CHAPTER 5 PROCEDURE FOR COMPUTING APPARENT PLACES

The barycentric direction of a star at any instant in Dynamical Barycentric Time (TDB) is calculated from its right ascension, declination, and space motion for the standard equinox and equator of J2000.0 in the FK5 system.

Note that for the computation of apparent places:

1) The distinction between TDB and Terrestrial Dynamical Time (TDT) is not significant.

2) The space-motion of the star is included but light-time is ignored.

3) The relativity term for light deflection is modified to the asymptotic case where the star is assumed to be at infinity.

STEP 1

Set TDB = TDT

STEP 2

Obtain the Earth's barycentric position $\vec{E}_B$ in astronomical units (au) and $\dot{\vec{E}}_B$ in au/day, at coordinate time t=TDB referred to the equinox and equator of J2000.0. Then, calculate the geocentric vector of the star at the required epoch in the J2000.0 frame.

When given the right ascension α, declination δ, proper motion in right ascension μ_α (in seconds of time per Julian century), proper motion in declination μ_δ (in arcseconds per Julian century), stellar parallax π (in arcseconds) and radial velocity v (in kilometers per second) at J2000.0, referred to the standard equinox and equator of J2000.0, calculate $\vec{P}$, the geocentric vector of the star at the required epoch t, from:

$$\vec{P} = \hat{r} + T\,\vec{v} - \mathrm{arc1''}\,\pi \vec{E}_B$$

where

$$\hat{r} = (\cos\delta\,\cos\alpha,\ \cos\delta\,\sin\alpha,\ \sin\delta),$$

$$\vec{v} = (v_x,\ v_y,\ v_z),$$

$$v_x = \mathrm{arc1''}(-\mu_\delta \sin\delta\,\cos\alpha - 15\mu_\alpha \cos\delta\,\sin\alpha + 21.094953\pi v\cos\delta\,\cos\alpha),$$

$v_y = \text{arc1}''(-\mu_\delta \sin\delta \sin\alpha + 15\mu_\alpha \cos\delta \cos\alpha + 21.094953\pi v \cos\delta \sin\alpha)$,

$v_z = \text{arc1}''(\ \mu_\delta \cos\delta \qquad\qquad\qquad\qquad + 21.094953\pi v \sin\delta)$,

T = (JD - 2451545.0)/36525, and JD is the Julian date at time t. The factor 21.094953 is [km/au]·[Julian century/sec].

STEP 3

Form the heliocentric position of the Earth and calculate the geocentric direction of the star, corrected for light deflection.

Form the heliocentric position of the Earth $\vec{E}$ from:

$$\vec{E} = \vec{E}_B - \vec{S},$$

where $\vec{S}$ is the barycentric position of the Sun at time t.

Form the geocentric direction $\hat{p}$ of the star and the unit vector $\hat{e}$ from:

$$\hat{p} = \vec{P}/|\vec{P}| \quad \text{and} \quad \hat{e} = \vec{E}/|\vec{E}|.$$

Calculate the geocentric direction $\hat{p}_1$ of the star, corrected for light deflection in the natural frame, from:

$$\vec{p}_1 = \hat{p} + (2\mu/c^2E)[(\hat{e}-(\hat{p}\cdot\hat{e})\hat{p})/(1+\hat{p}\cdot\hat{e})],$$

where the dot indicates a scalar product, μ/c^2 = the gravitational radius of the Sun = 9.87063×10^{-9} au and $E = |\vec{E}|$.

The vector $\vec{p}_1$ is a unit vector to order μ/c^2.

STEP 4

Calculate the proper direction of the star $\vec{p}_2$ in the geocentric inertial frame that is moving with the instantaneous velocity $\vec{V}$ of the Earth relative to the natural frame, from:

$$\vec{p}_2 = (\beta^{-1}\vec{p}_1 + \vec{V} + (\vec{p}_1\cdot\vec{V})\vec{V}/(1 + \beta^{-1}))/(1 + \vec{p}_1\cdot\vec{V}),$$

where $\vec{V} = \dot{\vec{E}}_B/c = 0.0057755\ \dot{\vec{E}}_B$ and $\beta = (1 - V^2)^{-\frac{1}{2}}$; the velocity $\vec{V}$ is expressed in units of the velocity of light and is equal to the Earth's velocity in the barycentric frame to order V^2.

STEP 5

Apply precession and nutation to the proper direction $\vec{p}_2$ by multiplying by the rotation matrices due to the precession [P] and the nutation [N] (Mueller, 1969), to obtain the apparent direction $\vec{p}_3$, from:

$$\vec{p}_3 = [N]\ [P]\ \vec{p}_2.$$

Here,

$$[P] = R_3(-Z_A)\ R_2(\theta_A)\ R_3(-\zeta_A),$$

$$[N] = R_1[-(\epsilon_A+\Delta\epsilon)]\ R_3(-\Delta\Psi)\ R_1(\epsilon_A),$$

where R_1, R_2, and R_3 represent rotations about the X, Y, and Z axes respectivley.

Rotation angles are defined by Lieske, et al. (1977) as follows:

$$\zeta_A = (2306.''2181 + 1.''39656\ T - 0.''000139\ T^2)t$$
$$+ (0.''30188 - 0.''000345\ T)t^2 + 0.''017998\ t^3,$$

$$\theta_A = (2004.''3109 - 0.''85330\ T - 0.''000217\ T^2)t$$
$$+ (-0.''42665 - 0.''000217\ T)t^2 - 0.''041833\ t^3,$$

$$Z_A = (2306.''2181 + 1.''39656\ T - 0.''000139\ T^2)t$$
$$+ (1.''09468 + 0.''000066\ T)t^2 + 0.''018203\ t^3,$$

$$\epsilon_A = (84381.''448 - 46.''8150\ T - 0.''00059\ T^2 + 0.''001813\ T^3)$$
$$+ (-46.''8150 - 0.''00117\ T + 0.''005439\ T^2)t$$
$$+ (-0.''00059 + 0.''005439\ T)t^2 + 0.''001813\ t^3,$$

where T represents Julian centuries from J2000.0 to an arbitrary epoch and t represents Julian centuries from the epoch to the date.

Note that the coefficient of Tt^2 in ζ_A has been changed to $-0.''000344$ by Lieske (1979), but this change has no effect in our case because we need to calculate the precession from J2000.0 to the date and hence T=0.

$\Delta\Psi$ and $\Delta\epsilon$ are the nutation in longitude and in obliquity, respectively (see Chapter 4).

STEP 6

Convert to spherical coordinates (α_t, δ_t) using:

$$\xi/\rho = \cos\delta_t \cos\alpha_t,$$

$$\eta/\rho = \cos\delta_t \sin\alpha_t,$$

$$\zeta/\rho = \sin\delta_t,$$

where $\vec{p}_3 = (\xi, \eta, \zeta)$ and $\rho = \sqrt{\xi^2 + \eta^2 + \zeta^2}$.

Useful references describing procedures in more detail are Smith, et al. (1989) and Yallop, et al. (1989).

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