CHAPTER 15 GENERAL RELATIVISTIC DYNAMICAL MODEL

The relativistic treatment of the near-Earth satellite orbit determination problem includes correction to the equations of motion, the time transformations, and the measurement model. The two coordinate systems generally used when including relativity in near-Earth orbit determination solutions are the solar system barycentric frame of reference and the geocentric or Earth-centered frame of reference.

Ashby and Bertotti (1986) constructed a locally inertial frame in the neighborhood of the gravitating Earth and demonstrated that the gravitational effects of the Sun, Moon, and other planets are basically reduced to their tidal forces, with very small relativistic corrections. Thus the main relativistic effects on a near-Earth satellite are those described by the Schwarzschild field of the Earth itself. This result makes the geocentric frame more suitable for describing the motion of a near-Earth satellite (Ries, et al., 1988).

The time coordinate in the inertial E-frame is Terrestrial Dynamical Time (TDT). This time coordinate is realized in practice by International Atomic Time (TAI), whose rate is defined by the atomic second in the International System of Units (SI).

EQUATIONS OF MOTION FOR AN ARTIFICIAL EARTH SATELLITE

The correction to the acceleration of an artificial Earth satellite $\Delta\vec{a}$ is

$$\Delta\vec{a} = - \frac{GM_\oplus}{c^2 r^3} \left[[2(\beta + \gamma) \frac{GM_\oplus}{r} - \gamma v^2]\vec{r} + [2(1 + \gamma) (\vec{r}\cdot\vec{v})\vec{v}] \right] \qquad (1)$$

where

c = speed of light,

β, γ = PPN parameters equal to 1 in General Relativity,

$\vec{r}, \vec{v}, \vec{a}$ = geocentric satellite position, velocity, and acceleration, respectively,

$GM_\oplus$ = gravitational parameter of the Earth.

The value of $GM_\oplus$ in the Newtonian two-body acceleration and in Eq.

(1) should be the geocentric value but the difference between the barycentric and geocentric mass of the Sun, Moon, and planets is not important when computing the indirect Newtonian pertubations. The effects of Lense-Thirring precession (frame-dragging), geodesic (de Sitter) precession, and the relativistic effects of the Earth's oblateness have been negelcted.

EQUATIONS OF MOTION IN THE BARYCENTRIC FRAME

The n-body equations of motion for the solar system frame of reference (the isotropic Parameterized Post-Newtonian system with TDB as the time coordinate) are required to describe the dynamics of the solar system and artificial probes moving about the solar system (for example, see Moyer, 1971). These are the equations applied to the Moon's motion for LLR (Newhall, Williams, and Dickey, 1987). In addition, relativistic corrections to the laser range measurement, the data timing, and the station coordinates are required (see Chapter 16).

SCALE EFFECT AND CHOICE OF TIME COORDINATE

Because the IAU definition of the time coordinate in the barycentric frame requires that only periodic differences exist between TDB and TDT (Kaplan, 1981), the spatial coordinates in the barycentric frame have effectively been rescaled to keep the speed of light unchanged between the barycentric and the geocentric frames (Misner, 1982; Hellings, 1986). Thus, when barycentric (or TDB) units of length are compared to geocentric (or TDT) units of length, a scale difference, L, appears. Noting that the mass parameter GM/c^2 or Gm/c^2 has units of length, the value for the mass parameter of a body in TDB units, GM, is related to its value in TDT units, Gm, according to GM = (1-L) Gm.

It can be shown that the value of the scale difference does not include the contribution of the gravitational and rotational potential of the Earth (Guinot and Seidelmann, 1988; Huang, et al., 1989) so that the value of the scale difference between the two frames is $L = 1.4808 \times 10^{-8}$ (Fukushima, et al., 1986).

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