the most recent Annual Report of the IERS.

REFERENCES

Boucher, C., 1989, "Definition and realization of terrestrial reference systems for monitoring Earth rotation," Variations in Earth Rotation.

CHAPTER 4 NUTATION

The standard nutation series is the IAU 1980 Theory of Nutation (Seidelmann, 1982; Wahr, 1981). The constants defining this theory are given in Table 4.1. The Wahr model is based on geophysical model 1066A of Gilbert and Dziewonski (1975) and therefore includes the effects of a solid inner core and a liquid outer core and a "distribution of elastic parameters inferred from a large set of seismological data." Wahr's work is based on a modification of a rigid Earth theory published by Kinoshita (1977).

VLBI and LLR observations have shown that there are deficiencies in the IAU 1976 Precession and in the IAU 1980 Theory of Nutation. However, these models are kept as part of the IERS Standards and the observed differences with respect to the conventional celestial pole position defined by the models are monitored and reported by the IERS. A preliminary evaluation of the correction to the IAU models has been published by W. E. Carter and T. Herring in the Annual Report of the BIH for 1987, p. D-105. A FORTRAN function for computing the corresponding effect on Ψ and ϵ is available from the Central Bureau on request.

FUNDAMENTAL ARGUMENTS

The fundamental arguments of the nutation series, referred to the epoch JD 2 451 545.0 = 2000 January 1.5 are given by the expressions:

$$
\begin{aligned}
l &= \text{Mean Anomaly of the Moon}\\
&= 134^\circ\ 57'\ 46''.733 + (1325^r + 198^\circ\ 52'\ 02''.633)\ T\\
&\quad + 31''.310\ T^2 + 0''.064\ T^3
\end{aligned}
$$

$$
\begin{aligned}
l' &= \text{Mean Anomaly of the Sun}\\
&= 357^\circ\ 31'\ 39''.804 + (99^r + 359^\circ\ 03'\ 01''.224)\ T\\
&\quad - 0''.577\ T^2 - 0''.012\ T^3
\end{aligned}
$$

$$
\begin{aligned}
F &= L - \Omega \text{ where } L = \text{Mean Longitude of the Moon}\\
&= 93^\circ\ 16'\ 18''.877 + (1342^r + 82^\circ\ 01'\ 03''.137)\ T\\
&\quad - 13''.257\ T^2 + 0''.011\ T^3
\end{aligned}
$$

$$
\begin{aligned}
D &= \text{Mean Elongation of the Moon from the Sun}\\
&= 297^\circ\ 51'\ 01''.307 + (1236^r + 307^\circ\ 06'\ 41''.328)\ T\\
&\quad - 6''.891\ T^2 + 0''.019\ T^3
\end{aligned}
$$

$$
\begin{aligned}
\Omega &= \text{Mean Longitude of the Ascending Node of the Moon}\\
&= 125^\circ\ 02'\ 40''.280 - (5^r + 134^\circ\ 08'\ 10''.539)\ T\\
&\quad + 7''.455\ T^2 + 0''.008\ T^3
\end{aligned}
$$

where T is measured in Julian Centuries of 36525 days of 86400 seconds of Dynamical Time since J2000.0.

Table 4.1. Series for nutation in longitude $\Delta\Psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0.

ARGUMENT l	l'	F	D	Ω	PERIOD (days)	LONGITUDE (0.0001")		OBLIQUITY (0.0001")	
0	0	0	0	1	6798.4	-171996	-174.2T	92025	8.9T
0	0	2	-2	2	182.6	-13187	-1.6T	5736	-3.1T
0	0	2	0	2	13.7	-2274	-0.2T	977	-0.5T
0	0	0	0	2	3399.2	2062	0.2T	-895	0.5T
0	1	0	0	0	365.2	1426	-3.4T	54	-0.1T
1	0	0	0	0	27.6	712	0.1T	-7	0.0T
0	1	2	-2	2	121.7	-517	1.2T	224	-0.6T
0	0	2	0	1	13.6	-386	-0.4T	200	0.0T
1	0	2	0	2	9.1	-301	0.0T	129	-0.1T
0	-1	2	-2	2	365.3	217	-0.5T	-95	0.3T
1	0	0	-2	0	31.8	-158	0.0T	-1	0.0T
0	0	2	-2	1	177.8	129	0.1T	-70	0.0T
-1	0	2	0	2	27.1	123	0.0T	-53	0.0T
1	0	0	0	1	27.7	63	0.1T	-33	0.0T
0	0	0	2	0	14.8	63	0.0T	-2	0.0T
-1	0	2	2	2	9.6	-59	0.0T	26	0.0T
-1	0	0	0	1	27.4	-58	-0.1T	32	0.0T
1	0	2	0	1	9.1	-51	0.0T	27	0.0T
2	0	0	-2	0	205.9	48	0.0T	1	0.0T
-2	0	2	0	1	1305.5	46	0.0T	-24	0.0T
0	0	2	2	2	7.1	-38	0.0T	16	0.0T
2	0	2	0	2	6.9	-31	0.0T	13	0.0T
2	0	0	0	0	13.8	29	0.0T	-1	0.0T
1	0	2	-2	2	23.9	29	0.0T	-12	0.0T
0	0	2	0	0	13.6	26	0.0T	-1	0.0T
0	0	2	-2	0	173.3	-22	0.0T	0	0.0T
-1	0	2	0	1	27.0	21	0.0T	-10	0.0T
0	2	0	0	0	182.6	17	-0.1T	0	0.0T
0	2	2	-2	2	91.3	-16	0.1T	7	0.0T
-1	0	0	2	1	32.0	16	0.0T	-8	0.0T
0	1	0	0	1	386.0	-15	0.0T	9	0.0T
1	0	0	-2	1	31.7	-13	0.0T	7	0.0T
0	-1	0	0	1	346.6	-12	0.0T	6	0.0T
2	0	-2	0	0	1095.2	11	0.0T	0	0.0T
-1	0	2	2	1	9.5	-10	0.0T	5	0.0T
1	0	2	2	2	5.6	-8	0.0T	3	0.0T
0	-1	2	0	2	14.2	-7	0.0T	3	0.0T
0	0	2	2	1	7.1	-7	0.0T	3	0.0T
1	1	0	-2	0	34.8	-7	0.0T	0	0.0T
0	1	2	0	2	13.2	7	0.0T	-3	0.0T
-2	0	0	2	1	199.8	-6	0.0T	3	0.0T
0	0	0	2	1	14.8	-6	0.0T	3	0.0T
2	0	2	-2	2	12.8	6	0.0T	-3	0.0T
1	0	0	2	0	9.6	6	0.0T	0	0.0T

Table 4.1 (continued)

ARGUMENT					PERIOD	LONGITUDE		OBLIQUITY	
l	l'	F	D	Ω	(days)	(0.0001")		(0.0001")	
1	0	2	-2	1	23.9	6	0.0T	-3	0.0T
0	0	0	-2	1	14.7	-5	0.0T	3	0.0T
0	-1	2	-2	1	346.6	-5	0.0T	3	0.0T
2	0	2	0	1	6.9	-5	0.0T	3	0.0T
1	-1	0	0	0	29.8	5	0.0T	0	0.0T
1	0	0	-1	0	411.8	-4	0.0T	0	0.0T
0	0	0	1	0	29.5	-4	0.0T	0	0.0T
0	1	0	-2	0	15.4	-4	0.0T	0	0.0T
1	0	-2	0	0	26.9	4	0.0T	0	0.0T
2	0	0	-2	1	212.3	4	0.0T	-2	0.0T
0	1	2	-2	1	119.6	4	0.0T	-2	0.0T
1	1	0	0	0	25.6	-3	0.0T	0	0.0T
1	-1	0	-1	0	3232.9	-3	0.0T	0	0.0T
-1	-1	2	2	2	9.8	-3	0.0T	1	0.0T
0	-1	2	2	2	7.2	-3	0.0T	1	0.0T
1	-1	2	0	2	9.4	-3	0.0T	1	0.0T
3	0	2	0	2	5.5	-3	0.0T	1	0.0T
-2	0	2	0	2	1615.7	-3	0.0T	1	0.0T
1	0	2	0	0	9.1	3	0.0T	0	0.0T
-1	0	2	4	2	5.8	-2	0.0T	1	0.0T
1	0	0	0	2	27.8	-2	0.0T	1	0.0T
-1	0	2	-2	1	32.6	-2	0.0T	1	0.0T
0	-2	2	-2	1	6786.3	-2	0.0T	1	0.0T
-2	0	0	0	1	13.7	-2	0.0T	1	0.0T
2	0	0	0	1	13.8	2	0.0T	-1	0.0T
3	0	0	0	0	9.2	2	0.0T	0	0.0T
1	1	2	0	2	8.9	2	0.0T	-1	0.0T
0	0	2	1	2	9.3	2	0.0T	-1	0.0T
1	0	0	2	1	9.6	-1	0.0T	0	0.0T
1	0	2	2	1	5.6	-1	0.0T	1	0.0T
1	1	0	-2	1	34.7	-1	0.0T	0	0.0T
0	1	0	2	0	14.2	-1	0.0T	0	0.0T
0	1	2	-2	0	117.5	-1	0.0T	0	0.0T
0	1	-2	2	0	329.8	-1	0.0T	0	0.0T
1	0	-2	2	0	32.8	-1	0.0T	0	0.0T
1	0	-2	-2	0	9.5	-1	0.0T	0	0.0T
1	0	2	-2	0	32.8	-1	0.0T	0	0.0T
1	0	0	-4	0	10.1	-1	0.0T	0	0.0T
2	0	0	-4	0	15.9	-1	0.0T	0	0.0T
0	0	2	4	2	4.8	-1	0.0T	0	0.0T
0	0	2	-1	2	25.4	-1	0.0T	0	0.0T
-2	0	2	4	2	7.3	-1	0.0T	1	0.0T
2	0	2	2	2	4.7	-1	0.0T	0	0.0T
0	-1	2	0	1	14.2	-1	0.0T	0	0.0T
0	0	-2	0	1	13.6	-1	0.0T	0	0.0T
0	0	4	-2	2	12.7	1	0.0T	0	0.0T
0	1	0	0	2	409.2	1	0.0T	0	0.0T

Table 4.1 (continued)

ARGUMENT					PERIOD	LONGITUDE		OBLIQUITY	
l	l'	F	D	Ω	(days)	(0.0001")		(0.0001")	
1	1	2	-2	2	22.5	1	0.0T	-1	0.0T
3	0	2	-2	2	8.7	1	0.0T	0	0.0T
-2	0	2	2	2	14.6	1	0.0T	-1	0.0T
-1	0	0	0	2	27.3	1	0.0T	-1	0.0T
0	0	-2	2	1	169.0	1	0.0T	0	0.0T
0	1	2	0	1	13.1	1	0.0T	0	0.0T
-1	0	4	0	2	9.1	1	0.0T	0	0.0T
2	1	0	-2	0	131.7	1	0.0T	0	0.0T
2	0	0	2	0	7.1	1	0.0T	0	0.0T
2	0	2	-2	1	12.8	1	0.0T	-1	0.0T
2	0	-2	0	1	943.2	1	0.0T	0	0.0T
1	-1	0	-2	0	29.3	1	0.0T	0	0.0T
-1	0	0	1	1	388.3	1	0.0T	0	0.0T
-1	-1	0	2	1	35.0	1	0.0T	0	0.0T
0	1	0	1	0	27.3	1	0.0T	0	0.0T

$$\epsilon_{J2000} = 23^\circ\ 26'\ 21''\!.448$$
$$\sin \epsilon_{J2000} = 0.39777716$$

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CHAPTER 5 PROCEDURE FOR COMPUTING APPARENT PLACES

The barycentric direction of a star at any instant in Dynamical Barycentric Time (TDB) is calculated from its right ascension, declination, and space motion for the standard equinox and equator of J2000.0 in the FK5 system.

Note that for the computation of apparent places:

1) The distinction between TDB and Terrestrial Dynamical Time (TDT) is not significant.

2) The space-motion of the star is included but light-time is ignored.

3) The relativity term for light deflection is modified to the asymptotic case where the star is assumed to be at infinity.

STEP 1

Set TDB = TDT

STEP 2

Obtain the Earth's barycentric position $\vec{E}_B$ in astronomical units (au) and $\dot{\vec{E}}_B$ in au/day, at coordinate time t=TDB referred to the equinox and equator of J2000.0. Then, calculate the geocentric vector of the star at the required epoch in the J2000.0 frame.

When given the right ascension α, declination δ, proper motion in right ascension μ_α (in seconds of time per Julian century), proper motion in declination μ_δ (in arcseconds per Julian century), stellar parallax π (in arcseconds) and radial velocity v (in kilometers per second) at J2000.0, referred to the standard equinox and equator of J2000.0, calculate $\vec{P}$, the geocentric vector of the star at the required epoch t, from:

$$\vec{P} = \hat{r} + T\,\vec{v} - \text{arc}1''\,\pi\vec{E}_B$$

where

$$\hat{r} = (\cos\delta\,\cos\alpha,\ \cos\delta\,\sin\alpha,\ \sin\delta),$$

$$\vec{v} = (v_x,\ v_y,\ v_z),$$

$$v_x = \text{arc}1''(-\mu_\delta\sin\delta\,\cos\alpha - 15\mu_\alpha\cos\delta\,\sin\alpha + 21.094953\pi v\cos\delta\,\cos\alpha),$$

v_y = arc1"($-\mu_\delta \sin\delta \sin\alpha + 15\mu_\alpha \cos\delta \cos\alpha + 21.094953\pi v \cos\delta \sin\alpha$),

v_z = arc1"($\mu_\delta \cos\delta \qquad\qquad + 21.094953\pi v \sin\delta$),

T = (JD - 2451545.0)/36525, and JD is the Julian date at time t. The factor 21.094953 is [km/au]•[Julian century/sec].

STEP 3

Form the heliocentric position of the Earth and calculate the geocentric direction of the star, corrected for light deflection.

Form the heliocentric position of the Earth $\vec{E}$ from:

$$\vec{E} = \vec{E}_B - \vec{S},$$

where $\vec{S}$ is the barycentric position of the Sun at time t.

Form the geocentric direction $\hat{p}$ of the star and the unit vector $\hat{e}$ from:

$$\hat{p} = \vec{P}/|\vec{P}| \quad \text{and} \quad \hat{e} = \vec{E}/|\vec{E}|.$$

Calculate the geocentric direction $\hat{p}_1$ of the star, corrected for light deflection in the natural frame, from:

$$\vec{p}_1 = \hat{p} + (2\mu/c^2E)[(\hat{e}-(\hat{p}\cdot\hat{e})\hat{p})/(1+\hat{p}\cdot\hat{e})],$$

where the dot indicates a scalar product, μ/c^2 = the gravitational radius of the Sun = 9.87063×10^{-9} au and $E = |\vec{E}|$.

The vector $\vec{p}_1$ is a unit vector to order μ/c^2.

STEP 4

Calculate the proper direction of the star $\vec{p}_2$ in the geocentric inertial frame that is moving with the instantaneous velocity $\vec{V}$ of the Earth relative to the natural frame, from:

$$\vec{p}_2 = (\beta^{-1}\vec{p}_1 + \vec{V} + (\vec{p}_1\cdot\vec{V})\vec{V}/(1 + \beta^{-1}))/(1 + \vec{p}_1\cdot\vec{V}),$$

where $\vec{V} = \dot{\vec{E}}_B/c = 0.0057755\ \dot{\vec{E}}_B$ and $\beta = (1 - V^2)^{-\frac{1}{2}}$; the velocity $\vec{V}$ is expressed in units of the velocity of light and is equal to the Earth's velocity in the barycentric frame to order V^2.

STEP 5

Apply precession and nutation to the proper direction $\vec{p}_2$ by multiplying by the rotation matrices due to the precession [P] and the nutation [N] (Mueller, 1969), to obtain the apparent direction $\vec{p}_3$, from:

$$\vec{p}_3 = [N]\ [P]\ \vec{p}_2.$$

Here,

$$[P] = R_3(-Z_A)\ R_2(\theta_A)\ R_3(-\zeta_A),$$

$$[N] = R_1[-(\epsilon_A+\Delta\epsilon)]\ R_3(-\Delta\Psi)\ R_1(\epsilon_A),$$

where R_1, R_2, and R_3 represent rotations about the X, Y, and Z axes respectivley.

Rotation angles are defined by Lieske, <u>et</u> <u>al</u>. (1977) as follows:

$$\zeta_A = (2306''.2181 + 1''.39656\ T - 0''.000139\ T^2)t$$
$$+ (0''.30188 - 0''.000345\ T)t^2 + 0''.017998\ t^3,$$

$$\theta_A = (2004''.3109 - 0''.85330\ T - 0''.000217\ T^2)t$$
$$+ (-0''.42665 - 0''.000217\ T)t^2 - 0''.041833\ t^3,$$

$$Z_A = (2306''.2181 + 1''.39656\ T - 0''.000139\ T^2)t$$
$$+ (1''.09468 + 0''.000066\ T)t^2 + 0''.018203\ t^3,$$

$$\epsilon_A = (84381''.448 - 46''.8150\ T - 0''.00059\ T^2 + 0''.001813\ T^3)$$
$$+ (-46''.8150 - 0''.00117\ T + 0''.005439\ T^2)t$$
$$+ (-0''.00059 + 0''.005439\ T)t^2 + 0''.001813\ t^3.$$

where T represents Julian centuries from J2000.0 to an arbitrary epoch and t represents Julian centuries from the epoch to the date.

Note that the coefficient of Tt^2 in ζ_A has been changed to $-0''.000344$ by Lieske (1979), but this change has no effect in our case because we need to calculate the precession from J2000.0 to the date and hence T=0.

$\Delta\Psi$ and $\Delta\epsilon$ are the nutation in longitude and in obliquity, respectively (see Chapter 4).

<u>STEP 6</u>

Convert to spherical coordinates (α_t, δ_t) using:

$$\xi/\rho = \cos\delta_t \cos\alpha_t,$$

$$\eta/\rho = \cos\delta_t \sin\alpha_t,$$

$$\zeta/\rho = \sin\delta_t,$$

where $\vec{p}_3 = (\xi,\eta,\zeta)$ and $\rho = \sqrt{\xi^2 + \eta^2 + \zeta^2}$.

Useful references describing procedures in more detail are Smith, et al. (1989) and Yallop, et al. (1989).

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CHAPTER 6 SOLID EARTH TIDES

The solid Earth tide model is based on an abbreviated form of the Wahr model (Wahr, 1981) using the Earth model 1066A of Gilbert and Dziewonski (1975).

The Love numbers for the induced free space potential, k, and for the vertical and horizontal displacements, h and ℓ, have been taken from Wahr's thesis, Tables 13 and 16. The long period, diurnal, and semi-diurnal terms are included. Third degree terms are neglected.

CALCULATIONS OF THE POTENTIAL COEFFICIENTS

The solid tide induced free space potential is most easily modelled as variations in the standard geopotential coefficients C_{nm} and S_{nm} (Eanes, et al., 1983). The Wahr model (or any other having frequency dependent Love numbers) is most efficiently computed in two steps. The first step uses a frequency independent Love number k_2 and an evaluation of the tidal potential in the time domain from a lunar and solar ephemeris. The second step corrects those arguments of a harmonic expansion of the tide generating potential for which the error from using the k_2 of Step 1 is above some cutoff.

The changes in normalized second degree geopotential coefficients for Step 1 are:

$$\Delta\overline{C}_{20} = \frac{1}{\sqrt{5}} k_2 \frac{R_e^3}{GM_\oplus} \sum_{j=2}^{3} \frac{GM_j}{r_j^3} P_{20} (\sin \phi_j), \tag{1a}$$

$$\Delta\overline{C}_{21} - i\Delta\overline{S}_{21} = \frac{1}{3} \sqrt{\frac{3}{5}} k_2 \frac{R_e^3}{GM_\oplus} \sum_{j=2}^{3} \frac{GM_j}{r_j^3} P_{21} (\sin \phi_j) e^{-i\lambda_j}, \tag{1b}$$

$$\Delta\overline{C}_{22} - i\Delta\overline{S}_{22} = \frac{1}{12} \sqrt{\frac{12}{5}} k_2 \frac{R_e^3}{GM_\oplus} \sum_{j=2}^{3} \frac{GM_j}{r_j^3} P_{22} (\sin \phi_j) e^{-i2\lambda_j}, \tag{1c}$$

where

k_2 = nominal second degree Love number,

R_e = equatorial radius of the Earth,

$GM_{\oplus}$ = gravitational parameter for the Earth,

GM_j = gravitational parameter for the Moon (j=2) and Sun (j=3),

r_j = distance from geocenter to Moon or Sun,

ϕ_j = body fixed geocentric latitude of Moon or Sun,

λ_j = body fixed east longitude (from Greenwich) of Sun or Moon.

The changes in normalized coefficients from Step 2 are:

$$\Delta\overline{C}_{nm} - i\Delta S_{nm} = A_M \sum_{s(\bar{n},m)} \delta k_s \; H_s \; \left(\begin{smallmatrix}1\\-i\end{smallmatrix}\right)_{\substack{n+m \; \text{odd}}}^{\substack{n+m \; \text{even}}} \; e^{i\theta_s}, \tag{2}$$

where

$$A_m = \frac{(-1)^m}{R_e\sqrt{4\pi}\;(2-\delta_{om})}, \qquad \delta_{om} = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases},$$

δk_s = difference between Wahr model for k at frequency s and the nominal value k_2 in the sense $k_s - k_2$,

H_s = amplitude (m) of term at frequency s from the Cartwright and Taylor (1971) and Cartwright and Edden (1973) harmonic expansion of the tide generating potential,

$$\theta_s = \bar{n} \cdot \bar{\beta} = \sum_{i=1}^{6} n_i\beta_i$$

where

$\bar{n}$ = six vector of multipliers of the Doodson variables,

$\bar{\beta}$ = the Doodson variables, and

δS_{20} = 0.

The Doodson variables are related to the fundamental arguments of the nutation series (see Chapter 4) by:

$s = F + \Omega = \beta_2$ (Moon's mean longitude), (3)

$h = s - D = \beta_3$ (Sun's mean longitude),

$p = s - \ell = \beta_4$ (Longitude of Moon's mean perigee),

$N' = -\Omega = \beta_5$ (Negative longitude of Moon's mean node),

$p_1 = s - D - \ell' = \beta_6$ (Longitude of Sun's mean perigee), and

$\tau = \theta_g + \pi - s = \beta_1$ (Time angle in lunar days reckoned from lower transit) where

θ_g = mean sidereal time of the conventional zero meridian.

The normalized geopotential coefficients ($\bar{C}_{nm}$, $\bar{S}_{nm}$) are related to the unnormalized coefficients (C_{nm}, S_{nm}) by the following:

$$C_{nm} = N_{nm}\,\bar{C}_{nm},\quad S_{nm} = N_{nm}\,\bar{S}_{nm},$$

$$N_{nm} = \left[\frac{(n-m)!\,(2n+1)\,(2-\delta_{om})}{(n+m)!}\right]^{\frac{1}{2}}.$$

Using a nominal k_2 of 0.3 and an amplitude cutoff of 9×10^{-12} change in normalized geopotential coefficients, the summation $S(n,\underline{m})$ requires six terms for the diurnal species (n=2, m=1) modifying $\bar{C}_{21}$ and $\bar{S}_{21}$ and two semi-diurnal terms (n=2, m=2) modifying $\bar{C}_{22}$ and $\bar{S}_{22}$. With the exception of the zero frequency tide, no long period terms are necessary. Table 6.1 gives required quantities for correcting the (2,1) and (2,2) coefficients. The correction to $\bar{C}_{20}$ is discussed in more detail below.

The Step 2 correction due to the K_1 constituent is given below as an example.

$$\begin{aligned}(\Delta\bar{C}_{21} \times 10^{12})_{K_1} &= 507.4 \sin(\tau+s)\\ &= 507.4 \sin(\theta_g+\pi)\\ &= -507.4 \sin(\theta_g)\end{aligned}$$

$$(\Delta\bar{S}_{21} \times 10^{12})_{K_1} = -507.4 \cos(\theta_g)$$

The total variation in geopotential coefficients due to the solid tide is obtained by adding the results of Step 2 (Eq. 2) to those of Step 1 (Eq. 1).

Table 6.1

Step 2 Solid Tide Corrections When k_2 = .3 in Step 1

Using a Cutoff Amplitude of 9×10^{-12} for $A_m \delta k_s H_s$

Long Period Tides (n=2, m=0)

None except zero frequency tide.

Diurnal Tides (n=2, m=1)

Doodson Number	τ	s	h	p	N'	p_1	$A_m \delta k_s H_s \times 10^{12}$
			$\bar{n}$, argument multipliers				
145.555 (O_1)	1	-1	0	0	0	0	-16.4
163.555 (P_1)	1	1	-2	0	0	0	-49.6
165.545	1	1	0	0	-1	0	-9.4
165.555 (K_1)	1	1	0	0	0	0	507.4
165.565	1	1	0	0	1	0	73.5
166.554 (ψ_1)	1	1	1	0	0	-1	-15.2

Semi-Diurnal Tides (n=2, m=2)

Doodson Number	τ	s	h	p	N'	p_1	$A_m \delta k_s H_s \times 10^{12}$
			$\bar{n}$, argument multipliers				
255.555 (M_2)	2	0	0	0	0	0	39.5
273.555 (S_2)	2	2	-2	0	0	0	18.4

TREATMENT OF THE PERMANENT TIDE

The mean value of $\Delta\bar{C}_{20}$ from Eq. 1a is not zero, and this permanent tide deserves special attention. Ideally, the mean value of the correction should be included in the adopted value of $\bar{C}_{20}$ and hence not included in the $\Delta\bar{C}_{20}$. The practical situation is not so clear because satellite derived values of $\bar{C}_{20}$ as in the GEM geopotentials have been obtained using a mixture of methods, some applying the corrections and others not applying it. There is no way to ensure consistency in this regard short of re-estimating $\bar{C}_{20}$ with a consistent technique. If this is done the inclusion of the zero frequency term in Eq. 1a should be avoided because k_2 is not the appropriate Love number to use for such a term.

The zero frequency change in $\bar{C}_{20}$ can be removed by computing $\Delta\bar{C}_{20}$ as

The zero frequency change in $\bar{C}_{20}$ can be removed by computing $\Delta\bar{C}_{20}$ as

$$\Delta\bar{C}_{20}{}^{*} = \Delta\bar{C}_{20} \text{ (Eq. 1a) } - \langle\Delta\bar{C}_{20}\rangle , \tag{4}$$

where

$$\begin{aligned}\langle\Delta\bar{C}_{20}\rangle &= A_0 H_0 k_2 \\ &= (4.4228 \times 10^{-8})(-0.31455)k_2 \\ &= -1.39119 \times 10^{-8} k_2 .\end{aligned}$$

Using $k_2 = 0.3$ then

$$\langle\Delta\bar{C}_{20}\rangle = -4.1736 \times 10^{-9}$$

or

$$\langle\Delta J_2\rangle = -\langle\Delta\bar{C}_{20}\rangle \sqrt{5} = 9.3324 \times 10^{-9} .$$

The decision to remove or not to remove the mean from the corrections depends on whether the adopted $\bar{C}_{20}$ does or does not already contain it and on whether k_2 is a potential 'solve for' parameter. If k_2 is to be estimated then it must not multiply the zero frequency term in the correction. In the most recent data reductions leading to GEM-T1, the total tide correction was applied. If we assume the more recent data has most of the weight in the determination of $\bar{C}_{20}$ then we conclude that the permanent deformation is not included in the GEM-T1 value of $\bar{C}_{20}$. Hence, if k_2 is to be estimated, first $\langle\Delta\bar{C}_{20}\rangle$ must be added to $\bar{C}_{20}$ and then $\Delta\bar{C}_{20}{}^{*}$ should be used in place of $\Delta\bar{C}_{20}$ of Eq. 1a. The k_2 used for restoring the permanent tide should match what was used in deriving the adopted value of $\bar{C}_{20}$.

The GEM-T1 value of $\bar{C}_{20}$ is $-484.16499 \times 10^{-6}$ and does not include the permanent deformation. The tidal corrections employed in the computations leading to GEM-T1 were equivalent to Eq. 1a with $k_2 = 0.29$. Let $\bar{C}_{20}{}^{*}$ denote the coefficient which includes the zero frequency term; then the GEM-T1 values of $\bar{C}_{20}$ with the permanent tide restored are:

$$\bar{C}_{20}{}^{*}(\text{GEM-T1}) = -484.16499 \times 10^{-6} - (1.39119 \times 10^{-8}) \times 0.29,$$

$$\bar{C}_{20}{}^{*}(\text{GEM-T1}) = -484.169025. \tag{5}$$

These values for $\bar{C}_{20}{}^{*}$ are recommended for use with the respective gravity field and should be added to the periodic tidal correction given as $\Delta\bar{C}_{20}^{*}$ in Eq. 4 to get the total time dependent value of $\bar{C}_{20}$. Note that this recommendation is not precisely equivalent to using the unmodified C_{20} value with the unmodified ΔC_{20} from Eq. 1a because the recommendation for k_2 is 0.3 instead of 0.29 which was used in deriving GEM-T1.

SOLID TIDE EFFECT ON STATION COORDINATES

The variations of station coordinates caused by solid Earth tides predicted using Wahr's theory are also most efficiently implemented using a two-step procedure. Only the second degree tides are necessary to retain 0.01 m precision. Also terms proportional to y, h+, h-, z, l+, w+, and w- are ignored. The first step uses frequency independent Love and Shida numbers and a computation of the tidal potential in the time domain. A convenient formulation of the displacement is given in the documentation for the GEODYN program. The vector displacement of the station due to tidal deformation for Step 1 can be computed from

$$\Delta\vec{r} = \sum_{j=2}^{3} \left[\frac{GM_j\ r^4}{GM_\oplus\ R_j^3}\right] \{\ [3\ell_2(\hat{R}_j \cdot \hat{r})]\ \hat{R}_j$$

$$+ \quad [3(\frac{h_2}{2} - \ell_2)(\hat{R}_j \cdot \hat{r})^2 - \frac{h_2}{2}]\ \hat{r}\ \}, \qquad (6)$$

GM_j = gravitational parameter for the Moon (j=2) or the Sun (j=3),

$GM_\oplus$ = gravitational parameter for the Earth,

$\hat{R}_j, R_j$ = unit vector from the geocenter to Moon or Sun and the magnitude of that vector,

$\hat{r}, r$ = unit vector from the geocenter to the station and the magnitude of that vector,

h_2 = nominal second degree Love number,

ℓ_2 = nominal Shida number.

If nominal values for h_2 and ℓ_2 of 0.6090 and 0.0852 respectively are used with a cutoff of 0.005m of radial displacement, only one term needs to be corrected in Step 2. This is the K_1 frequency where h from Wahr's theory is 0.5203. Only the radial displacement needs to be corrected and to sufficient accuracy this can be implemented as a periodic change in station height given by

$$\delta h_{STA} = \delta h_{K_1} \; H_{K_1} \left(-\sqrt{\frac{5}{24\pi}}\right) 3 \sin\phi \cos\phi \sin(\theta_{K_1} + \lambda),$$

where

δh_{K_1} = h_{K_1} (Wahr) - h_2 (Nominal) = -0.0887,

H_{K_1} = amplitude of K_1 term (165.555) in the harmonic expansion of the tide generating potential = 0.36878 m,

ϕ = geocentric latitude of station ,

λ = east longitude of station ,

θ_{K_1} = K_1 tide argument = $\tau + s = \theta_g + \pi$,

or simplifying

$$\delta h_{STA} = -0.0253 \sin \phi \cos \phi \sin (\theta_g + \lambda).$$

The effect is maximum at $\phi = 45°$ where the amplitude is 0.013 m.

There is also a zero frequency station displacement which may or may not be included in the nominal station coordinates. The mean correction could be removed analogously to the discussion above. When baselines or coordinates are compared at the few cm level, care must be taken that the correction was handled consistently. It is essential that published station coordinates identify how the zero frequency contribution was included.

If nominal Love and Shida numbers of 0.6090 and 0.0852 respectively are used with Eq. 6, the permanent deformation introduced is in the radial direction.

$$\Delta\vec{r} \cdot \hat{r} = \sqrt{\frac{5}{4\pi}} (0.6090)(-0.31455)\left(\frac{3}{2} \sin^2 \phi - \frac{1}{2}\right) \quad (7)$$

$$= -0.12083 \left(\frac{3}{2} \sin^2 \phi - \frac{1}{2}\right) \text{ meters,}$$

and in the north direction

$$\Delta\vec{r} \cdot \hat{e}_p = \sqrt{\frac{5}{4\pi}} (0.0852)(-0.31455) \; 3 \cos \phi \sin \phi \quad (8)$$

$$= -0.05071 \cos \phi \sin \phi \text{ meters.}$$

ROTATIONAL DEFORMATION DUE TO POLAR MOTION

The variation of station coordinates caused by the polar tide is recommended to be taken into account. Let us choose $\hat{x}$, $\hat{y}$, and $\hat{z}$ as a terrestrial system of reference. The $\hat{z}$ axis is oriented along the Earth's mean rotation axis, the $\hat{x}$ axis is in the direction of the adopted origin of longitude and $\hat{y}$ axis is oriented along the 90° E meridian.

The centrifugal potential caused by the Earth's rotation is

$$V = \frac{1}{2}\,[r^2|\vec{\Omega}|^2 - (\vec{r}\cdot\vec{\Omega})^2], \qquad (9)$$

where $\vec{\Omega} = \Omega(m_1\hat{x} + m_2\hat{y} + (1 + m_3)\hat{z})$. Ω is the mean angular velocity of rotation of the Earth, m_i are small dimensionless parameters, m_1, m_2 describing polar motion and m_3 describing variation in the rotation rate, r is the radial distance to the station.

Neglecting the variations in m_3 which induce displacements that are below the mm level, the m_1 and m_2 terms give a first order perturbation in the potential V (Wahr, 1985)

$$\Delta V(r,\theta,\lambda) = -\frac{\Omega^2 r^2}{2}\sin 2\theta\ (m_1\cos\lambda + m_2\sin\lambda), \qquad (10)$$

where θ is the co-latitude, and λ is the eastward longitude.

Let us define the radial displacement S_r, the horizontal displacements S_θ and S_λ, positive upwards, south and east respectively, in a horizon system at the station due to ΔV using the formulation of tidal Love numbers (W. Munk and G. MacDonald, 1960).

$$S_r = h\,\frac{\Delta V}{g},$$

$$S_\theta = -\frac{\ell}{g}\,\partial_\theta \Delta V, \qquad (11)$$

$$S_\lambda = \frac{\ell}{g}\,\frac{1}{\sin\theta}\,\partial_\lambda \Delta V,$$

where g is the gravitational acceleration at the Earth's surface, h, ℓ are the second-order bodytide displacement Love Numbers.

In general, these computed displacements have a non-zero average over any given time span because m_1 and m_2, used to find ΔV, have a non-zero average. Consequently, the use of these results will lead to a change in the estimated mean station coordinates. When mean coordinates produced by different users are compared at the centimeter level, it is important to ensure that this effect has been handled consistently. It is recommended that m_1 and m_2 used in eq. 10 be replaced by parameters defined to be zero for the Terrestrial Reference Frame discussed in Chapter 3.

Thus, define

$$\begin{aligned} x_p &= m_1 - \bar{x} \\ y_p &= -m_2 - \bar{y} \end{aligned} \tag{12}$$

where $\bar{x}$ and $\bar{y}$ are the values of m_1 and $-m_2$ for the Chapter 3 Terrestrial Reference Frame. Then, using $h = 0.6$, $\ell = 0.085$, and $r = a = 6.4 \times 10^6$m,

$$\begin{aligned} S_r &= -\ 32 \sin 2\theta\ (x_p \cos\lambda - y_p \sin\lambda)\ \text{mm}, \\ S_\theta &= -9 \cos 2\theta\ (x_p \cos\lambda - y_p \sin\lambda)\ \text{mm}, \\ S_\lambda &= 9 \cos\theta\ (x_p \sin\lambda + y_p \cos\lambda)\ \text{mm}. \end{aligned} \tag{13}$$

for x_p and y_p in seconds of arc.

Taking into account that x_p and y_p vary, at most, 0.8 arcsec, the maximum radial displacement is approximately 25 mm, and the maximum horizontal displacement is about 7 mm.

If X, Y, and Z are Cartesian coordinates of a station in a right-hand equatorial coordinate system, we have the displacements of coordinates

$$[dX,\ dY,\ dZ]^T = R^T\ [S_\theta,\ S_\lambda,\ S_r]^T, \tag{14}$$

where

$$R = \begin{bmatrix} \cos\theta\cos\lambda & \cos\theta\sin\lambda & -\sin\theta \\ -\sin\lambda & \cos\lambda & 0 \\ \sin\theta\cos\lambda & \sin\theta\sin\lambda & \cos\theta \end{bmatrix}.$$

The formula (13) can be used for determination of the corrections to station coordinates due to polar tide.

The deformation caused by the polar tide also leads to time-dependent perturbations in the C_{21} and S_{21} geopotential coefficients. The change in the external potential caused by this deformation is $k\Delta V$, where ΔV is given by eq. 10, and k is the degree 2 potential Love number. Using $k = 0.3$ gives

$$\bar{C}_{21} = -1.3 \times 10^{-9}\ (x_p),$$

$$\bar{S}_{21} = -1.3 \times 10^{-9}\ (-y_p),$$

where x_p and y_p are in seconds of arc and are used instead of m_1 and $-m_2$ so that no mean is introduced into $\bar{C}_{21}$ and $\bar{S}_{21}$ when making this correction.

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CHAPTER 7 OCEAN TIDE MODEL

The dynamical effect of ocean tides is most easily implemented as periodic variations in the normalized geopotential coefficients. The variations can be written as (Eanes, et al., 1983):

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} \sum_{+}^{-} \left(C^{\pm}_{snm} \mp iS^{\pm}_{snm} \right) e^{\pm i\theta_s}, \qquad (1)$$

where

$$F_{nm} = \frac{4\pi G\rho_w}{g} \left[\frac{(n+m)!}{(n+m)!(2n+1)(2-\delta_{om})}\right]^{\frac{1}{2}} \frac{1+k'_n}{2n+1},$$

$g = GE/a_e^2 = 9.798261\ \mathrm{ms}^{-2}$,

G = The universal gravitational constant = $6.673 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$,

ρ_w = density of seawater = 1025 kg m^{-3},

k'_n = load deformation coefficients ($k'_2 = -0.3075$, $k'_3 = -0.195$, $k'_4 = -0.132$, $k'_5 = -0.1032$, $k'_6 = -0.0892$),

$C^{\pm}_{snm}$, $S^{\pm}_{snm}$ = ocean tide coefficients in m for the tide constituent s (see Table 7.1),

θ_s = argument of the tide constituent s as defined in the solid tide model (Chapter 6).

The summation, $\sum_{+}^{-}$, implies addition of the expression using the top signs (the prograde waves C^{+}_{snm} and S^{+}_{snm}) to that using the bottom signs (the retrograde waves C^{-}_{snm} and S^{-}_{snm}). The ocean tide coefficients $C^{\pm}_{snm}$ and $S^{\pm}_{snm}$ as used here are related to the Schwiderski (1983) ocean tide amplitude and phase by:

$$C^{\pm}_{snm} - iS^{\pm}_{snm} = -i\hat{C}^{\pm}_{snm}\, e^{i(\epsilon^{\pm}_{snm} + \chi_s)}, \qquad (2)$$

where

$\hat{C}^{\pm}_{snm}$ = ocean tide amplitude for constituent s using the Schwiderski notation,

$\epsilon^{\pm}_{snm}$ = ocean tide phase for constituent s,

$$\chi_s = \begin{cases} 0 & \text{Semidiurnal, and long period} \\ \pi/2 & K_1\text{; (or in general for constituents with } H_s>0), \\ -\pi/2 & O_1, P_1, Q_1\text{; (or in general for constituents with } H_s<0) \end{cases}$$

For clarity, equation 1 is rewritten in two forms below:

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [(C^+_{snm} + C^-_{snm}) \cos\theta_s + (S^+_{snm} + S^-_{snm}) \sin\theta_s] \qquad (3a)$$

or

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^+_{snm} \sin(\theta_s+\varepsilon^+_{snm}+\chi_s) + \hat{C}^-_{snm} \sin(\theta_s+\varepsilon^-_{snm}+\chi_s)], \qquad (3b)$$

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [(S^+_{snm} + S^-_{snm}) \cos\theta_s - (C^+_{snm} + C^-_{snm}) \sin\theta_s] \qquad (3c)$$

or

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^+_{snm} \cos(\theta_s+\varepsilon^+_{snm}+\chi_s) - \hat{C}^-_{snm} \cos(\theta_s+\varepsilon^-_{snm}+\chi_s)]. \qquad (3d)$$

The summation over s(n,m) should include all constituents for which Schwiderski has computed a model. Except for cases of near resonance, the retrograde terms do not produce long period (> 1 day) orbit perturbations for the diurnal and semi-diurnal tides. The rms of the along-track perturbations on Lageos due to the combination of all of the retrograde waves is less than 5 cm.

For computing inclination and node perturbations, only the even degree terms are required, but for the eccentricity and periapsis the odd degree terms are not negligible. Long period perturbations are only produced when the degree (n) is greater than 1 and the order (m) is 0 for long period tides, 1 for diurnal tides, and 2 for semi-diurnal tides. Finally, the ocean tide amplitudes and their effect on satellite orbits decrease with increasing degree, so truncation above degree 6 is justified for Lageos.

Thus, for the diurnal tides (Q_1, O_1, P_1, K_1) only the n = 2, 3, 4, 5, 6 and m = 1 terms need be computed. For the semi-diurnal tides (N_2, M_2, S_2, K_2) only n = 2, 3, 4, 5, 6 and m = 2 terms need be computed. For the long period tides (S_{sa}, M_m, M_f) only n = 2, 3, 4, 5, 6 and m = 0 terms need be computed. Table 7.1 gives the values required for each of the constituents for which Schwiderski has computed a model. Note that the units in Table 7.1 are cm and hence must be scaled to m for use with the constants given on page 32.

The n = 2, m = 2 term for the S_2 argument can be modified to account for the atmospheric tide using the results of Chapman and Lindzen (1970). The modified values to be used instead of those in Table 7.1 are:

$$C^{+}_{22} = -0.537 \text{ (cm)}, \quad S^{+}_{22} = 0.321 \text{ (cm)}.$$

For the most precise applications, more than the 11 terms listed in Table 7.1 need to be modelled. This can be accomplished by assuming that the ocean tide admittance varies smoothly with frequency and by using the Schwiderski values as a guide to the interpolation to other frequencies.

Table 7.1

OCEAN TIDE COEFFICIENTS FROM THE SCHWIDERSKI MODEL

ARGUMENT NUMBER		n	m	$\hat{C}^{+}_{snm}$ (cm)	ε^{+}_{snm} (deg)	C^{+}_{snm} (cm)	S^{+}_{snm} (cm)
057.555	S_{sa}	2	0	.6215	221.672	-.8264	-.9284
057.555	S_{sa}	3	0	.0311	1.735	.0019	.0621
057.555	S_{sa}	4	0	.1624	92.674	.3244	-.0152
057.555	S_{sa}	5	0	.2628	251.737	-.4991	-.1647
057.555	S_{sa}	6	0	.4363	145.744	.4912	-.7213
065.455	M_m	2	0	.5313	258.900	-1.0428	-.2046
065.455	M_m	3	0	.0317	94.298	.0632	-.0047
065.455	M_m	4	0	.0998	69.054	.1863	.0713
065.455	M_m	5	0	.2279	292.291	-.4218	.1729
065.455	M_m	6	0	.0660	39.882	.0847	.1014
075.555	M_f	2	0	.8525	251.956	-1.6211	-.5281
075.555	M_f	3	0	.0951	148.236	.1001	-.1617
075.555	M_f	4	0	.2984	102.723	.5822	-.1315
075.555	M_f	5	0	.2960	223.167	-.4050	-.4318
075.555	M_f	6	0	.0880	107.916	.1675	-.0542
135.655	Q_1	2	1	.5373	313.735	-.3715	-.3882
135.655	Q_1	3	1	.3136	107.346	.0935	.2994
135.655	Q_1	4	1	.2930	288.992	-.0953	-.2770
135.655	Q_1	5	1	.2209	112.383	.0841	.2042
135.655	Q_1	6	1	.0396	287.824	-.0121	-.0377
145.555	O_1	2	1	2.4186	313.716	-1.6715	-1.7481
145.555	O_1	3	1	1.3161	83.599	-.1467	1.3079
145.555	O_1	4	1	1.4301	276.282	-.1565	-1.4215
145.555	O_1	5	1	.9505	109.128	.3115	.8980
145.555	O_1	6	1	.1870	282.623	-.0409	-.1825

Table 7.1 (continued)

ARGUMENT NUMBER		n	m	$\hat{C}^{+}_{snm}$ (cm)	ε^{+}_{snm} (deg)	C^{+}_{snm} (cm)	S^{+}_{snm} (cm)
163.555	P_1	2	1	.9020	313.912	-.6256	-.6498
163.555	P_1	3	1	.2976	39.958	-.2281	.1911
163.555	P_1	4	1	.6346	258.311	.1286	-.6215
163.555	P_1	5	1	.4130	104.438	.1030	.4000
163.555	P_1	6	1	.0583	276.591	-.0067	-.0579
165.555	K_1	2	1	2.8158	315.113	1.9950	1.9872
165.555	K_1	3	1	.8925	33.752	.7421	-.4959
165.555	K_1	4	1	1.9121	254.229	-.5197	1.8401
165.555	K_1	5	1	1.2111	104.672	-.3068	-1.1716
165.555	K_1	6	1	.1645	281.867	.0338	.1610
245.655	N_2	2	2	.6516	321.788	-.4030	.5120
245.655	N_2	3	2	.1084	171.923	.0152	-.1074
245.655	N_2	4	2	.2137	141.779	.1322	-.1679
245.655	N_2	5	2	.0836	5.034	.0073	.0832
245.655	N_2	6	2	.0674	346.544	-.0157	.0656
255.555	M_2	2	2	2.9551	310.553	-2.2453	1.9213
255.555	M_2	3	2	.3610	168.623	.0712	-.3539
255.555	M_2	4	2	1.0066	124.755	.8270	-.5738
255.555	M_2	5	2	.2751	356.561	-.0165	.2746
255.555	M_2	6	2	.4130	329.056	-.2124	.3542
273.555	S_2	2	2	.9291	314.011	-.6682	.6456
273.555	S_2	3	2	.2633	201.968	-.0985	-.2442
273.555	S_2	4	2	.3716	103.027	.3621	-.0838
273.555	S_2	5	2	.1365	3.772	.0090	.1362
273.555	S_2	6	2	.1726	280.381	-.1698	.0311
275.555	K_2	2	2	.2593	315.069	-.1832	.1836
275.555	K_2	3	2	.0943	195.007	-.0244	-.0911
275.555	K_2	4	2	.1059	103.521	.1029	-.0247
275.555	K_2	5	2	.0382	.411	.0003	.0382
275.555	K_2	6	2	.0467	281.357	-.0458	.0092

NOTES:

1. The Doodson variable multipliers ($\bar{n}$) are coded into the argument number (A) after Doodson (Proc. R. Soc. A., **100**, pp. 305-329, 1921) as:

   $A = n_1(n_2+5)(n_3+5) \;.\; (n_4+5)(n_5+5)(n_6+5).$

2. For the long period tides (m = 0), the value of $\hat{C}^{+}_{snm}$ used to

compute C^{+}_{snm} and S^{+}_{snm} was twice that shown to account for the combined effect of the retrograde and prograde waves.

3. The spherical harmonic decomposition of Schwiderski's models was computed by C. Goad of the Ohio State University.

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Schwiderski, E., 1983, "Atlas of Ocean Tidal Charts and Maps, Part I: The Semidiurnal Principal Lunar Tide M_2," Marine Geodesy, **6**, pp. 219-256. (See also Chapter 8).

OCEAN LOADING

In the past decade, displacements due to ocean loading have been computed by several authors (Goad, 1980; Agnew, 1983; Scherneck, 1983; Pagiatakis, et al., 1984; Sato and Hanada, 1985), most of them using methods similar to that described by Farrell (1972) and based on global representations of the ocean tides like those by Schwiderski (1980).

In all of these models, the locally referenced displacement vector $\vec{E}$ in the vertical (radial), N-S and E-W local coordinate system at time t can be computed as a sum of the contributions of n individual ocean tides

$$\vec{E}(t) = \sum_{i=1}^{n} \left\{ \begin{array}{l} A_i^r \cdot \cos(\omega_i t + \phi_i - \delta_i^r) \\ A_i^{NS} \cdot \cos(\omega_i t + \phi_i - \delta_i^{NS}) \\ A_i^{EW} \cdot \cos(\omega_i t + \phi_i - \delta_i^{EW}) \end{array} \right.$$

where ω_i is the frequency of the tidal constituents and ϕ_i the corresponding astronomical argument. The amplitudes A_i^r, A_i^{NS}, A_i^{EW}, and the Greenwich phase lags δ_i^r, δ_i^{NS}, δ_i^{EW} of each tidal component are determined by the particular model assumed for the deformation of the Earth.

The site displacements resulting from the ocean tides have been compiled by Clyde Goad. Tidal loading radial (height) displacement amplitude and phase values for laser and VLBI station locations have been computed for the M_2, S_2, K_2, N_2, O_1, K_1, P_1, Q_1, M_f, M_m, and S_{sa} ocean tides using models generated by Schwiderski (1978). The technique of Goad (1980) which uses integrated Green's Functions was employed. The load deformation coefficients used in generating the Green's function integrals were taken from Farrell (1972).

Table 8.1 gives the amplitude and phase values for the eleven main tides. The phase convention of Schwiderski has been preserved. His phases for the diurnal tides are different by ± 90° with the standard Doodson definition. This chapter also contains a FORTRAN subroutine which will return the proper angular argument to be used with the Schwiderski phases. The height displacement, in centimeters, can be found for a given constituent i as follows:

$$h(i) = amp(i) \times \cos(\, arg(i, t) - phase\,(i)\,),$$

where arg(i,t) is generated by the subroutine ARG below. The total tidal displacement is found by summing over all 9 constituents. A negative h(i) means that the surface at the predicted point has been lowered.

ATMOSPHERIC LOADING

The procedure described below is taken from the publication of Sovers and Fanselow (1987). A time varying atmospheric pressure distribution can induce crustal deformation. Rabbel and Schuh (1986) estimate the effects of atmospheric loading on VLBI baseline determinations, and conclude that they may amount to many millimeters of seasonal variation. In contrast to ocean tidal effects, analysis of the situation in the atmospheric case does not benefit from the presence of a well-understood periodic driving force. Otherwise, estimation of atmospheric loading via Green's function techniques is analogous to methods used to calculate ocean loading effects. Rabbel and Schuh recommend a simplified form of the dependence of the vertical crustal displacement on pressure distribution. It involves only the instantaneous pressure at the site in question, and an average pressure over a circular region C with a 2000 km radius surrounding the site. The expression for the vertical displacement (mm) is

$$\Delta r = -0.35p - 0.55\bar{p}, \qquad (1)$$

where p is the local pressure anomaly, and $\bar{p}$ the pressure anomaly within 2000 km circular region mentioned above (both quantites are in mbar). Note that the reference point for this displacement is the site location at standard pressure (1013 mbar).

An additional mechanism for characterizing $\bar{p}$ may be applied. The two-dimensional surface pressure distribution surrounding a site is described by

$$p(x,y) = A_0 + A_1x + A_2y + A_3x^2 + A_4xy + A_5y^2, \qquad (2)$$

where x and y are the local East and North distances of the point in question from the VLBI site. The pressure anamoly $\bar{p}$ may be evaluated by the simple integration

$$\bar{p} = \int\int_C dx\, dy\, p(x,y) \;/\; \int\int_C dx\, dy \qquad (3)$$

giving

$$\bar{p} = A_0 + (A_3 + A_5)R^2/4, \qquad (4)$$

where $R^2=(x^2 + y^2)$.

It remains the task of the data analyst to perform a quadratic fit to the available weather data to determine the coefficients A_{0-5}. Future advances in understanding the atmosphere-crust elastic interaction can probably be accomodated by adjusting the coefficients in Eq. (1). Furthermore, expansion of eq. (1) might be required for stations close to the coast.

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```
C     SUBROUTINE ARG(IYEAR,DAY,ANGLE)
C
C   COMPUTES THE ANGULAR ARGUMENT WHICH DEPENDS ON TIME FOR 11
C   TIDAL ARGUMENT CALCULATIONS
C
C
C     ORDER OF THE 11 ANGULAR QUANTITIES IN VECTOR ANGLE
C
C   01-M2
C   02-S2
C   03-N2
C   04-K2
C   05-K1
C   06-O1
C   07-P1
C   08-Q1
C   09-Mf
C   10-Mm
C   11-Ssa
C
C   TAKEN FROM 'TABLE 1 CONSTANTS OF MAJOR TIDAL MODES'
C        WHICH DR. SCHWIDERSKI SENDS ALONG WITH HIS TAPE OF TIDAL
C        AMPLITUDES AND PHASES
C
C
C   INPUT--
C
C      IYEAR - EX. 79 FOR 1979
C      DAY - DAY OF YEAR GREENWICH TIME
C                      EXAMPLE 32.5 FOR FEB 1   12 NOON
C                              1.25 FOR JAN 1    6 AM
C
C   OUTPUT--
C
C      ANGLE - ANGULAR ARGUMENT FOR SCHWIDERSKI COMPUTATION
C
C***********************************************************
C
C
C            C A U T I O N
C
C   SCHWIDERSKI MODIFIES THE ANGULAR ARGUMENTS OF THE DIURNAL
C   TERMS BY ± 90 DEGREES.  THEREFORE HIS DIURNAL PHASES
C   CANNOT BE USED WITH THE STANDARD DOODSON OR CARTWRIGHT
C   CONVENTIONS
C
C   THIS SUBROUTINE IS VALID ONLY FOR DATES AFTER 1973.
C
C***********************************************************
C
C
```

```
      SUBROUTINE ARG(IYEAR,DAY,ANGLE)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL ANGFAC(4,11)
      DIMENSION ANGLE(11),SPEED(11)
C
C  SPEED OF ALL TERMS IN RADIANS PER SEC
C
      EQUIVALENCE (SPEED(1),SIGM2),(SPEED(2),SIGS2),(SPEED(3),SIGN2)
      EQUIVALENCE (SPEED(4),SIGK2),(SPEED(5),SIGK1),(SPEED(6),SIGO1)
      EQUIVALENCE (SPEED(7),SIGP1),(SPEED(8),SIGQ1),(SPEED(9),SIGMF)
      EQUIVALENCE (SPEED(10),SIGMM),(SPEED(11),SIGSSA)
      DATA SIGM2/1.40519D-4/
      DATA SIGS2/1.45444D-4/
      DATA SIGN2/1.37880D-4/
      DATA SIGK2/1.45842D-4/
      DATA SIGK1/.72921D-4/
      DATA SIGO1/.67598D-4/
      DATA SIGP1/.72523D-4/
      DATA SIGQ1/.64959D-4/
      DATA SIGMF/.053234D-4/
      DATA SIGMM/.026392D-4/
      DATA SIGSSA/.003982D-4/
      DATA ANGFAC/2.E0,-2.E0,0.E0,0.E0,4*0.E0,
     .            2.E0,-3.E0,1.E0,0.E0,2.E0,3*0.E0,
     .            1.E0,2*0.E0,.25E0,1.E0,-2.E0,0.E0,-.25E0,
     .            -1.E0,2*0.E0,-.25E0,1.E0,-3.E0,1.E0,-.25E0,
     .            0.E0,2.E0,2*0.E0,0.E0,1.E0,-1.E0,0.E0,
     .            2.E0,3*0.E0/
      DATA TWOPI/6.28318530718D0/
      DATA DTR/.174532925199D-1/
C
C  DAY OF YEAR
C
      ID=DAY
C
C  FRACTIONAL PART OF DAY IN SECONDS
C
      FDAY=(DAY-ID)*86400.D0
      ICAPD=ID+365*(IYEAR-75)+((IYEAR-73)/4)
      CAPT=(27392.500528D0+1.000000035D0*ICAPD)/36525.D0
C
C MEAN LONGITUDE OF SUN AT BEGINNING OF DAY
C
      H0=(279.69668D0+(36000.768930485D0+3.03D-4*CAPT)*CAPT)*DTR
C
C MEAN LONGITUDE OF MOON AT BEGINNING OF DAY
C
      S0=(((1.9D-6*CAPT-.001133D0)*CAPT+481267.88314137D0)*CAPT
     .  +270.434358D0)*DTR
C
C MEAN LONGITUDE OF LUNAR PERIGEE AT BEGINNING OF DAY
```

```
C
      P0=(((-1.2D-5*CAPT-.010325D0)*CAPT+4069.0340329577D0)*CAPT
     .  +334.329653D0)*DTR
      DO 500 K=1,11
      ANGLE(K)=SPEED(K)*FDAY+ANGFAC(1,K)*H0+ANGFAC(2,K)*S0
     .  +ANGFAC(3,K)*P0+ANGFAC(4,K)*TWOPI
      ANGLE(K)=DMOD(ANGLE(K),TWOPI)
      IF(ANGLE(K).LT.0.D0)ANGLE(K)=ANGLE(K)+TWOPI
  500 CONTINUE
      RETURN
      END
```

Table 8.1. Displacement due to ocean loading (cm in amplitude and degree in phase).

	M_2		S_2		K_1		O_1		N_2		P_1		K_2		Q_1		M_f		M_m		S_{sa}	
	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS
Chlboltn	1.37	-46.1	.47	-11.1	.31	-58.5	.10	-127.5	.31	-63.7	.10	-58.9	.12	-5.4	.01	106.4	.09	-4.2	.03	-26.8	.04	-27.5
Madrid64	1.37	-86.4	.46	-61.4	.23	-70.0	.03	-151.3	.29	-106.3	.07	-69.4	.12	-63.8	.02	48.1	.04	-25.5	.02	-98.1	.01	-102.4
Robled32	1.37	-86.4	.46	-61.5	.23	-70.0	.03	-151.0	.29	-106.3	.07	-69.4	.12	-63.8	.02	48.0	.04	-25.5	.02	-98.1	.01	-102.5
Cebrer26	1.37	-87.0	.45	-62.0	.23	-70.0	.03	-149.7	.28	-106.6	.07	-69.5	.12	-64.6	.02	48.1	.04	-25.2	.02	-97.8	.01	-102.8
Haystack	.96	-176.8	.26	-154.8	.39	-5.4	.26	-4.8	.21	165.2	.12	-3.2	.07	-161.6	.05	-1.7	.04	10.5	.02	59.2	.04	-96.0
Westford	.96	-176.9	.26	-154.8	.39	-5.4	.26	-4.8	.21	165.2	.12	-3.2	.07	-161.6	.05	-1.7	.04	10.5	.02	59.2	.04	-96.1
Marpoint	.95	158.7	.24	-170.4	.33	-1.9	.22	-1.6	.16	141.7	.11	.7	.06	-175.0	.05	2.3	.02	17.7	.01	74.3	.05	-117.3
NRAO 140	.68	152.1	.20	-173.2	.29	.6	.20	-.9	.11	133.7	.10	3.6	.05	-177.1	.04	2.9	.01	20.5	.00	66.4	.04	-121.8
Richmond	.82	165.9	.24	-162.3	.18	15.2	.12	25.1	.14	146.6	.06	13.1	.06	-169.7	.03	35.0	.05	154.8	.02	160.5	.18	-122.2
HRAS 085	.08	-168.8	.15	-128.1	.47	32.5	.32	17.1	.05	-69.3	.15	30.8	.04	-109.1	.07	10.0	.02	-167.7	.01	157.3	.01	137.3
Plattvil	.22	111.6	.13	-168.3	.47	39.5	.31	25.2	.02	-11.2	.15	38.7	.03	-151.3	.06	17.8	.01	6.5	.01	49.2	.02	92.7
VLA	.04	169.6	.14	-131.9	.57	38.0	.37	22.8	.06	-67.0	.18	36.8	.04	-108.8	.08	14.6	.01	-138.3	.01	123.5	.02	90.2
Vernal	.22	96.5	.13	-170.8	.58	44.5	.37	29.9	.03	-18.3	.18	43.6	.02	-148.3	.07	21.5	.01	-1.9	.01	49.3	.03	83.0
Flagstaf	.01	-2.8	.14	-129.2	.69	41.6	.44	26.3	.07	-63.9	.22	40.5	.04	-103.0	.09	17.6	.01	-88.3	.01	105.0	.03	79.9
Yuma	.26	-57.2	.21	-98.3	.90	40.1	.57	24.8	.13	-70.8	.28	38.9	.07	-83.7	.11	15.9	.01	-117.4	.01	145.2	.05	73.7
Ely	.24	70.2	.12	-168.2	.76	47.4	.48	32.8	.06	-27.1	.24	46.6	.02	-132.5	.09	23.9	.01	-16.7	.01	55.0	.04	77.6
BlkButte	.28	-42.4	.19	-99.9	.96	41.5	.61	26.3	.13	-65.6	.30	40.6	.07	-82.7	.12	17.4	.01	-100.7	.01	137.7	.05	72.9
Ocotillo	.42	-49.5	.26	-86.6	1.06	39.6	.67	24.3	.17	-67.8	.33	38.4	.09	-76.2	.13	15.6	.01	-110.5	.01	154.8	.06	71.6
DeadManL	.28	-34.2	.18	-99.2	1.00	42.1	.63	27.0	.13	-62.5	.31	41.2	.07	-81.2	.12	18.1	.01	-89.6	.01	130.4	.06	73.4
Mon Peak	.51	-45.7	.27	-83.1	1.12	39.7	.71	24.5	.19	-66.2	.35	38.7	.10	-73.6	.13	15.7	.01	-106.4	.01	157.0	.06	71.4
PinFlats	.38	-39.5	.22	-89.7	1.07	41.0	.68	25.9	.16	-63.5	.33	40.1	.08	-76.2	.13	17.1	.01	-96.2	.01	143.1	.06	72.6
GoldVenu	.22	-13.4	.15	-108.7	.98	43.6	.62	28.6	.12	-56.7	.31	42.8	.05	-84.0	.12	19.7	.01	-69.8	.01	109.4	.06	73.8
Mojave12	.22	-11.3	.15	-107.5	.99	43.6	.62	28.7	.12	-55.3	.31	42.9	.05	-82.5	.12	19.8	.01	-67.2	.01	107.7	.06	73.9
GoldEcho	.22	-11.7	.15	-108.6	.98	43.6	.62	28.7	.12	-56.5	.31	42.8	.05	-83.7	.12	19.8	.01	-68.8	.01	108.5	.06	73.9
Otay	.70	-45.0	.35	-74.0	1.29	38.2	.81	23.1	.24	-64.7	.40	37.1	.12	-68.2	.15	14.6	.02	-104.4	.01	167.0	.07	71.1
GoldPion	.22	-10.3	.14	-108.6	.99	43.7	.62	28.8	.12	-55.0	.31	42.9	.05	-82.8	.12	19.9	.01	-66.1	.01	106.3	.06	74.1
GoldMars	.23	-8.5	.14	-109.7	.99	43.8	.62	28.9	.12	-54.3	.31	43.1	.05	-83.1	.12	20.0	.01	-64.4	.01	104.9	.06	74.2
LaJolla	.84	-41.1	.36	-71.8	1.36	38.4	.85	23.3	.25	-63.2	.43	37.3	.13	-66.3	.16	14.8	.02	-99.4	.01	167.6	.08	71.3
PBlossom	.40	-23.8	.19	-89.1	1.15	42.3	.72	27.3	.17	-57.4	.36	41.5	.07	-73.4	.14	18.5	.01	-77.2	.01	131.3	.07	72.6
JPL MV3	.51	-27.0	.21	-84.7	1.21	41.9	.76	26.9	.18	-57.9	.38	41.2	.08	-71.1	.14	18.1	.01	-81.2	.01	141.3	.07	71.9
JPL MV1	.51	-27.0	.21	-84.7	1.21	41.9	.76	26.9	.18	-57.9	.38	41.2	.08	-71.1	.14	18.1	.01	-81.2	.01	141.3	.07	71.9
JPL MV2	.51	-27.0	.21	-84.7	1.21	41.9	.76	26.9	.18	-57.9	.38	41.2	.08	-71.1	.14	18.1	.01	-81.2	.01	141.3	.07	71.9
OVRO 130	.28	32.6	.10	-140.6	1.01	46.8	.63	32.1	.10	-37.6	.31	46.3	.03	-91.4	.12	23.2	.01	-38.4	.01	73.4	.06	74.4
OVRO 90	.28	32.7	.10	-140.6	1.01	46.8	.63	32.1	.10	-37.7	.31	46.3	.03	-91.5	.12	23.2	.01	-38.5	.01	73.5	.06	74.4
PVerdes	.80	-31.7	.32	-69.9	1.42	39.9	.90	24.9	.25	-58.3	.45	39.0	.12	-63.4	.17	16.2	.02	-84.2	.01	165.3	.08	71.9
Malibu	.64	-27.1	.27	-72.8	1.37	40.8	.86	25.8	.23	-56.5	.43	40.1	.10	-64.2	.16	17.0	.02	-80.4	.01	158.0	.08	71.7
SaddlePk	.64	-26.4	.27	-73.5	1.36	40.9	.86	25.9	.23	-56.5	.43	40.2	.10	-64.5	.16	17.1	.02	-80.3	.01	157.1	.08	71.7
Gorman	.45	-13.3	.18	-84.6	1.25	43.0	.78	28.1	.18	-51.4	.39	42.6	.07	-68.2	.14	19.3	.01	-67.9	.01	128.7	.07	72.3
MammothL	.35	39.7	.09	-155.2	1.02	47.8	.64	33.2	.10	-30.5	.32	47.3	.02	-95.6	.12	24.3	.02	-33.5	.01	65.9	.06	74.2
SanPaula	.59	-20.6	.24	-73.3	1.38	41.6	.86	26.7	.22	-53.4	.43	41.1	.09	-63.2	.16	17.9	.02	-74.2	.01	150.7	.08	72.1
Vandenbg	.92	-11.2	.27	-59.1	1.65	42.0	1.03	27.2	.28	-45.7	.51	42.1	.11	-52.4	.19	18.3	.02	-64.6	.01	162.9	.10	71.8
Vndnberg	.97	-11.3	.26	-58.7	1.67	42.0	1.04	27.3	.28	-45.0	.52	42.3	.11	-51.6	.19	18.2	.02	-64.3	.01	164.9	.10	71.6
Quincy	.62	56.2	.13	150.3	1.11	51.5	.69	37.4	.12	-2.5	.34	51.1	.01	138.9	.13	28.6	.03	-20.1	.02	49.4	.07	74.2
HatCreek	.74	59.8	.16	138.8	1.15	52.7	.71	38.7	.14	5.5	.35	52.1	.02	115.6	.13	29.9	.03	-17.3	.02	45.8	.07	74.1
Fort Ord	.89	20.3	.04	-53.4	1.55	46.4	.96	31.9	.23	-25.1	.47	46.8	.05	-34.1	.18	23.1	.03	-41.2	.01	92.8	.09	73.1
Vacavill	.74	42.0	.07	133.3	1.37	49.2	.85	35.1	.18	-10.8	.42	49.4	.02	1.4	.16	26.3	.03	-30.1	.01	62.6	.08	73.9
SanFranc	1.04	33.8	.05	105.5	1.53	48.4	.95	34.2	.22	-12.9	.47	48.8	.03	-4.1	.18	25.4	.03	-33.6	.01	71.1	.09	73.8
Presidio	1.04	33.8	.05	105.5	1.53	48.4	.95	34.2	.22	-12.9	.47	48.8	.03	-4.1	.18	25.4	.03	-33.6	.01	71.1	.09	73.8
Pt Reyes	1.19	36.4	.10	81.4	1.69	48.6	1.05	34.5	.27	-8.7	.51	49.1	.04	16.7	.19	25.6	.04	-32.5	.02	66.3	.10	73.9
Yakataga	2.49	100.7	.89	137.4	1.32	87.4	.83	75.0	.43	78.1	.43	85.7	.23	133.1	.15	66.9	.10	4.7	.09	32.1	.13	59.6
GilCreek	.78	100.7	.32	140.1	.51	96.3	.35	88.1	.10	87.5	.16	95.3	.08	134.7	.06	79.4	.09	16.7	.06	23.1	.10	71.7
Kodiak	2.82	113.1	1.07	147.3	1.61	98.0	1.05	85.7	.51	94.5	.52	96.9	.27	144.7	.19	79.7	.12	.8	.10	38.8	.15	56.6
Kauai	.95	-111.3	.38	-130.0	1.15	61.9	.66	56.2	.21	-132.3	.33	61.9	.12	-138.7	.10	58.4	.05	-140.1	.03	-170.8	.06	112.5
SndPoint	1.94	127.6	.80	150.9	1.45	112.9	1.06	104.1	.33	122.2	.49	115.6	.21	151.1	.19	102.8	.13	-4.7	.09	56.9	.15	63.6
Kwajal26	3.03	-46.7	1.60	-28.1	.95	-125.9	.66	-151.2	.48	-45.2	.31	-127.2	.39	-30.3	.13	-165.3	.10	-162.3	.04	179.4	.11	-161.9
Tidbin64	.91	125.1	.12	175.5	.26	116.5	.27	65.2	.19	95.2	.10	108.8	.03	160.7	.08	50.5	.01	-18.0	.00	-26.6	.03	-126.8
Kashima	.87	51.5	.46	75.4	1.14	-138.3	.89	-157.6	.13	64.7	.35	-138.5	.13	78.7	.18	-163.0	.02	-12.2	.05	42.9	.10	105.6

Table 8.1 (continued)

	M_2		S_2		K_1		O_1		N_2		P_1		K_2		Q_1		M_f		M_m		S_{sa}	
	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS	AMP	PHAS
Johani26	1.63	-127.4	.74	-102.4	.09	128.3	.14	116.8	.33	-135.6	.03	127.6	.20	-101.8	.03	101.6	.02	-171.6	.03	112.6	.07	146.0
Wettzell	.52	-64.4	.14	-36.6	.18	-57.9	.08	-97.7	.11	-88.3	.05	-54.1	.04	-32.7	.01	-40.5	.05	6.1	.03	11.9	.05	75.4
Onsala60	.39	-62.3	.10	-29.2	.22	-50.4	.11	-103.3	.08	-84.2	.07	-47.7	.02	-12.6	.00	-130.0	.08	13.2	.05	23.1	.06	51.5
Onsala85	.39	-62.3	.10	-29.5	.22	-50.2	.11	-103.2	.08	-84.3	.07	-47.6	.02	-12.7	.00	-129.6	.08	13.2	.05	23.0	.06	51.5
Werthovn	.66	-59.7	.19	-28.6	.20	-58.0	.09	-92.5	.14	-81.1	.06	-55.4	.04	-25.5	.01	-35.4	.06	3.3	.04	6.5	.03	52.0
Eflsberg	.64	-60.7	.18	-31.5	.21	-58.2	.09	-93.0	.14	-82.2	.06	-55.8	.04	-30.2	.01	-36.9	.06	2.7	.04	5.9	.03	50.6
Orroral	.91	125.6	.13	174.3	.28	116.5	.28	66.1	.20	97.0	.11	109.1	.03	160.0	.08	51.6	.01	-16.3	.00	-24.0	.03	-120.6
Yaragade	.37	168.9	.13	-105.4	.81	13.5	.64	6.5	.07	94.7	.26	13.4	.02	-95.0	.13	-2.6	.01	-129.5	.04	-131.7	.08	-63.8
Graz	.53	-65.1	.14	-37.6	.16	-60.3	.07	-99.0	.11	-89.1	.05	-55.6	.04	-32.9	.01	-42.7	.04	6.1	.03	13.5	.05	82.0
Plana	.42	-63.9	.11	-39.9	.10	-71.8	.06	-104.4	.09	-90.3	.03	-64.2	.03	-34.3	.01	-51.7	.03	7.8	.03	20.2	.06	91.0
Kralov	.52	-65.1	.14	-37.5	.17	-58.1	.08	-98.3	.11	-89.0	.05	-54.1	.03	-32.8	.01	-46.0	.05	6.8	.03	13.2	.05	76.7
Ondrejov	.50	-65.1	.14	-33.6	.17	-57.8	.08	-98.9	.11	-87.2	.05	-53.6	.04	-27.8	.01	-55.5	.05	8.0	.03	13.6	.05	75.7
Helwan	.37	-67.0	.11	-52.3	.08	-170.9	.05	-150.5	.09	-93.4	.02	-174.8	.03	-48.1	.00	-124.4	.00	-42.3	.02	30.7	.07	99.6
Wettzell	.53	-63.8	.14	-36.6	.18	-57.8	.08	-97.5	.11	-88.2	.05	-54.0	.04	-32.7	.01	-39.9	.05	6.1	.03	12.0	.05	75.4
Metsahov	.28	-59.8	.07	-23.3	.17	-52.2	.11	-102.9	.06	-89.2	.05	-48.4	.02	-1.4	.01	-110.3	.07	19.8	.05	16.8	.07	74.8
Grasse	.73	-71.4	.21	-45.7	.18	-62.0	.06	-97.9	.15	-93.3	.05	-58.3	.05	-44.8	.01	5.5	.04	-4.5	.02	-2.0	.03	83.2
Huahine	.46	-107.5	.23	167.4	.04	142.9	.10	-82.2	.10	-122.5	.01	132.1	.07	174.6	.03	-99.4	.08	-158.2	.04	-151.5	.05	-141.4
Potsdam	.51	-63.3	.13	-31.3	.19	-55.5	.09	-98.4	.10	-84.2	.06	-52.3	.03	-24.4	.01	-62.3	.06	9.0	.04	14.9	.05	67.4
Dionysos	.42	-65.3	.11	-44.0	.07	-85.5	.05	-109.1	.09	-91.3	.02	-76.9	.03	-39.7	.01	-46.6	.02	3.6	.02	22.8	.06	95.1
Matera	.49	-65.9	.13	-39.4	.15	-60.4	.07	-98.5	.11	-90.2	.04	-55.2	.03	-34.8	.01	-42.3	.04	6.6	.03	14.7	.05	83.5
Dodair	.72	65.1	.37	84.4	.97	-135.2	.76	-154.4	.12	81.8	.30	-135.4	.11	88.3	.15	-159.9	.01	-10.1	.04	46.0	.09	110.9
Simosato	1.25	87.6	.57	107.0	1.11	-126.5	.86	-145.5	.25	95.7	.34	-126.6	.17	108.9	.17	-153.0	.01	-32.2	.05	43.7	.11	113.0
Mazatlan	.74	-89.2	.53	-88.2	.81	20.0	.56	5.2	.22	-81.6	.25	15.5	.16	-85.0	.12	-2.6	.05	-158.7	.03	-171.5	.06	48.5
Kootwijk	.60	-36.6	.19	-7.1	.20	-63.1	.10	-69.5	.15	-63.1	.06	-62.3	.04	-1.6	.02	-38.0	.08	1.3	.05	15.0	.04	12.0
Shanghai	.58	169.9	.14	-178.2	.62	-97.8	.51	-113.2	.17	163.6	.19	-92.8	.04	172.2	.10	-115.7	.01	105.3	.03	95.7	.09	146.8
Arequipa	.37	124.2	.10	95.2	.50	-140.1	.26	-160.2	.13	96.5	.15	-141.7	.04	99.4	.04	-177.2	.04	-176.8	.01	126.8	.03	105.0
Borowiec	.44	-61.4	.12	-28.8	.17	-56.1	.09	-99.9	.09	-85.2	.05	-52.0	.03	-19.7	.01	-73.1	.05	11.2	.04	15.5	.06	74.9
San Fern	2.29	-105.7	.77	-84.4	.24	-89.5	.07	141.5	.48	-128.4	.07	-90.9	.21	-86.0	.04	53.0	.03	-47.7	.03	-124.5	.03	-108.1
Zimmerwa	.68	-68.9	.21	-41.4	.20	-59.2	.07	-99.9	.15	-90.7	.06	-56.3	.05	-40.0	.01	-1.9	.05	-.2	.02	2.5	.03	71.9
Herstmon	.44	-88.0	.11	-74.6	.30	-60.2	.10	-111.6	.06	-4.4	.08	-61.8	.03	-132.9	.01	-141.7	.09	1.0	.05	-18.7	.03	-25.3
Ft. Davi	.08	-164.8	.15	-127.2	.48	32.5	.32	17.2	.05	-70.3	.15	30.8	.04	-108.3	.07	10.0	.02	-167.9	.01	157.8	.01	135.4
Mojave	.22	-10.7	.15	-107.6	.99	43.6	.62	28.7	.12	-55.2	.31	42.9	.05	-82.5	.12	19.9	.01	-67.1	.01	107.7	.06	73.9
Greenbel	.94	159.1	.25	-169.9	.34	-2.3	.23	-1.9	.17	143.6	.11	.1	.07	-174.4	.05	2.0	.02	19.5	.01	68.9	.05	-115.5
Maui, HI	1.18	-117.1	.48	-134.4	1.28	57.7	.73	49.2	.28	-138.7	.37	57.5	.15	-135.1	.11	47.2	.06	-143.7	.03	-169.1	.05	109.5
Monument	.51	-45.7	.27	-83.5	1.12	39.8	.71	24.5	.18	-66.2	.35	38.7	.10	-73.8	.13	15.8	.01	-106.4	.01	156.7	.06	71.4
Plattevi	.23	111.4	.13	-168.1	.47	39.5	.31	25.2	.02	-11.7	.15	38.7	.03	-151.0	.06	17.8	.01	6.3	.01	49.2	.02	92.7
Quincy	.62	56.3	.13	150.8	1.11	51.5	.69	37.4	.12	-2.6	.34	51.0	.01	140.8	.13	28.6	.03	-20.1	.02	49.3	.07	74.2
Simeiz	.34	-55.1	.09	-27.6	.07	-86.3	.07	-112.4	.08	-87.0	.02	-77.0	.02	-19.4	.01	-85.5	.03	16.1	.03	22.2	.07	91.0
Zvenigor	.28	-53.1	.07	-16.7	.12	-64.1	.09	-105.2	.06	-87.6	.04	-57.9	.02	-2.1	.01	-94.4	.05	21.4	.05	19.7	.08	85.5

CHAPTER 9 PLATE MOTION MODEL

One of the factors which can affect Earth rotation results is the motion of the tectonic plates which make up the Earth's surface. As the plates move, fixed coordinates for the observing stations will become inconsistent with each other. The rates of relative motions for some regular observing sites are believed to be 5 cm per year or larger. The observations of plate motions so far by Satellite Laser Ranging and Very Long Baseline Interferometry appear to be roughly consistent with the average rates over the last few million years derived from the geological record and other geophysical information. Thus, in order to reduce inconsistencies in the station coordinates and to make the results from different techniques more directly comparable, a model for plate motions based on the relative plate motion model RM-2 of Minster and Jordan (1978) is recommended.

From the RM-2 model, Minster and Jordan (1978) derive four different absolute plate motion models. The two which have been discussed most widely are AM0-2, which has zero net rotation of the Earth's surface, and AM1-2, which minimizes the motion of a set of hot spots. Kaula (1975) and others have discussed alternate geophysical constraints which can be used in order to investigate the plate motions with respect to the bulk of the mantle. For convenience, when future changes are made in the relative motion model, and to avoid dependence on the choice of the hot spots to be held fixed, the (AM0-2) model is recommended. The Cartesian rotation vector for each of the major plates is given in Table 9.1. A subroutine called ABSMOV, provided by J. B. Minster, is also included. It computes the new site position at time t from the old site position at time t0 using the recommended plate motion model AM0-2.

Future improvements are planned for future IERS Standards including: (1) replacement of RM-2 by a new model, new velocities and new plates; and (2) adoption of a vertical motion model (post-glacial rebound). In any case, the AM0-2 model should be used as a default, for stations which appear to follow reasonably its values. For some stations, particularly in the vicinity of plate boundaries, users may benefit by estimating velocities or using specififc values not derived from AM0-2. This is also a way to take into account now some non-negligible vertical motions. Published station coordinates should include the epoch associated with the coordinates.

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Table 9.1

CARTESIAN ROTATION VECTOR FOR EACH PLATE USING KINEMATIC PLATE MODEL AM0-2 (NO NET ROTATION)

Plate Name	Ω_X deg/My.	Ω_Y deg/My.	Ω_Z deg/My.
Pacific	-0.12276	0.31163	-0.65537
Cocos	-0.63726	-1.33142	0.72556
Nazca	-0.09086	-0.53281	0.63061
Caribbean	-0.02787	-0.05661	0.10780
South America	-0.05604	-0.10672	-0.08642
Antarctica	-0.05286	-0.09492	0.21570
India/Australia	0.48372	0.25011	0.43132
Africa	0.05660	-0.19249	0.24016
Arabia	0.27885	-0.16744	0.37359
Eurasia	-0.03071	-0.15865	0.19605
North America	0.03299	-0.22828	-0.01427

SUBROUTINE ABSMOV

NAME

absmov - compute new site position at time t from old site position at time t0 using plate motion model AM0-2

SYNOPSIS

```
call absmov(psit,t0,x0,y0,z0,t,x,y,z)
real*S t0,x0,y0,z0,t,x,y,z
character*4 psit
```

DESCRIPTION

Absmov takes a site specified by its initial coordinates x0,y0,z0 at epoch time t0, and computes its updated position x,y,z, at epoch t, based on the geological "absolute", (no net rotation) plate motion model AM0-2 (Minister and Jordan, 1978).

Its intended use is to account for the geologically determined component of relative movements between geodetic sites in the processing of VLBI or SLR data.

The routine uses a geocentric Cartesian coordinate system with the x-axis at 0° longitude, and the z-axis along the axis of rotation of the Earth. The epoch times t0 and t are assumed to be given in years (only the difference t-t0 is used in the calculation).

The name of the plate on which the site is located is specified in the calling window by the 4 character code: psit

There are 11 valid plate names, namely: afrc (Africa), anta (Antarctica), arab (Arabia), carb (Caribbean), coco (Cocos), eura (Eurasia), indi (India/Australia), nazc (Nazca), noam (north America), pcfc (Pacific), and soam (South America).

(AM0-2 is specified internally in the form of a data statement containing the Cartesian Euler vectors for these plates)

AUTHOR

J.B. Minster, Science Horizons

DIAGNOSTICS

The only diagnostic arises if the plate name psit is not recognized. In that case the returned coordinates x,y,z are all set to zero. It is the responsibility of the calling program to verify that the returned coordinates are nonzero.

BUGS

Absmov does not know anything about the geometry of plate boundaries. To include such information would require considerable code, and seriously impair performance. Since there is only a limited number of sites, it is much more efficient to associate a plate name with each site in the input.

```
c
c     Subroutine absmov: compute new site position x,y,z at time t
c      from old site position x0,y0,z0 at time t0 using plate
c     motion model AM0-2
c     Author: J. Bernard Minster, Science Horizons, September,1985
c
           subroutine absmov (psit,t0,x0,y0,z0,t,x,y,z)
           real*8 t0,x0,y0,z0,t,x,y,z
           character*4 psit
c
c     specification of model AM0-2
c
           parameter (NPLT = 11)
```

```
      character*4 pnm(NPLT)
      real omx(NPLT),omy(NPLT),omz(NPLT)
      save pnm,omx,omy,omz
      data ( pnm(i),   omx(i),   omy(i),   omz(i), i=1,NPLT
     *      /'pcfc', -0.12276, -0.31163, -0.65537,
     *       'coco', -0.63726, -1.33142,  0.72556,
     *       'nazc', -0.09086, -0.53281,  0.63061,
     *       'carb', -0.02787, -0.05661,  0.10780,
     *       'soam', -0.05604, -0.10672, -0.08642,
     *       'anta', -0.05286, -0.09492,  0.21570,
     *       'indi',  0.48372,  0.25011,  0.43132,
     *       'afrc',  0.05660, -0.19249,  0.24061,
     *       'arab',  0.27885, -0.16744,  0.37359,
     *       'eura', -0.03071, -0.15865,  0.19605,
     *       'noam',  0.03299, -0.22828, -0.01427/
c
c    First initialize things properly
c
      ipsit = -1
      x = 0.0d00
      y = 0.0d00
      z = 0.0d00
c
c    Then look-up plate in the list
c    If plate name is blank, use the default reference plate
c
      do 20 i = 1,NPLT
20    if ( psit .eq. pnm(i) ) ipsit = i
c
c    If plate name is not recognized, return the new position as
c    zero
c
      if ( ipsit .eq. -1 ) return
c
c    Now perform the conversion form degree/My to radians/year.
c
      orx = omx(ipsit) * 1.7453292e-08
      ory = omy(ipsit) * 1.7453292e-08
      orz = omz(ipsit) * 1.7453292e-08
c
c    Finally compute the new coordinates at time t from the old
c    coordinates at time t0.
c
      x = x0 + ( ory*z0 - orz*y0 ) * ( t-t0 )
      y = y0 + ( orz*x0 - orx*z0 ) * ( t-t0 )
      z = z0 + ( orx*y0 - ory*x0 ) * ( t-t0 )
c
      return
      end
```

CHAPTER 10 TROPOSPHERIC MODEL

SATELLITE LASER RANGING

The formulation of Marini and Murray (1973) is commonly used in laser ranging. The formula has been tested by comparison with ray-tracing radiosonde profiles.

The correction to one-way range is

$$\Delta R = \frac{f(\lambda)}{f(\phi,H)} \cdot \frac{A+B}{\sin E + \dfrac{B/(A+B)}{\sin E + 0.01}}, \tag{1}$$

where

$$A = 0.002357P_0 + 0.000141e_0, \tag{2}$$

$$B = (1.084 \times 10^{-8})P_0\, T_0\, K + (4.734 \times 10^{-8})\, \frac{P_0^2}{T_0}\, \frac{2}{(3-1/K)}, \tag{3}$$

$$K = 1.163 - 0.00968 \cos 2\phi - 0.00104\, T_0 + 0.00001435P_0, \tag{4}$$

where

ΔR = range correction (meters),
E = true elevation of satellite,
P_0 = atmospheric pressure at the laser site (millibars),
T_0 = atmospheric temperature at the laser site (degrees Kelvin),
e_0 = water vapor pressure at the laser site (millibars),
$f(\lambda)$ = laser frequency parameter (λ = wavelength in micrometers),
$f(\phi,H)$ = laser site function,

Additional definitions of these parameters are available. The water vapor pressure, e_0, can be calculated from a relative humidity measurement, $R_h(\%)$ by

$$e_0 = \frac{R_h}{100} \times 6.11 \times 10^{\frac{7.5(T_0 - 273.15)}{237.3 + (T_0 - 273.15)}}.$$

The laser frequency parameter, $f(\lambda)$, is

$$f(\lambda) \equiv 0.9650 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4}.$$

$f(\lambda) = 1$ for a ruby laser, i. e. $f(0.6943) = 1$.

The laser site function is

$$f(\phi,H) = 1-0.0026 \cos 2\phi - 0.00031\,H,$$

where ϕ is the latitude and H is the geodetic height (km).

VERY LONG BASELINE INTERFEROMETRY

The most serious problem in practical atmospheric modelling is that of unmeasured atmospheric parameters. The differences between mathematical models are often less than the errors which would be introduced by the character and distribution of the wet component and breakdowns in azimuthal symmetry. For this reason, it is customary in the data reduction to determine the zenith atmospheric delay as a parameter and use models only for the mapping function which is the ratio of delay at a given zenith angle to the zenith delay. Accordingly, the IERS standard applies only to the mapping function which is the ratio of delay at a given zenith angle to the zenith delay.

Some standard models are available: CFA2.2 (Davis, et al., 1985), Chao (1974), Saastamoinen (1972), Black (1984), Marini (1972), Hopfield (1969), Yionoulis (1970), Goldfinger (1980), Matsakis, et al., (1986), Baby, et al. (1988), and Lanyi (1984). The reader should be aware of typographical errors in the published versions of the last three works cited. The models differ in their allowance for Earth curvature, atmospheric boundary structure, scale heights, and bending. Of these, the model which attempts to address all these aspects, particularly bending, in the most complete manner, is that of Lanyi. Some of the other models can be duplicated by dropping terms from the Lanyi model. Its abundance of adjustable parameters could prove useful for experimental applications. It is recommended that the lapse rate and the wet scale height parameter be adjusted for site dependence and seasonal variation (see Askne and Nordius, 1987). As pointed out by Davis, et al. (1985), the mapping function is considerably less sensitive than the zenith delay to the wet component.

There is some discrepancy in the reported literature concerning the numerical values of the refractivity coefficients for the wet delay. Measurements of the refractivity at radio wavelengths at different temperatures have been fit to a linear slope (in 1/T) by Boudouris (1963) and Birnbaum and Chatterjee (1952). Thayer (1974), using the same data, extrapolated values from the optical to derive slightly different values (Table 10.1) that were still within the measurement errors of the previous authors. The theoretical basis of this extrapolation was criticized by Hill, et al. (1982). The coefficients reported by Birnbaum and Chatterjee are actually a weighted average of their data and those of three other works, going back to 1935. Two works of these are based entirely upon data above 100° C; the coefficients of Boudouris are intermediate between the remaining two measurements. Within the range of atmospheric temperature variations, all three sets of coefficients are consistent with the data (Table 10.1) and the choice has little effect on the mapping function.

Table 10.1. Values of refractivity coefficients K2 and K3 for radio frequencies.

	K2	(error)	K3	(error)
Birnbaum and Chatterjee	71.40	(5.8)	3.747×10^5	(.03)
Boudouris	72.00	(10.5)	3.754×10^5	(.03)
Thayer	64.79	(.08)	3.776×10^5	(.004)

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CHAPTER 11 TIDAL VARIATIONS IN UT1

To properly compare UT1 results obtained at different epochs, the high frequency tidal effects must be accounted for. Thus, a standard expression for the tidal variations in UT1 is required. The periodic variations in UT1 due to tidal deformation of the polar moment of inertia have been rederived (Yoder, et al., 1981) including the tidal deformation of the Earth with a decoupled core. This model leads to effective Love numbers that differ from the bulk value of 0.301 because of the oceans and the fluid core which give rise to different theoretical values of the ratio k/C for the fortnightly and monthly terms. However, Yoder, et al., recommend the value of 0.94 for k/C for both cases. Periodic terms in UT1 are given in Table 11.1 and are discussed in Explanatory Supplement to the IERS Bulletins A and B, March 1989 as well as the Annual Reports of the IERS. Table 11.1 includes terms due to zonal tides, with periods up to 35 days, with k/C = 0.94. Table 11.2 lists the remainder of the tidal terms (with periods greater than 35 days).

UT1R, ΔR, and ωR represent the regularized forms of UT1 (including periods up to 35 days), of the duration of the day Δ, and of the angular velocity of the Earth, ω. The units are 10^{-4} s for UT, 10^{-5} s for Δ, and 10^{-14} rad/s for ω.

It should be mentioned that the oceanic tides with short periods cause variations in UT1 which are only partially represented by the model given below. According to Brosche, et al., (1989) the contribution of the oceanic tides should be split into a part which is in phase with the solid Earth tides and into an out-of-phase part. The discrepency between the model of Yoder, et al., (which already contains a general influence of the oceans) and the model of Brosche, et al., amounts to a maximum of ± 0.08 msec for the fortnightly and monthly terms. The influence of the ocean tides with periods less than or equal to 24^h is at the level of ± 0.1 msec.

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Brosche, P., Seiler, U., Suendermann, J., and Wuensch, J., 1989, "Periodic Changes in Earth's Rotation due to Oceanic Tides," paper accepted by Astronomy and Astrophysics.

Yoder, C. F., Williams, J. G., Parke, M. E., 1981, "Tidal Variations of Earth Rotation," J. Geophys. Res., **86**, pp. 881-891.

Table 11.1. Zonal Tide Terms With Periods Up to 35 Days.

ARGUMENT*					PERIOD	UT1-UT1R	Δ-ΔR	ω-ωR
						Coefficient of	Coefficient of	
l	l'	F	D	Ω	Days	Sin (Argument)	Cos (Argument)	
1	0	2	2	2	5.64	-0.02	0.3	-0.2
2	0	2	0	1	6.85	-0.04	0.4	-0.3
2	0	2	0	2	6.86	-0.10	0.9	-0.8
0	0	2	2	1	7.09	-0.05	0.4	-0.4
0	0	2	2	2	7.10	-0.12	1.1	-0.9
1	0	2	0	0	9.11	-0.04	0.3	-0.2
1	0	2	0	1	9.12	-0.41	2.8	-2.4
1	0	2	0	2	9.13	-0.99	6.8	-5.8
3	0	0	0	0	9.18	-0.02	0.1	-0.1
-1	0	2	2	1	9.54	-0.08	0.5	-0.5
-1	0	2	2	2	9.56	-0.20	1.3	-1.1
1	0	0	2	0	9.61	-0.08	0.5	-0.4
2	0	2	-2	2	12.81	0.02	-0.1	0.1
0	1	2	0	2	13.17	0.03	-0.1	0.1
0	0	2	0	0	13.61	-0.30	1.4	-1.2
0	0	2	0	1	13.63	-3.21	14.8	-12.5
0	0	2	0	2	13.66	-7.76	35.7	-30.1
2	0	0	0	-1	13.75	0.02	-0.1	0.1
2	0	0	0	0	13.78	-0.34	1.5	-1.3
2	0	0	0	1	13.81	0.02	-0.1	0.1
0	-1	2	0	2	14.19	-0.02	0.1	-0.1
0	0	0	2	-1	14.73	0.05	-0.2	0.2
0	0	0	2	0	14.77	-0.73	3.1	-2.6
0	0	0	2	1	14.80	-0.05	0.2	-0.2
0	-1	0	2	0	15.39	-0.05	0.2	-0.2
1	0	2	-2	1	23.86	0.05	-0.1	0.1
1	0	2	-2	2	23.94	0.10	-0.3	0.2
1	1	0	0	0	25.62	0.04	-0.1	0.1
-1	0	2	0	0	26.88	0.05	-0.1	0.1
-1	0	2	0	1	26.98	0.18	-0.4	0.3
-1	0	2	0	2	27.09	0.44	-1.0	0.9
1	0	0	0	-1	27.44	0.53	-1.2	1.0
1	0	0	0	0	27.56	-8.26	18.8	-15.9
1	0	0	0	1	27.67	0.54	-1.2	1.0
0	0	0	1	0	29.53	0.05	-0.1	0.1
1	-1	0	0	0	29.80	-0.06	0.1	-0.1
-1	0	0	2	-1	31.66	0.12	-0.2	0.2
-1	0	0	2	0	31.81	-1.82	3.6	-3.0
-1	0	0	2	1	31.96	0.13	-0.3	0.2
1	0	-2	2	-1	32.61	0.02	0.0	0.0
-1	-1	0	2	0	34.85	-0.09	0.2	-0.1

* l = 134°96 + 13°064993(MJD-51544.5) Mean Anomaly of the Moon
l' = 357°53 + 0°985600(MJD-51544.5) Mean Anomaly of the Sun
F = 93°27 + 13°229350(MJD-51544.5) L-Ω: L: Mean Longitude of the Moon
D = 297°85 + 12°190749(MJD-51544.5) Mean Elongation of the Moon from the Sun
Ω = 125°04 - 0°052954(MJD-51544.5) Mean Longitude of the Ascending Node of the Moon

Table 11.2. Zonal Tide Terms With Periods Greater than 35 Days.

ARGUMENT*					PERIOD	UT1-UT1R	Δ-ΔR	ω-ωR
						Coefficient of	Coefficient of	
l	l'	F	D	Ω	Days	Sin (Argument)	Cos (Argument)	
0	2	2	-2	2	91.31	-0.06	0.0	0.0
0	1	2	-2	1	119.61	0.03	0.0	0.0
0	1	2	-2	2	121.75	-1.88	1.0	-0.8
0	0	2	-2	0	173.31	0.25	-0.1	0.1
0	0	2	-2	1	177.84	1.17	-0.4	0.3
0	0	2	-2	2	182.62	-48.25	16.6	-14.0
0	2	0	0	0	182.63	-0.19	0.1	-0.1
2	0	0	-2	-1	199.84	0.05	0.0	0.0
2	0	0	-2	0	205.89	-0.55	0.2	-0.1
2	0	0	-2	1	212.32	0.04	0.0	0.0
0	-1	2	-2	1	346.60	-0.05	0.0	0.0
0	1	0	0	-1	346.64	0.09	0.0	0.0
0	-1	2	-2	2	365.22	0.83	-0.1	0.1
0	1	0	0	0	365.26	-15.36	2.6	-2.2
0	1	0	0	1	386.00	-0.14	0.0	0.0
1	0	0	-1	0	411.78	0.03	0.0	0.0
2	0	-2	0	0	1095.17	-0.14	0.0	0.0
-2	0	2	0	1	1305.47	0.42	0.0	0.0
-1	1	0	1	0	3232.85	0.04	0.0	0.0
0	0	0	0	2	3399.18	7.90	0.1	-0.1
0	0	0	0	1	6790.36	-1617.27	-14.9	12.6

* l = 134°96 + 13°064993(MJD-51544.5) Mean Anomaly of the Moon
l' = 357°53 + 0°985600(MJD-51544.5) Mean Anomaly of the Sun
F = 93°27 + 13°229350(MJD-51544.5) L-Ω: L: Mean Longitude of the Moon
D = 297°85 + 12°190749(MJD-51544.5) Mean Elongation of the Moon from the Sun
Ω = 125°04 - 0°052954(MJD-51544.5) Mean Longitude of the Ascending Node of the Moon

CHAPTER 12 LUNAR AND PLANETARY EPHEMERIDES

The planetary and lunar ephemerides recommended for the IERS standards are the JPL Development Ephemeris DE200 and the Lunar Ephemeris LE200. These have formed the basis for the Astronomical Almanac since 1984. DE200/LE200 should be used in the analysis of SLR and VLBI. However, in LLR analysis, parameters of the Earth-Moon system should be fit or a more recent lunar ephemeris should be used.

The ephemerides, DE200/LE200, were created from the 1950-based ephemerides, DE118/LE62. Their orientation with respect to the dynamical equinox of J2000.0 was described by Standish (1982).

Associated with the ephemerides is the set of astronomical constants used in the creation of the ephemerides which are listed in Table 12.1. Many of these values do not agree exactly with those of the IAU 1976. The difference, listed in Table 12.2, has been necessary in order to provide a best fit of the ephemerides to the observational data. These constants are provided directly with the ephemerides and should be considered to be an integral part of them.

Table 12.1. JPL Planetary and Lunar Ephemerides DE200/LE200

Scale (km/au)	149597870.66
Speed of light (km/sec)	299792.458
Obliquity of the ecliptic	23° 26' 21."4119
Earth-Moon mass ratio	81.300587
GM(Sun)/GM(Mercury)	6023600
GM(Sun)/GM(Venus)	408523.5
GM(Sun)/GM(Mars)	3098710
GM(Sun)/GM(Jupiter)	1047.350
GM(Sun)/GM(Saturn)	3498.0
GM(Sun)/GM(Uranus)	22960
GM(Sun)/GM(Neptune)	19314
GM(Sun)/GM(Pluto)	130000000
GM(Sun)/GM(Earth+Moon)	328900.55

Table 12.2. IAU Values Which Differ From Those of DE200/LE200

Scale (sec/au)	449.004782
(149597870.15...km/au)	
Moon-Earth mass ratio	0.0123002
(E/M = 81.30068...)	
Obliquity of the ecliptic	23° 26' 21".448
GM(Sun)/GM(Jupiter)	1047.355
GM(Sun)/GM(Saturn)	3498.5
GM(Sun)/GM(Uranus)	22869
GM(Sun)/GM(Pluto)	3000000
GM(Sun)/GM(Earth+Moon)	328900.5

REFERENCES

Standish, E. M., 1982, "Orientation of the JPL Ephemerides, DE200/LE200, to the Dynamical Equinox of J2000," Astron. Astrophys., **114**, pp. 297-302.

CHAPTER 13 GEOPOTENTIAL

The recommended geopotential field is the GEM-T1 model given in the following table.

The $GM_\oplus$ and a_e values reported with GEM-T1 (398600.436 km^3/s^2 and 6378137 m) should be used as scale parameters with the geopotential coefficients. The recommended $GM_\oplus$ = 398600.44 should be used with the two-body term. Although the GEM-T1 is given with terms through degree and order 36, only terms through degree and order twenty are required for Lageos.

Values for the C_{21} and S_{21} coefficients are not included in the GEM-T1 model (they were constrained to be zero in the solution), and so they should be handled separately.

The C_{21} and S_{21} coefficients describe the position of the Earth's figure axis. When averaged over many years, the figure axis should closely coincide with the observed position of the rotation pole averaged over the same time period. Any differences between the mean figure and mean rotation pole averaged would be due to long-period fluid motions in the atmosphere, oceans, or Earth's fluid core (Wahr, 1987, 1990). At present, there is no independent evidence that such motions are important. So, it is recommended that the mean values used for C_{21} and S_{21} give a mean figure axis that corresponds to the mean pole position of the Chapter 3 Terrestrial Reference Frame.

The BIH Circular D pole positions from 1982 through 1988 are consistent with the IERS Reference Pole to within ± 0".005 corresponding to an uncertainty of $\pm 0.01 \times 10^{-9}$ in $\bar{C}_{21}$(IERS) and $\bar{S}_{21}$ (IERS).

This choice for C_{21} and S_{21} is realized as follows. First, to use the geopotential coefficients to solve for a satellite orbit, it is necessary to rotate from the Earth-fixed frame, where the coefficients are pertinent, to an inertial frame, where the satellite motion is computed. This transformation between frames should include polar motion. We assume the polar motion parameters used are relative to the IERS Reference Pole. Then, if $C_{21} = S_{21} = 0$ were used, the assumed mean figure axis would coincide with the IERS Reference Pole.

If $\bar{x}$ and $\bar{y}$ are the angular displacements of the Chapter 3 Terrestrial Reference Frame relative to the IERS Reference Pole then the values

$$C_{21} = \bar{x}C_{20},$$

$$S_{21} = -\bar{y}C_{20}$$

(Lambeck, 1970) should be added to the geopotential model, so that the mean figure axis coincides with the Chapter 3 pole. This gives normalized coefficients of

$$\bar{C}_{21}(\text{IERS}) = -0.17 \times 10^{-8},$$

$$\bar{S}_{21}(\text{IERS}) = 1.19 \times 10^{-9}.$$

For consistency with the desired Terrestrial Reference Frame, the $\bar{C}_{21}$(IERS) and $\bar{S}_{21}$(IERS) are recommended for use in place of $\bar{C}_{21}$(GEM-T1) and $\bar{S}_{21}$(GEM-T1).

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Lambeck, K., 1971, "Determination of the Earth's Pole of Rotation from Laser Range Observations to Satellites," Bulletin Geodesique, **101**, pp. 263-280.

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Wahr, J., 1987, "The Earth's C_{21} and S_{21} gravity coefficients and the rotation of the core," Geophys. J. R. Astr. Soc., **88**, pp. 265-276. Corrections can be found in Geophys. J. Int., in press.

Wahr, J., 1990, in press.

GEM-T1 NORMALIZED COEFFICIENTS

($\times 10^6$)

ZONALS

INDEX N M	VALUE	INDEX N M	VALUE	INDEX N M	VALUE	INDEX N M	VALUE	INDEX N M	VALUE
* 2 0	-484.1649906	3 0	0.9572357	4 0	0.5387322	5 0	0.0687802	6 0	-0.1481004
7 0	0.0905337	8 0	0.0459023	9 0	0.0283764	10 0	0.0572211	11 0	-0.0512619
12 0	0.0320806	13 0	0.0422319	14 0	-0.0197327	15 0	0.0018731	16 0	-0.0093772
17 0	0.0203968	18 0	0.0112912	19 0	-0.0046084	20 0	0.0153150	21 0	0.0097754
22 0	-0.0048440	23 0	-0.0241260	24 0	-0.0009556	25 0	0.0068867	26 0	0.0018391
27 0	0.0041234	28 0	-0.0058541	29 0	-0.0039091	30 0	-0.0002749	31 0	0.0051154
32 0	0.0000819	33 0	0.0022286	34 0	-0.0024803	35 0	0.0012731	36 0	0.0007396

SECTORIALS AND TESSERALS

INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S
** 3	1	2.0297737	0.2495946	4	1	-0.5334272	-0.4751189	5	1	-0.0589503	-0.0955435
6	1	-0.0813751	0.0238900	7	1	0.2770971	0.0978177	8	1	0.0288561	0.0547223
9	1	0.1480477	0.0245251	10	1	0.0769655	-0.1381110	11	1	0.0095019	-0.0278111
12	1	-0.0492610	-0.0496520	13	1	-0.0540617	0.0434555	14	1	-0.0187462	0.0232244
15	1	0.0082868	0.0142124	16	1	0.0317099	0.0173493	17	1	-0.0309381	-0.0268459
18	1	-0.0002253	-0.0456055	19	1	-0.0115942	0.0053764	20	1	0.0145119	-0.0212711
21	1	-0.0153942	0.0417459	22	1	0.0083946	-0.0147250	23	1	0.0008657	0.0145970
24	1	0.0081178	-0.0291987	25	1	0.0037145	0.0043498	26	1	0.0049741	-0.0172518
27	1	0.0005230	0.0066113	28	1	0.0065294	-0.0100251	29	1	0.0034699	0.0024176
30	1	-0.0016171	-0.0090886	31	1	0.0051782	0.0023829	32	1	-0.0091529	-0.0092451
33	1	0.0012672	0.0021258	34	1	-0.0015608	-0.0091011	35	1	-0.0019069	0.0020110
36	1	0.0028774	-0.0058408	2	2	2.4389280	-1.3998397	3	2	0.9035491	-0.6204198
4	2	0.3470021	0.6640304	5	2	0.6557902	-0.3234056	6	2	0.0516096	-0.3749956
7	2	0.3177108	0.0916083	8	2	0.0703801	0.0684494	9	2	0.0311365	-0.0323882
10	2	-0.0805212	-0.0513356	11	2	0.0090541	-0.0992414	12	2	0.0076400	0.0349183
13	2	0.0534361	-0.0575844	14	2	-0.0348122	-0.0060681	15	2	-0.0216258	-0.0364425
16	2	-0.0156437	0.0245431	17	2	-0.0057800	0.0171247	18	2	0.0084083	0.0168428
19	2	0.0084369	-0.0104744	20	2	0.0198772	0.0032259	21	2	0.0009874	-0.0026067
22	2	-0.0142925	0.0020958	23	2	-0.0005313	-0.0017780	24	2	-0.0058515	0.0052022
25	2	0.0037220	0.0052068	26	2	-0.0052887	0.0002522	27	2	0.0102174	-0.0028223
28	2	-0.0084276	-0.0115524	29	2	0.0094632	-0.0043311	30	2	-0.0040515	-0.0053645
31	2	0.0066008	0.0008362	32	2	0.0018649	0.0043749	33	2	-0.0010231	0.0009548
34	2	0.0035817	0.0051783	35	2	-0.0025529	0.0010277	36	2	0.0001790	0.0012922
3	3	0.7209866	1.4131694	4	3	0.9909779	-0.2006215	5	3	-0.4482036	-0.2151363
6	3	0.0619709	0.0046430	7	3	0.2507429	-0.2091639	8	3	-0.0199664	-0.0869367
9	3	-0.1553742	-0.0840158	10	3	-0.0013119	-0.1614824	11	3	-0.0288895	-0.1324963
12	3	0.0324198	0.0179438	13	3	-0.0140259	0.0836615	14	3	0.0369311	0.0224222
15	3	0.0446271	0.0265447	16	3	-0.0320841	-0.0450272	17	3	0.0101214	0.0098939
18	3	-0.0010020	-0.0070483	19	3	0.0014391	0.0141955	20	3	0.0082691	0.0137151
21	3	0.0019941	0.0226923	22	3	0.0067253	-0.0080913	23	3	-0.0045642	-0.0119458

* $\bar{C}_{20}$ does not include the zero frequency term; see Chapter 6 (Eq. 5) for the adjusted value.

** $\bar{C}_{21}$ and $\bar{S}_{21}$ should be the IERS values; see this chapter for recommended values.

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
24	3	0.0069148	-0.0105925	25	3	-0.0032642	-0.0031087	26	3	-0.0002621	-0.0037215
27	3	-0.0051035	-0.0018585	28	3	-0.0002646	0.0011662	29	3	-0.0043864	-0.0017609
30	3	-0.0016075	0.0014770	31	3	-0.0018668	-0.0040769	32	3	-0.0006663	0.0029992
33	3	-0.0017072	-0.0028776	34	3	-0.0005145	0.0022065	35	3	0.0006733	0.0006861
36	3	-0.0008129	-0.0013852	4	4	-0.1900348	0.3084595	5	4	-0.2948236	0.0524087
6	4	-0.0927975	-0.4733069	7	4	-0.2737404	-0.1220207	8	4	-0.2460639	0.0677453
9	4	-0.0128303	0.0232637	10	4	-0.0973123	-0.0693825	11	4	-0.0332108	-0.0700036
12	4	-0.0653020	-0.0030125	13	4	-0.0088182	-0.0003732	14	4	-0.0088329	0.0018783
15	4	-0.0443760	0.0126416	16	4	0.0365123	0.0438559	17	4	0.0125878	0.0312242
18	4	0.0434167	0.0060924	19	4	0.0025699	0.0076747	20	4	-0.0017951	0.0008281
21	4	-0.0002550	0.0069894	22	4	-0.0094462	0.0167100	23	4	-0.0100336	-0.0016696
24	4	0.0060580	0.0181145	25	4	0.0063503	-0.0015335	26	4	0.0053222	0.0048968
27	4	0.0029164	-0.0006401	28	4	0.0028786	-0.0024855	29	4	-0.0063641	0.0007289
30	4	-0.0021102	-0.0039250	31	4	-0.0049509	-0.0016392	32	4	0.0020179	-0.0033825
33	4	-0.0002362	0.0003938	34	4	0.0028502	-0.0018773	35	4	0.0027442	0.0014864
36	4	0.0001181	-0.0058408	5	5	0.1777563	-0.6660281	6	5	-0.2657650	-0.5377472
7	5	0.0034750	0.0196519	8	5	-0.0249335	0.0853003	9	5	-0.0141122	-0.0600627
10	5	-0.0504401	-0.0438269	11	5	0.0459086	0.0552848	12	5	0.0306040	-0.0014745
13	5	0.0596176	0.0574460	14	5	0.0227952	-0.0116078	15	5	0.0160742	0.0108864
16	5	-0.0077242	-0.0016773	17	5	-0.0111472	-0.0056031	18	5	0.0017426	0.0211276
19	5	-0.0024457	0.0173789	20	5	-0.0104182	0.0003034	21	5	0.0177593	-0.0158911
22	5	-0.0046335	-0.0001257	23	5	0.0019941	-0.0079407	24	5	-0.0140838	-0.0079805
25	5	-0.0024585	-0.0023514	26	5	0.0043007	0.0106472	27	5	-0.0015884	0.0037103
28	5	0.0029222	-0.0002361	29	5	0.0034464	0.0035422	30	5	0.0033916	0.0006983
31	5	0.0009096	0.0014649	32	5	-0.0000480	-0.0027949	33	5	-0.0005132	0.0032353
34	5	-0.0012952	0.0000401	35	5	-0.0002351	-0.0007374	36	5	-0.0011798	0.0003403
6	6	0.0090593	-0.2363344	7	6	-0.3578527	0.1509175	8	6	-0.0664178	0.3128323
9	6	0.0705263	0.2166285	10	6	-0.0347366	-0.0777189	11	6	0.0084723	0.0242910
12	6	0.0013881	0.0458322	13	6	-0.0223869	-0.0118360	14	6	-0.0031868	0.0065119
15	6	0.0272318	-0.0517077	16	6	0.0179949	-0.0267835	17	6	0.0002920	-0.0204166
18	6	0.0311991	-0.0085569	19	6	-0.0062545	0.0039169	20	6	0.0127607	0.0009662
21	6	0.0042146	-0.0083501	22	6	0.0146261	0.0024348	23	6	0.0099988	0.0049568
24	6	-0.0003055	-0.0006641	25	6	0.0059358	-0.0067430	26	6	0.0085388	0.0031968
27	6	0.0017746	-0.0021342	28	6	-0.0083296	0.0020029	29	6	-0.0002960	-0.0024663
30	6	-0.0032177	0.0041679	31	6	-0.0005622	0.0007667	32	6	-0.0038310	0.0002190
33	6	0.0013824	-0.0012542	34	6	0.0005808	-0.0003220	35	6	0.0008304	-0.0014752
36	6	-0.0005757	-0.0008988	7	7	0.0015976	0.0220013	8	7	0.0704248	0.0748626
9	7	-0.1186233	-0.1005510	10	7	0.0097468	-0.0042901	11	7	0.0096093	-0.0918891
12	7	-0.0126975	0.0348291	13	7	0.0035736	-0.0066171	14	7	0.0374843	-0.0043588
15	7	0.0667130	0.0114545	16	7	0.0030511	-0.0090737	17	7	0.0229520	-0.0119809
18	7	-0.0007957	0.0067159	19	7	0.0051443	-0.0016620	20	7	-0.0077913	0.0048958
21	7	-0.0122279	-0.0014058	22	7	0.0127538	0.0013042	23	7	-0.0023282	0.0026452
24	7	-0.0025113	0.0050662	25	7	0.0002632	0.0034596	26	7	0.0054524	0.0025831
27	7	0.0069279	-0.0028217	28	7	-0.0046459	-0.0015444	29	7	0.0011830	-0.0072940
30	7	-0.0001720	-0.0001291	31	7	0.0014817	-0.0017479	32	7	-0.0030832	0.0018611
33	7	-0.0000405	0.0017756	34	7	0.0023148	0.0001028	35	7	0.0000748	0.0015282
36	7	-0.0001662	-0.0004199	8	8	-0.1188827	0.1223320	9	8	0.1844954	-0.0018494
10	8	0.0437468	-0.0924808	11	8	-0.0063530	0.0225827	12	8	-0.0212177	0.0169046
13	8	-0.0122964	-0.0110925	14	8	-0.0329416	-0.0131814	15	8	-0.0406660	0.0247325
16	8	-0.0134376	0.0022804	17	8	0.0311564	0.0087750	18	8	0.0457191	0.0004305

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
19	8	0.0148626	-0.0113273	20	8	-0.0020109	-0.0012912	21	8	-0.0181008	0.0025208
22	8	-0.0098173	-0.0068267	23	8	0.0042209	-0.0067954	24	8	-0.0024330	0.0075574
25	8	0.0014256	-0.0041241	26	8	0.0030889	-0.0021407	27	8	-0.0041794	-0.0044059
28	8	-0.0005655	-0.0031778	29	8	-0.0064245	0.0025406	30	8	0.0029536	0.0005720
31	8	0.0000872	-0.0011845	32	8	0.0008818	0.0033226	33	8	-0.0000823	0.0015750
34	8	0.0007083	-0.0007860	35	8	0.0002679	0.0001772	36	8	-0.0010487	-0.0005378
9	9	-0.0555457	0.0975889	10	9	0.1281797	-0.0481860	11	9	-0.0387774	0.0402849
12	9	0.0469380	0.0132223	13	9	0.0203827	0.0457820	14	9	0.0371609	0.0179332
15	9	0.0134441	0.0410187	16	9	-0.0165750	-0.0509825	17	9	-0.0032015	-0.0343246
18	9	-0.0135216	0.0192446	19	9	0.0017566	0.0086593	20	9	0.0228121	0.0072350
21	9	0.0173205	-0.0093913	22	9	0.0125106	-0.0094816	23	9	-0.0040274	-0.0103916
24	9	-0.0038940	-0.0014302	25	9	-0.0060297	0.0098442	26	9	0.0025191	-0.0006592
27	9	0.0003990	0.0021553	28	9	0.0029414	-0.0030550	29	9	-0.0016394	0.0024790
30	9	0.0001665	-0.0039351	31	9	-0.0038941	-0.0017016	32	9	0.0019461	0.0007681
33	9	-0.0002939	0.0018691	34	9	0.0012660	0.0015093	35	9	0.0010826	-0.0020731
36	9	-0.0003170	0.0005484	10	10	0.0945596	-0.0201041	11	10	-0.0520582	-0.0176126
12	10	-0.0091273	0.0316782	13	10	0.0433028	-0.0380383	14	10	0.0369953	-0.0027966
15	10	0.0095928	0.0160812	16	10	-0.0104171	0.0066056	17	10	0.0021050	0.0201193
18	10	0.0090063	-0.0108619	19	10	-0.0353538	-0.0026556	20	10	-0.0224201	-0.0080926
21	10	0.0036543	0.0018356	22	10	0.0050062	0.0203830	23	10	0.0199758	-0.0037585
24	10	0.0173535	0.0092954	25	10	0.0056907	-0.0044669	26	10	-0.0048370	0.0016645
27	10	-0.0083100	0.0060112	28	10	-0.0072826	0.0012046	29	10	0.0000602	0.0060144
30	10	0.0012265	-0.0010402	31	10	0.0025178	-0.0037447	32	10	0.0008308	-0.0019507
33	10	0.0002341	-0.0009449	34	10	-0.0014879	-0.0000093	35	10	-0.0014313	-0.0008354
36	10	-0.0003570	0.0004671	11	11	0.0543322	-0.0547288	12	11	0.0054143	-0.0095228
13	11	-0.0401906	0.0055015	14	11	0.0080835	-0.0413614	15	11	0.0017171	0.0289322
16	11	0.0140156	-0.0064368	17	11	-0.0171108	0.0175019	18	11	-0.0127989	-0.0005971
19	11	0.0164804	0.0134748	20	11	0.0113787	-0.0239305	21	11	0.0092806	-0.0367834
22	11	-0.0093740	-0.0183775	23	11	0.0038490	0.0136794	24	11	0.0127396	0.0121180
25	11	0.0055793	-0.0012766	26	11	0.0032045	0.0050467	27	11	-0.0011883	-0.0030951
28	11	-0.0006214	-0.0008290	29	11	-0.0093226	0.0004548	30	11	-0.0016207	0.0050752
31	11	-0.0016631	0.0058521	32	11	-0.0024010	-0.0006463	33	11	0.0055568	-0.0004803
34	11	0.0012406	-0.0038325	35	11	0.0006785	-0.0036583	36	11	0.0005880	-0.0007308
12	12	-0.0035280	-0.0117964	13	12	-0.0280059	0.0864102	14	12	0.0089681	-0.0320668
15	12	-0.0283317	0.0124872	16	12	0.0208803	0.0057370	17	12	0.0342734	0.0172570
18	12	-0.0261819	-0.0165262	19	12	0.0032037	0.0043292	20	12	-0.0040581	0.0172980
21	12	0.0028236	0.0127066	22	12	0.0074377	-0.0078426	23	12	0.0215777	-0.0166812
24	12	0.0123406	-0.0095152	25	12	-0.0055425	0.0110100	26	12	-0.0196457	0.0054338
27	12	-0.0004228	-0.0017182	28	12	0.0004024	0.0024269	29	12	-0.0008339	-0.0049254
30	12	0.0037915	-0.0034240	31	12	0.0001491	0.0046159	32	12	-0.0017118	0.0041940
33	12	0.0052208	0.0041268	34	12	0.0004550	0.0024876	35	23	0.0014841	-0.0020619
36	12	-0.0002182	-0.0016068	13	13	-0.0615483	0.0682661	14	13	0.0315333	0.0446234
15	13	-0.0281051	-0.0049829	16	13	0.0130754	0.0006134	17	13	0.0169075	0.0201122
18	13	-0.0065815	-0.0351551	19	13	-0.0060894	-0.0291709	20	13	0.0266491	0.0048913
21	13	-0.0181694	0.0115969	22	13	-0.0169455	0.0178453	23	13	-0.0104578	-0.0075112
24	13	-0.0036235	-0.0003824	25	13	0.0073795	-0.0151883	26	13	0.0027230	0.0014151
27	13	-0.0059813	-0.0041287	28	13	0.0000983	0.0035308	29	13	-0.0011458	-0.0019753
30	13	0.0146742	-0.0000192	31	13	0.0056871	0.0013250	32	13	0.0072556	0.0002229
33	13	0.0036684	0.0067767	34	13	-0.0080803	0.0012768	35	13	-0.0011849	0.0044812
36	13	0.0007685	0.0037948	14	14	-0.0505657	-0.0063741	15	14	0.0061707	-0.0256132

INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S
16	14	-0.0191226	-0.0382895	17	14	-0.0133370	0.0117613	18	14	-0.0092828	-0.0109400
19	14	-0.0051227	-0.0126448	20	14	0.0103228	-0.0117620	21	14	0.0187760	0.0086994
22	14	0.0087280	0.0102407	23	14	0.0046108	-0.0032737	24	14	-0.0186436	0.0014570
25	14	-0.0219418	0.0132058	26	14	0.0039290	0.0056350	27	14	0.0119702	0.0066379
28	14	-0.0021064	-0.0065025	29	14	-0.0051525	0.0019409	30	14	-0.0000327	-0.0025522
31	14	-0.0072828	0.0012481	32	14	0.0046569	0.0069216	33	14	0.0092319	0.0025099
34	14	-0.0010431	-0.0002878	35	14	-0.0004836	-0.0001238	36	14	-0.0048384	-0.0040665
15	15	-0.0180948	-0.0080854	16	15	-0.0125321	-0.0322958	17	15	0.0049435	0.0057493
18	15	-0.0377619	-0.0198247	19	15	-0.0183164	-0.0127675	20	15	-0.0227306	-0.0004135
21	15	0.0166205	0.0149837	22	15	0.0279373	0.0031033	23	15	0.0177318	-0.0022813
24	15	0.0098097	-0.0135286	25	15	-0.0019899	-0.0022710	26	15	-0.0113797	0.0047010
27	15	-0.0043373	0.0001002	28	15	-0.0082106	0.0053751	29	15	-0.0012718	-0.0024918
30	15	0.0028146	-0.0092780	31	15	0.0004541	-0.0043732	32	15	0.0039107	-0.0049375
33	15	-0.0030055	0.0021726	34	15	0.0007643	0.0030005	35	15	0.0002609	0.0028304
36	15	-0.0018223	0.0018634	16	16	-0.0324114	-0.0043686	17	16	-0.0290683	0.0018848
18	16	0.0097880	0.0050024	19	16	-0.0199047	-0.0119326	20	16	-0.0106685	0.0016919
21	16	0.0087331	-0.0051553	22	16	0.0000892	-0.0049265	23	16	0.0049029	0.0117671
24	16	-0.0004908	0.0062766	25	16	0.0030419	-0.0127972	26	16	0.0058241	-0.0041510
27	16	0.0065849	-0.0041072	28	16	-0.0083073	-0.0076860	29	16	-0.0021980	-0.0055327
30	16	0.0006189	0.0056597	31	16	-0.0045141	0.0048013	32	16	0.0029187	0.0041010
33	16	-0.0003879	0.0019724	34	16	0.0011337	-0.0026578	35	16	0.0001314	-0.0013369
36	16	0.0013405	-0.0020055	17	17	-0.0383106	-0.0206234	18	17	0.0061142	0.0087663
19	17	0.0279459	-0.0108837	20	17	0.0042934	-0.0089776	21	17	-0.0067459	0.0008396
22	17	0.0138079	-0.0111258	23	17	-0.0072125	-0.0066031	24	17	-0.0084625	0.0018817
25	17	-0.0083083	0.0005500	26	17	-0.0048890	0.0082820	27	17	0.0055505	0.0015914
28	17	0.0045201	-0.0042606	29	17	0.0045708	-0.0027849	30	17	0.0010057	-0.0015438
31	17	-0.0059335	0.0025686	32	17	-0.0036602	0.0018909	33	17	-0.0021426	0.0030432
34	17	0.0003626	0.0025764	35	17	0.0033863	-0.0024391	36	17	0.0021679	-0.0008490
18	18	-0.0044492	-0.0050647	19	18	0.0216467	-0.0031131	20	18	0.0105771	0.0013024
21	18	0.0168304	-0.0065691	22	18	0.0070311	-0.0102955	23	18	-0.0019056	-0.0063023
24	18	0.0043072	-0.0050725	25	18	-0.0013004	-0.0106697	26	18	-0.0090148	0.0075516
27	18	-0.0051898	0.0059394	28	18	0.0003645	-0.0008295	29	18	-0.0020095	-0.0001378
30	18	0.0003035	0.0008530	31	18	0.0025645	0.0008491	32	18	0.0022105	-0.0014795
33	18	0.0008702	-0.0012393	34	18	-0.0027232	-0.0000869	35	18	0.0015003	-0.0005937
36	18	0.0000693	0.0005306	19	19	0.0064638	0.0104244	20	19	-0.0070980	0.0084586
21	19	-0.0209515	0.0158790	22	19	0.0066210	-0.0046952	23	19	-0.0086827	0.0074916
24	19	0.0005274	-0.0150179	25	19	0.0091802	0.0021317	26	19	0.0016260	0.0007199
27	19	0.0009254	-0.0062305	28	19	0.0044182	0.0138003	29	19	-0.0021920	0.0015059
30	19	-0.0056266	-0.0026384	31	19	0.0020172	0.0028162	32	19	0.0030881	-0.0015146
33	19	0.0016796	0.0002369	34	19	0.0005953	-0.0010032	35	19	-0.0029031	0.0005932
36	19	-0.0002562	-0.0000695	20	20	0.0017085	-0.0135051	21	20	-0.0190411	0.0185361
22	20	-0.0133152	0.0147789	23	20	0.0172248	-0.0090475	24	20	-0.0060619	-0.0003298
25	20	-0.0037315	-0.0066217	26	20	0.0094830	-0.0109488	27	20	0.0029525	0.0030106
28	20	-0.0009805	0.0011271	29	20	-0.0047996	0.0031625	30	20	-0.0000276	0.0035666
31	20	0.0018947	0.0005595	32	20	-0.0015881	0.0014199	33	20	0.0020822	-0.0007525
34	20	0.0008917	-0.0003975	35	20	-0.0007745	-0.0008763	36	20	-0.0009128	-0.0008661
21	21	0.0024775	-0.0068510	22	21	-0.0132244	0.0075983	23	21	0.0108195	0.0076428
24	21	0.0105744	0.0011197	25	21	0.0053968	0.0031404	26	21	-0.0003907	-0.0024147
27	21	0.0020712	-0.0045432	28	21	0.0024900	0.0002697	29	21	-0.0093986	-0.0059494
30	21	-0.0074518	-0.0030982	31	21	0.0023358	0.0035767	32	21	0.0011690	0.0058848

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
33	21	0.0007319	-0.0008394	34	21	0.0013743	-0.0006590	35	21	0.0013843	0.0027357
36	21	0.0007411	-0.0021175	22	22	-0.0014623	0.0047182	23	22	-0.0009034	-0.0021445
24	22	-0.0017322	-0.0013033	25	22	-0.0018741	-0.0017578	26	22	0.0109119	0.0091610
27	22	-0.0001430	0.0029341	28	22	-0.0048465	0.0005118	29	22	0.0096562	0.0044395
30	22	0.0032458	-0.0055410	31	22	-0.0062664	-0.0057325	32	22	-0.0046595	0.0007455
33	22	-0.0040992	-0.0010974	34	22	0.0008294	0.0004221	35	22	0.0000159	0.0033003
36	22	0.0005732	0.0006864	23	23	0.0008446	0.0002030	24	23	-0.0021435	-0.0090055
25	23	0.0045743	-0.0024633	26	23	0.0023537	0.0089516	27	23	-0.0053881	-0.0027372
28	23	-0.0026403	0.0063891	29	23	-0.0050118	-0.0000780	30	23	-0.0015833	-0.0053377
31	23	0.0095047	0.0056544	32	23	0.0038573	0.0004759	33	23	-0.0006277	-0.0043975
34	23	-0.0009300	-0.0021403	35	23	-0.0023783	-0.0015551	36	23	-0.0012168	-0.0005518
24	24	0.0023438	-0.0012129	25	24	0.0036065	-0.0038584	26	24	-0.0013736	0.0121837
27	24	-0.0019361	0.0026238	28	24	0.0068762	-0.0150841	29	24	-0.0025342	0.0037024
30	24	-0.0025120	-0.0000377	31	24	-0.0038298	-0.0019101	32	24	-0.0065905	0.0053251
33	24	0.0039903	-0.0004821	34	24	0.0067267	0.0008511	35	24	0.0025277	0.0021953
36	24	0.0006589	-0.0014201	25	25	0.0049455	0.0040141	26	25	-0.0039876	0.0082488
27	25	0.0118014	0.0031453	28	25	0.0011295	-0.0048096	29	25	0.0083057	0.0036286
30	25	0.0088475	-0.0056062	31	25	-0.0077449	-0.0002757	32	25	-0.0131730	0.0077043
33	25	-0.0012628	-0.0048224	34	25	0.0062516	-0.0082080	35	25	-0.0033892	0.0015231
36	25	0.0000270	0.0086477	26	26	0.0034281	-0.0042690	27	26	-0.0050080	0.0040035
28	26	0.0034421	0.0016811	29	26	0.0062584	-0.0036878	30	26	-0.0032087	0.0081399
31	26	-0.0046923	0.0003639	32	26	-0.0010511	-0.0015337	33	26	0.0081343	0.0055386
34	26	0.0010816	-0.0090061	35	26	-0.0143438	-0.0001412	36	26	0.0084469	0.0110849
27	27	0.0068945	0.0034538	28	27	-0.0099247	0.0013336	29	27	-0.0074725	-0.0021507
30	27	-0.0019206	0.0078152	31	27	0.0069960	0.0122525	32	27	-0.0030949	-0.0030221
33	27	-0.0103323	-0.0021297	34	27	0.0068974	-0.0005442	35	27	0.0027787	-0.0191290
36	27	-0.0101912	0.0042305	28	28	0.0067689	0.0019493	29	28	0.0103226	-0.0019625
30	28	-0.0089658	-0.0051589	31	28	0.0003428	0.0017261	32	28	0.0015717	0.0023530
33	28	-0.0108851	0.0018952	34	28	0.0045502	-0.0081979	35	28	-0.0108936	-0.0233542
36	28	0.0069480	0.0056619	29	29	0.0086336	0.0031601	30	29	0.0048227	0.0001240
31	29	-0.0054215	-0.0059502	32	29	-0.0033428	0.0025024	33	29	-0.0213209	0.0001932
34	29	-0.0038113	-0.0044842	35	29	-0.0039791	0.0005664	36	29	-0.0013415	-0.0024474
30	30	-0.0015075	-0.0004221	31	30	-0.0024527	0.0084230	32	30	0.0082893	0.0016713
33	30	0.0025217	-0.0134935	34	30	-0.0061161	0.0000268	35	30	0.0037128	-0.0028852
36	30	-0.0015551	-0.0020261	31	31	-0.0002319	-0.0000737	32	31	-0.0007616	-0.0027125
33	31	0.0001745	0.0011566	34	31	0.0023534	0.0022983	35	31	0.0013305	0.0010339
36	31	-0.0035516	0.0013604	32	32	0.0007277	0.0005068	33	32	0.0024100	-0.0000219
34	32	-0.0008166	-0.0011610	35	32	-0.0039791	0.0005664	36	32	-0.0008804	-0.0003000
33	33	-0.0002676	-0.0003603	34	33	0.0010234	0.0013813	35	33	-0.0002881	0.0016655
36	33	-0.0023972	-0.0026170	34	34	-0.0002093	-0.0006039	35	34	-0.0005168	0.0001237
36	34	0.0008324	0.0017560	35	35	0.0000938	-0.0001450	36	35	0.0002152	-0.0007347
36	36	0.0001484	0.0004197								

CHAPTER 14 RADIATION PRESSURE REFLECTANCE MODEL

For a near-Earth satellite the solar radiation pressure acceleration, $\ddot{\bar{r}}$ is given by:

$$\ddot{\bar{r}} = \kappa \left[\frac{A}{R}\right]^2 C_R \frac{a}{m} \frac{\vec{R}}{R},$$

where

κ = 4.560 × 10^{-6} newtons/m^2 (1367 watts/m^2),

A = astronomical unit in meters,

R = heliocentric radius vector to the satellite,

a = cross-sectional area (m^2) of the satellite perpendicular to $\vec{R}$,

m = satellite mass,

C_R = reflectivity coefficient, usually an adjusted parameter.

The radiation pressure due to backscatter from the Earth is ignored. The model for the Earth's shadow should include the umbra and the penumbra (Haley, 1973).

Earth Radius	6402	km
Moon Radius	1738	km

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CHAPTER 15 GENERAL RELATIVISTIC DYNAMICAL MODEL

The relativistic treatment of the near-Earth satellite orbit determination problem includes correction to the equations of motion, the time transformations, and the measurement model. The two coordinate systems generally used when including relativity in near-Earth orbit determination solutions are the solar system barycentric frame of reference and the geocentric or Earth-centered frame of reference.

Ashby and Bertotti (1986) constructed a locally inertial frame in the neighborhood of the gravitating Earth and demonstrated that the gravitational effects of the Sun, Moon, and other planets are basically reduced to their tidal forces, with very small relativistic corrections. Thus the main relativistic effects on a near-Earth satellite are those described by the Schwarzschild field of the Earth itself. This result makes the geocentric frame more suitable for describing the motion of a near-Earth satellite (Ries, et al., 1988).

The time coordinate in the inertial E-frame is Terrestrial Dynamical Time (TDT). This time coordinate is realized in practice by International Atomic Time (TAI), whose rate is defined by the atomic second in the International System of Units (SI).

EQUATIONS OF MOTION FOR AN ARTIFICIAL EARTH SATELLITE

The correction to the acceleration of an artificial Earth satellite $\Delta\vec{a}$ is

$$\Delta\vec{a} = -\frac{GM_\oplus}{c^2r^3}\left[[2(\beta+\gamma)\frac{GM_\oplus}{r} - \gamma v^2]\vec{r} + [2(1+\gamma)(\vec{r}\cdot\vec{v})\vec{v}]\right] \qquad (1)$$

where

c = speed of light,

β, γ = PPN parameters equal to 1 in General Relativity,

$\vec{r}, \vec{v}, \vec{a}$ = geocentric satellite position, velocity, and acceleration, respectively,

$GM_\oplus$ = gravitational parameter of the Earth.

The value of $GM_\oplus$ in the Newtonian two-body acceleration and in Eq.

(1) should be the geocentric value but the difference between the barycentric and geocentric mass of the Sun, Moon, and planets is not important when computing the indirect Newtonian pertubations. The effects of Lense-Thirring precession (frame-dragging), geodesic (de Sitter) precession, and the relativistic effects of the Earth's oblateness have been negelcted.

EQUATIONS OF MOTION IN THE BARYCENTRIC FRAME

The n-body equations of motion for the solar system frame of reference (the isotropic Parameterized Post-Newtonian system with TDB as the time coordinate) are required to describe the dynamics of the solar system and artificial probes moving about the solar system (for example, see Moyer, 1971). These are the equations applied to the Moon's motion for LLR (Newhall, Williams, and Dickey, 1987). In addition, relativistic corrections to the laser range measurement, the data timing, and the station coordinates are required (see Chapter 16).

SCALE EFFECT AND CHOICE OF TIME COORDINATE

Because the IAU definition of the time coordinate in the barycentric frame requires that only periodic differences exist between TDB and TDT (Kaplan, 1981), the spatial coordinates in the barycentric frame have effectively been rescaled to keep the speed of light unchanged between the barycentric and the geocentric frames (Misner, 1982; Hellings, 1986). Thus, when barycentric (or TDB) units of length are compared to geocentric (or TDT) units of length, a scale difference, L, appears. Noting that the mass parameter GM/c^2 or Gm/c^2 has units of length, the value for the mass parameter of a body in TDB units, GM, is related to its value in TDT units, Gm, according to $GM = (1-L)\ Gm$.

It can be shown that the value of the scale difference does not include the contribution of the gravitational and rotational potential of the Earth (Guinot and Seidelmann, 1988; Huang, et al., 1989) so that the value of the scale difference between the two frames is $L = 1.4808 \times 10^{-8}$ (Fukushima, et al., 1986).

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CHAPTER 16 GENERAL RELATIVISTIC TERMS FOR PROPAGATION, TIME, AND COORDINATES

VLBI TIME DELAY

There are many papers dealing with relativistic effects which must be accounted for in VLBI processing; see Robertson (1975), Finkelstein, *et al*. (1983), Hellings (1986), Pavlov (1985), Cannon, *et al*. (1986), Soffel, *et al*. (1986), Zeller, *et al*. (1986), Zhu and Groten (1988). As pointed out by Boucher (1986), the relativistic correction models proposed in various articles are not quite compatible. Nearly all the models in these papers were derived in the barycentric or B-frame. The following material is taken largely from the article of Zhu and Groten (1988).

Event one is the arrival of a radio signal at antenna 1; event two is the same wave front at antenna 2. The delay expressed in coordinate time is,

$$\Delta t_{obs} = -[\vec{R}_1(t_1)-\vec{R}_2(t_2)] \cdot \hat{K}/c + \Delta t_{grav}, \tag{1}$$

[see Finkelstein, *et al*. (1983), Zeller, *et al*. (1986)]. Here $\hat{K}$ is the direction of the source seen from the barycenter. Δt_{grav} is called the gravitational time delay. $\vec{R}_1$ and $\vec{R}_2$ are barycenter vectors of antenna 1 and 2, respectively. In the derivation, it is assumed that $|\vec{R}_o| \gg |\vec{R}_i|$, where $|\vec{R}_o|$ is the distance of the solar system barycenter from the source. For radio stars in our galaxy, relevant corrections must be added.

Defining the baseline in the B-frame as,

$$\vec{b} = \vec{R}_1(t_1) - \vec{R}_2(t_1), \tag{2}$$

it is easy to obtain (Robertson, 1975),

$$\Delta t_{obs} = \Delta t_o \{1 + (\dot{\vec{R}}_2\cdot\hat{K})/c + (\dot{\vec{R}}_2\cdot\hat{K})^2/c^2 + (\ddot{\vec{R}}_2\cdot\hat{K})\ \Delta t_o/2c\} + \Delta t_{grav}, \tag{3}$$

where $\Delta t_o = -(\vec{b}\cdot\hat{K})/c$.

The barycentric position vector of antenna i can also be written as $\vec{R}_i = \vec{R} + \vec{r}_i$, where $\vec{R}$ is the barycentric position of the geocenter, and $\vec{R}_i$ is the geocentric position vector of the same antenna measured in the B-frame. Then equation (2) is equivalent to

$$\vec{b} = \vec{r}_1(t_1) - \vec{r}_2(t_1). \tag{2'}$$

Equation (3) is the delay expressed in coordinate time. It must be transformed to measurable atomic time. The baseline $\vec{b}$ in the above equations has to be changed into its counterpart expressed in the geocentric or E-frame. Define this baseline in the E-frame as $\vec{b} = \vec{r}_1(\tau_1) - \vec{r}_2(\tau_1)$.

Zhu and Groten (1988) use the approach similar to the one used in Zeller, et al. (1986) in order to account for these coordinate transformations. The coordinates in the E-frame are expressed by $(c\tau,\hat{x})$. The coordinate transformations between these two systems take the form (Fukushima, et al. 1986)

$$\Delta t = \{1+\langle\phi+v^2/2\rangle_p/c^2\}\Delta\tau + (\vec{v}\cdot\Delta\hat{x})c^2, \tag{4-a}$$

$$\Delta\vec{x} = \{ 1-(\phi + \langle\phi + v^2/2\rangle)/c^2 \} \Delta\hat{x} + \vec{v}\Delta\tau + (\vec{v}\cdot\Delta\hat{x}) \vec{v}/2c^2, \tag{4-b}$$

where ϕ and $\vec{v}$ are Newtonian potential and velocity at the center of mass of the Earth, respectively, $\langle\ \rangle$ denotes the long time average, $\langle\ \rangle_p$ means the periodic terms with the long time average being removed.

If $\dot{\vec{r}}_2 = \vec{v}_2$, the final $\Delta\tau_{obs}$ derived in the B-frame is

$$\begin{aligned}\Delta\tau^B_{obs} &= \Delta\tau_o\, [1 + (\vec{v}+\vec{v}_2)\cdot\hat{K}/c + (\vec{v}\cdot\hat{K})^2/c^2 + 2(\vec{v}\cdot\hat{K})(\vec{v}_2\cdot\hat{K})/c^2 - \phi/c^2 \\ &\quad - (\phi+v^2/2)/c^2 - (\vec{v}\cdot\vec{v}_2)/c^2] + (\vec{b}\cdot\vec{v})/c^2 + (\vec{v}\cdot\vec{v})(\hat{K}\cdot\vec{v})/2c^3 \\ &\quad + (\vec{b}\cdot\vec{v})(\vec{v}_2\cdot\hat{K})/c^3 + \Delta\tau_{grav},\end{aligned} \tag{5}$$

with

$$\Delta\tau_o = - (\vec{b}\cdot\hat{K})/c. \tag{6}$$

This result differs from that given in Zeller, et al. (1986) only by an additional term $-\Delta\tau\phi/c^2$ which is the consequence of the general relativistic effect on the spatial coordinate transformation. This term amounts to 10^{-8} (in which the secular part is at 10^{-8}, and the periodic part is at 10^{-10}). Therefore it should not be neglected.

As derived in Finkelstein, et al. (1983), Zeller, et al. (1986) etc. the gravitational time delay is

$$\Delta\tau_{grav} = 2 \sum_j (Gm_j/c^3)\ \ell n\ [(|\vec{R}_{1j}| - \vec{R}_{1j} \cdot \hat{K})/(|\vec{R}_{2j}| - \vec{R}_{2j} \cdot \hat{K})], \qquad (7)$$

in which

$$\vec{R}_{ij} = \vec{R}_i - \vec{R}_j$$

$$\begin{bmatrix} i = 1,2, & j = 1....n \\ i = \text{antenna}, & j = \text{attracting body} \end{bmatrix}$$

is the vector from body j to antenna i. If the attracting body is the Earth, equation (7) takes the form,

$$\Delta\tau_{grav}(E) = \frac{2GM_\oplus}{c^3}\ \ell n \left[\frac{1 + \sin E_1}{1 + \sin E_2}\right], \qquad (8)$$

in which E_i is the source elevation angle viewed at antenna i. In any other cases, since $|\vec{R}_{ij}| >> |\vec{b}|$ equation (7) reduces to

$$\Delta\tau_{grav}(j) \approx 2 \frac{Gm_j}{c^3|\vec{R}_{oj}|} \frac{-\vec{b}\cdot\hat{K} + \vec{b}\cdot\vec{n}_{oj}}{1 - \hat{n}_{oj}\cdot\hat{K}} \qquad (9)$$

where $\vec{R}_{oj}$ is the vector from body j to geocenter, and

$$\hat{n}_{oj} = \vec{R}_{oj}/|\vec{R}_{oj}|.$$

As pointed by Zeller, et al. (1986), when $\hat{n}_{oj}$ and $\hat{K}$ are nearly parallel, that is, the source is near the limb of the body, equation (9) will not be accurate enough. In any other case, equation (9) could be used instead of equation (7).

Another expression used for the VLBI delay observable (Shapiro, et al., 1989) is

$$\begin{aligned}\Delta\tau = &- (\hat{K}\cdot\vec{b})/c \\ &- [(\vec{b}\cdot\vec{v}) - (\hat{K}\cdot\vec{b})(\hat{K}\cdot\vec{v})]/c^2 \\ &+ \Delta\tau^{pm}\end{aligned}$$

$$+ (\hat{K}\cdot\vec{b})(\hat{K}\cdot\vec{r}_2)/c^2$$
$$+ [(\vec{b}\cdot\vec{v})(\hat{K}\cdot\vec{v})/2 + (\hat{K}\cdot\vec{b})(\vec{v}\cdot\vec{v})/2 - (\hat{K}\cdot\vec{b})(\hat{K}\cdot\vec{v})^2]/c^3$$
$$+ 2U(\hat{K}\cdot\vec{b})/c + (\vec{b}\cdot\vec{v})(\hat{K}\cdot\dot{\vec{r}}_2)/c^3,$$

where $\Delta\tau$ is the delay defined to be $\tau_2-\tau_1$ with τ_1 being the atomic time reading at site 1 when a wavefront arrives from a distant source, and τ_2 is the atomic time at site 2 when this same wavefront arrives there, $\hat{K}$ is the unit vector in the direction of the source as viewed from the solar system barycenter in the J2000.0 coordinate system; $\vec{b}$ is the baseline vector $\vec{r}_2-\vec{r}_1$, at atomic time τ_1; $\vec{r}_1$ and $\vec{r}_2$ are the vectors from the center of the Earth to site 1 and site 2, respectively; $\vec{v}$ is the velocity of the center of the Earth in solar system barycentric (SSB) coordinates at the SSB coordinate time corresponding to τ_1; $\dot{\vec{r}}_2$ is geocentric velocity of site 2 at τ_1; $\Delta\tau^{pm}$ is the propagation medium delay and light deflection defined below; U is the gravitational potential at site 1 computed at the SSB coordinate time corresponding to τ_1 and is given by:

$$U = \sum_k \frac{GM_k}{c^2 r_k},$$

where G is the gravitational constant, M_k is the mass of the k^{th} body, and r_k is the distance from the k^{th} body to site 1. The sum k includes all bodies except the Earth itself. In practice, only the Sun need be included.

The $\Delta\tau^{pm}$ term is given by

$$\Delta\tau^{pm} = \frac{(1+\gamma)r_o}{c}\,\ell n\left[\frac{r_e(1+\hat{r}_e\cdot\hat{K}) + \Delta\vec{r}_1\cdot(\hat{r}_e+\hat{K})}{r_e(1+r_e\cdot K) + \Delta\vec{r}_2\cdot(r_e+\hat{K})}\right]$$
$$+ \frac{(1+\gamma)r_\omega}{c}\,\ell n\left[\frac{r_1(1+\vec{r}_1\cdot\hat{K}/r_1)}{r_2(1+\vec{r}_2\cdot K/r_2)}\right]$$
$$+ \tau^a + \tau^i,$$

where

$$r_o = GM_\odot/c^2 \text{ and } r_\omega = GM_\oplus/c^2,$$

and $M_\odot$ and $M_\oplus$ are the masses of the Sun and the Earth, respectively; $\underset{\sim}{K}$ is the unit vector in the direction from Sun to the radio source; $\hat{r}_e$ is the unit vector in the direction from the Sun to the Earth; r_e is the distance from the Sun to the Earth; r_1 and r_2 are the magnitudes of these vectors, and τ^a and τ^i are the differential delays due to the propagation of the signals through the atmosphere and the ionosphere, respectively. The variable γ is the γ-factor in the parametrized post-Newtonian gravitational theory. Note: the above expressions are only valid for sources which are an infinite distance from the Earth, and, in particular, are not valid for signals from GPS satellites.

PROPAGATION CORRECTION FOR LASER RANGING

The space-time curvature near a massive body requires a correction to the Euclidean computation of range, ρ. This correction in seconds, Δt, is given by (Holdridge, 1967)

$$\Delta t = \frac{(1 + \gamma)GM}{c^3} \ln \left[\frac{R_1 + R_2 + \rho}{R_1 + R_2 - \rho}\right], \tag{10}$$

where

c = speed of light,

γ = PPN parameter equal to 1 in General Relativity,

R_1 = distance from the body's center to the beginning of the light path,

R_2 = distance from the body's center to the end of the light path,

GM = gravitational parameter of the deflecting body.

For near-Earth satellites, working in the geocentric frame of reference, the only body to be considered is the Earth (Ries, Huang, and Watkins, 1988). For lunar laser ranging, which is formulated in the solar system barycentric reference frame, the Sun and the Earth must be considered (Newhall, Williams, and Dickey, 1987).

In the computation of the instantaneous space-fixed positions of a station and a lunar reflector in the analysis of LLR data, the body-centered coordinates of the two sites are affected by a scale reduction and a Lorentz contraction effect (Martin, Torrence, and Misner, 1985). The scale effect is about 15 cm in the height of a tracking station, while the maximum value of the Lorentz effect is about 3 cm. The equation for the transformation of $\vec{r}$, the geocentric position vector of a station expressed in the geocentric frame, is

$$\vec{r}_b = \vec{r} \left[1 - \frac{\gamma\Phi}{c^2} - L \right] - \frac{1}{2}\left(\frac{\vec{V} \cdot \vec{r}}{c^2}\right)\vec{V}, \tag{11}$$

where

$\vec{r}_b$ = station position expressed in the barycentric frame,

Φ = gravitational potential at the geocenter (excluding the Earth's mass),

$\vec{V}$ = barycentric velocity of the Earth,

L = $\langle (\Phi + V^2/2)/c^2) \rangle = 1.4808 \times 10^{-8}$, where $\langle\rangle$ denotes the long time average.

A similar equation applies to the selenocentric reflector coordinates; the maximum value of the Lorentz effect is about 1 cm (Newhall, Williams, and Dickey, 1987).

EPOCH AND TIME INTERVALS

The transformation from solar system barycentric coordinate time t (i. e. TDB) to atomic clock time, τ (i. e. TAI), involves a constant offset, an annual and diurnal term, and other small periodic terms. This transformation has usually been given in the past in terms of the Earth's orbital elements, terrestrial coordinates, and UT. With the near universal availability of ephemeris tapes, a more convenient expression is given with sufficient accuracy by Moyer (1981)

$$t - \tau = \Delta T_A + \frac{2}{c^2}(\dot{\vec{r}}_B^S \cdot \vec{r}_B^S) + \frac{1}{c^2}(\dot{\vec{r}}_B^C \cdot \vec{r}_E^B)$$

$$+ \frac{1}{c^2}(\dot{\vec{r}}_E^C \cdot \vec{r}_A^E) + \frac{\mu_J}{c^2(\mu_S + \mu_J)}(\dot{\vec{r}}_J^S \cdot \vec{r}_J^S)$$

$$+ \frac{\mu_{SA}}{c^2(\mu_S + \mu_{SA})}(\dot{\vec{r}}_{SA}^S \cdot \vec{r}_{SA}^S) + \frac{1}{c^2}(\dot{\vec{r}}_S^C \cdot \vec{r}_B^S), \qquad (12)$$

where $\Delta T_A = 32\overset{s}{.}184$. The quantities $\vec{r}_x^y$ and $\dot{\vec{r}}_x^y$ are the position and velocity of point x relative to point y. The superscripts and subscripts are defined as follows:

A = location of station clock on Earth which reads International Atomic Time (TAI),

E = Earth,

B = Earth-Moon barycenter,

M = Moon,

S = Sun,

C = solar system barycenter,

J = Jupiter,

SA = Saturn.

The quantity μ_X is the gravitational coefficient of body X. An expression with higher accuracy is available in Fairhead, et al. (1988). No relativistic time coorections are required for satellite laser ranging in the geocentric frame (i. e. TDT $\equiv$ TAI + ΔT_A).

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IERS Technical Note No 3. IERS Standards (1989).

There are some major differences between of the IERS Standards(1989) and the Project MERIT Standards. The biggest changes have been the elimination of all references to optical observations and deletion of all information for artificial Earth satellites other than Lageos. These changes were made because these observations are no longer used operationally by the IERS. Included for the first time is a chapter on the tropospheric model.

Minor changes have also been made to many of the chapters. The following is a brief list of the modifications by chapter.

CHAPTER 1

equatorial radius of the Earth
dynamical form factor of the Earth
geocentric constant of gravitation
constant of gravitation
mass of the Earth-Moon system
mass of Saturn
geopotential - use of GEM-T1
change in radii for penumbra model
change in relativistic corrections
slight change in empirical force parameter for Lageos
slight change in lunar secular acceleration
plate motion model added

CHAPTER 2 - celestial reference frame is based on ICRF

CHAPTER 3 - conventional terrestrial reference frame is based on ITRF

CHAPTER 4 - same

CHAPTER 5 - same

CHAPTER 6 - addition of rotational deformation due to polar motion

CHAPTER 7 - same

CHAPTER 8 - addition of some horizontal site displacement
addition of site displacement due to atmospheric loading

CHAPTER 9 - same as Update #1

CHAPTER 10 - new

CHAPTER 11 - same
fixed errors in MERIT Standards in l', omega, and 386-day tide

CHAPTER 12 - rewritten

CHAPTER 13 - rewritten
uses GEM-T1
lists GEM-T1 coefficients by table

CHAPTER 14 - same

CHAPTER 15 - rewritten
now includes relativistic corrections used in SLR

CHAPTER 16 - rewritten
now includes multiple models for relativistic corrections in VLBI