3. CONNECTION OF THE HIPPARCOS FRAME

The Hipparcos mission

The satellite Hipparcos of the European Space Agency (ESA) was launched in August 1989. The mission will determine the positions, proper motions and parallaxes of about 120,000 stars brighter than the 13th magnitude. Hipparcos is the first space astrometric mission. In addition to the advantages of observations made above the atmosphere, the originality of this project lies on the principle of differential measurements over large field which allow to eliminate regional errors affecting ground-based measurements over narrow field. The instrument aboard the satellite, the principle of observations and the method of data analysis are described by Kovalevsky (1980,1984) and Perryman (1989).

In the first step of the data reduction, the star to star angles measured during one or two rotations of the satellite (several hours) are projected on the same mean great circle with an arbitrary origin. In the second step, the reconstruction of the stellar sphere unifies the great circles origins and the coordinates of each star at its epoch of observation are computed in a coordinate system near the ecliptic system. Finally, the astrometric parameters - positions, proper motions and parallaxes - are estimated from these coordinates using a physical model accounting for the motion of the satellite around the Earth and the Sun and the light deflection produced by the gravitational field of the Solar System.

The error budget (ESA 1978) indicates that the precisions should be at the level of 0.002" in positions, 0.002"/year in proper motions and 0.002" in parallaxes of stars brighter than the 9th magnitude. Nevertheless, the astrometric potential of Hipparcos stellar reference frame will be achieved only if it is linked to a non-rotating celestial reference frame, i.e., to the VLBI reference frame and to the planetary dynamical reference system. This link will allow

- to "stop" the rotation of the Hipparcos frame produced by galactic rotation,
- to densify the extragalactic frame by galactic objects accessible at optical wavelengths,
- to directly register the celestial objects structures obtained with radio and optical techniques at the same angular resolution.

Link objects between the Hipparcos and VLBI frames

The VLBI coordinates of extragalactic objects located at cosmological distance define a system of stable directions in space. The proper motions of quasars and active galactic nuclei are undetectable at the level of 20 microseconds of arc per year, as measured in the particular pair, but most probably representative, 3C345/NRAO512 (Bartel et al. 1986).

Having quasi point-like structures in the optical domain, quasars are well adapted for precise astrometric measurements but they are too faint for the Hipparcos satellite, and cannot be used to calibrate the proper motions of the stars. The brightest radio source is the quasar 3C273B ($m_v = 12.5$), but extragalactic objects have generally magnitudes larger than 17.

Froeschle and Kovalevsky (1982) present three methods to connect Hipparcos to the VLBI frame, based on

a) the positions measurements of bright radio stars with both the Hipparcos and VLBI techniques,
b) the observation of quasars with the VLBI technique, and of stars with the satellite, while their angular separations are measured with the Hubble Space Telescope (HST),
c) Hipparcos stars are connected by the HST to extragalactic objects whose radio emission is either non-existing or too weak to perform VLBI precise observations. This method prevents the Hipparcos frame to rotate but it does not contribute to the unification of the radio and the optical frames.

The use of radio stars as common objects (method a) to connect the two frames is the most direct method. However, the flux densities of radio stars are about 100 times weaker than those of the extragalactic radio sources normally observed with VLBI. Thus, radio stars are difficult to detect and their astrometric potential is weak if we use the classical "absolute" positioning VLBI technique with bandwidth synthesis delay observations. But milliarcsecond astrometric precision can be achieved by performing differential VLBI observations with a reference source angularly near the star (Lestrade et al., 1990). The radio link stars must be brighter than magnitude 11 for good observation conditions by the satellite. Their radio components size must be smaller 0.001" in order not to be resolved by the VLBI observations. The flux density must be of at least a few mJy to match the sensitivity of phase-referencing VLBI technique. Besides, the optical and radio counterparts should coincide within 0.002". Following these criteria, Lestrade et al. (1982) established an initial list of 22 radio stars candidate for the link. Walter (1977) had also proposed a list of about 30 bright stars selected for their radio emission to link the radio and the optical frames; not all of them have been retained because some presented an extended radio structure. The Input Catalogue of the Hipparcos mission contains 186 radio stars (Argue 1989) but not all of them are suitable for the link. Four link stars (Algol, HR5110, λ And and σ Cr B) are in the FK4 and FK5.

Modelling of the link

As no regional deformations are expected for the Hipparcos frame, the link between the two frames can be mathematically expressed by a global rotation at an arbitrary epoch represented by a matrix [R], and by a matrix [\dot{R}] which represents the angular velocity of rotation of one frame relative to the other.

For each radio star the VLBI technique provides a vector \( \mathbf{\sigma}_{Vi} \) at an epoch \( t_{Vi} \) of observation of the star, and the associated proper motion vector \( \mathbf{\dot{\sigma}}_{Vi} \). Similarly, the observations of the satellite give the vector \( \mathbf{\sigma}_{Hi} \) at an epoch \( t_{Hi} \) of observation, and the associated proper motion vector \( \mathbf{\dot{\sigma}}_{Hi} \).

At an instant \( t_i \), two link equations between the two frames can be written in vectorial form for each star:

\[
\mathbf{\sigma}_{Vi} (t_i) = [R(t_i)] \cdot \mathbf{\sigma}_{Hi} (t_i)
\]

\[
\mathbf{\dot{\sigma}}_{Vi} = [R(t_{Vi})] \cdot \mathbf{\dot{\sigma}}_{Hi} + [\dot{R}] \mathbf{\sigma}_{Hi} (t_i)
\]

The matrices [R] and [\dot{R}] are function of the rotation angles \( A_1, A_2, A_3 \) around the axes of one of the frames; the angular velocity of rotation is represented by the three time derivatives \( \dot{A}_1, \dot{A}_2, \dot{A}_3 \). There are six unknowns (the three rotation angles and their time derivatives), and so in principle two radio stars are sufficient to find a solution with the link equations (4) and (5) written in coordinates form. However, a solution with a larger number of stars to perform a least squares adjustment is preferable.

The director cosine vectors of a star and its proper motion are, in spherical equatorial coordinates

\[
\mathbf{\sigma} = \begin{bmatrix}
\cos \alpha \cos \delta \\
\sin \alpha \cos \delta \\
\sin \delta
\end{bmatrix}
\]
The positions, $\alpha$ and $\delta$, and proper motions, $\mu_\alpha$ and $\mu_\delta$, are measured by the Hipparcos instrument and by VLBI at a variety of times. However the global rotation $[R(t_0)]$ must be determined at an arbitrarily chosen but unique epoch $t_0$. So the VLBI and Hipparcos positions must be re-written in more general form:

$$
t_{\text{Hi}}(t_0) = t_{\text{Hi}}(t_{\text{Hi}}) + (t_0 - t_{\text{Hi}}) \dot{\delta}_{\text{Hi}}$$  

(8)

Inserting these relations into equations (4) and (5), one finds:

$$
t_{\text{Vi}}(t_{\text{Hi}}) + (t_0 - t_{\text{Hi}}) \dot{\delta}_{\text{Hi}} = [R(t_0)] \{ t_{\text{Hi}}(t_{\text{Hi}}) + (t_0 - t_{\text{Hi}}) \dot{\delta}_{\text{Hi}} \}$$

(10)

$$
\dot{t}_{\text{Vi}} = [R(t_0)] \dot{\delta}_{\text{Hi}} + [\dot{R}] \{ t_{\text{Hi}}(t_{\text{Hi}}) + (t_0 - t_{\text{Hi}}) \dot{\delta}_{\text{Hi}} \}$$

(11)

We parametrize the matrix $[R(t_0)]$ by three successive individual rotations around each axis. These three rotations are represented by the matrices $[R_x],[R_y],[R_z]$, the angle $A_1$, around the direction of the right ascension origin, the angle $A_2$ around the axis at 90° in the equator, and the angle $A_3$ around the direction of the celestial pole.

$$
[R_x] =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos A_1 & \sin A_1 \\
0 & -\sin A_1 & \cos A_1
\end{bmatrix}
$$

$$
[R_y] =
\begin{bmatrix}
\cos A_2 & 0 & -\sin A_2 \\
0 & 1 & 0 \\
\sin A_2 & 0 & \cos A_2
\end{bmatrix}
$$

$$
[R_z] =
\begin{bmatrix}
\cos A_3 & \sin A_3 & 0 \\
-\sin A_3 & \cos A_3 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

Developing the product of these three matrices, we obtain the rotation matrix

$$
[R] = [R_z],[R_y],[R_x]
$$

$$
[R] =
\begin{bmatrix}
\cos A_2 \cos A_3 & \cos A_1 \sin A_3 + \sin A_1 \sin A_2 \cos A_3 & \sin A_1 \sin A_3 - \cos A_1 \sin A_2 \cos A_3 \\
-\cos A_2 \sin A_3 & \cos A_1 \cos A_3 - \sin A_1 \sin A_2 \sin A_3 & \sin A_1 \cos A_3 + \cos A_1 \sin A_2 \sin A_3 \\
\sin A_2 & -\sin A_1 \cos A_2 & \cos A_1 \cos A_2
\end{bmatrix}
$$

(12)
This matrix could be used to link the ecliptic coordinate system of Hipparcos to the equatorial VLBI coordinate system. Nevertheless, it is easier to transform the Hipparcos ecliptic coordinates into equatorial coordinates before the link. If we proceed in this way, the angles \( A_1, A_2, A_3 \) will be small quantities of the order of some tens of milliarcseconds \((\sim 10^{-7} \, \text{rad})\), and we can approximate \( \cos A_i \sim 1, \sin A_i^{-\text{Ai}} \). The matrix \([R(t_0)]\) becomes:

\[
[R(t_0)] = \begin{bmatrix}
1 & A_3 & -A_2 \\
-A_3 & 1 & A_1 \\
A_2 & -A_1 & 1
\end{bmatrix}
\] (13)

\([\dot{R}]\) is the matrix of elements \( \dot{A}_1, \dot{A}_2, \dot{A}_3 \) which expresses the relative angular velocity between the two frames. Assuming that this velocity is constant, we get

\[
[\dot{R}] = \begin{bmatrix}
0 & \dot{A}_3 & -\dot{A}_2 \\
-\dot{A}_3 & 0 & \dot{A}_1 \\
\dot{A}_2 & -\dot{A}_1 & 0
\end{bmatrix}
\] (14)

Introducing (6), (7), (13) and (14), equations (10) and (11) can be written, in equatorial coordinates, as:

\[
\begin{bmatrix}
\cos\alpha_v \cos\delta_v \\
\sin\alpha_v \cos\delta_v \\
\sin\delta_v
\end{bmatrix} + \Delta t_v =
\begin{bmatrix}
-\sin\alpha_v \cos\delta_v \mu_{\alpha v} - \cos\alpha_v \sin\delta_v \mu_{\delta v} \\
\cos\alpha_v \cos\delta_v \mu_{\alpha v} - \sin\alpha_v \sin\delta_v \mu_{\delta v} \\
\cos\delta_v \mu_{\delta v}
\end{bmatrix}
\begin{bmatrix}
\cos\alpha_H \cos\delta_H + (-\sin\alpha_H \cos\delta_H \mu_{\alpha H} - \cos\alpha_H \sin\delta_H \mu_{\delta H}) \Delta t_H \\
\sin\alpha_H \cos\delta_H + (\cos\alpha_H \cos\delta_H \mu_{\alpha H} - \sin\alpha_H \sin\delta_H \mu_{\delta H}) \Delta t_H \\
\sin\delta_H + \cos\delta_H \mu_{\delta H} \Delta t_H
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\sin\alpha_v \cos\delta_v \mu_{\alpha v} - \cos\alpha_v \sin\delta_v \mu_{\delta v} \\
\cos\alpha_v \cos\delta_v \mu_{\alpha v} - \sin\alpha_v \sin\delta_v \mu_{\delta v}
\end{bmatrix} =
\begin{bmatrix}
\cos\alpha_v \cos\delta_v \mu_{\delta v} \\
\cos\delta_v \mu_{\delta v} \Delta t_H
\end{bmatrix}
\] (15)

\[
\begin{bmatrix}
1 & A_3 & -A_2 \\
-A_3 & 1 & A_1 \\
A_2 & -A_1 & 1
\end{bmatrix}
\begin{bmatrix}
-\sin\alpha_H \cos\delta_H \mu_{\alpha H} - \cos\alpha_H \sin\delta_H \mu_{\delta H} \\
\cos\alpha_H \cos\delta_H \mu_{\alpha H} - \sin\alpha_H \sin\delta_H \mu_{\delta H} +
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & A_3 & -A_2 \\
-A_3 & 1 & A_1 \\
A_2 & -A_1 & 1
\end{bmatrix}
\begin{bmatrix}
\cos\alpha_H \cos\delta_H \mu_{\delta H} \\
\cos\delta_H \mu_{\delta H}
\end{bmatrix}
\] (16)
where \( \Delta t_{vi} = t_0 - t_{vi} \), \( \Delta t_{Hi} = t_0 - t_{Hi} \), \((\alpha_{vi}, \delta_{vi})\), \((\alpha_{Hi}, \delta_{Hi})\) are the equatorial coordinates of the \( i \)-th radio star obtained with VLBI and Hipparcos respectively, and \((\mu_{\alpha vi}, \mu_{\delta vi})\), \((\mu_{\alpha Hi}, \mu_{\delta Hi})\) are the components of their respective proper motions.

For \( N \) radio stars observed with both techniques, the matrix equations (15) and (16) give \( 4N \) independent equations with the six unknowns \( A_1(t_0), A_2(t_0), A_3(t_0), A_1, A_2, A_3 \), which represent the link between the Hipparcos and the VLBI frames.

**Simulation of Hipparcos data**

We have simulated Hipparcos and VLBI astrometric data for a realistic set of stars to test the quality of the link. We have made the hypothesis that the parallax effects have been eliminated in Hipparcos as well as in VLBI data.

Equations (10) and (11) can be re-arranged for the simulation of data:

\[
\begin{align*}
\dot{\varphi}_{Hi}(t_{Hi}) &= \{ [R(t_0)] + (t_{Hi} - t_0) [\dot{R}] \}^{-1} \{ \varphi_{vi}(t_{vi}) + (t_{Hi} - t_{vi}) \dot{\varphi}_{vi} \}, \\
\dot{\varphi}_{Hi} &= \{ [R(t_0)] + (t_{Hi} - t_0) [\dot{R}] \}^{-1} \{ \varphi_{vi} - [\dot{R}] \varphi_{Hi} (t_{Hi}) \}.
\end{align*}
\]

(17) (18)

Adopting VLBI positions and proper motions at epochs \( t_{vi} \), one can determine the corresponding Hipparcos coordinates at epochs \( t_{Hi} \), after choosing the rotation angles \( A_1, A_2, A_3 \) and the angular velocities \( \dot{A}_1, \dot{A}_2, \dot{A}_3 \).

It is expected that less than 14 radio stars will be observed at multiple epochs by the VLBI technique. The initial VLBI observations of 10 stars are presented by Lestrade et al. (1988) and White et al. (1990). The simulation presented here concerns 14 radio stars. The simulated positions and proper motions used are listed in table 19; their distribution on the sky is shown on figure 19.

All these stars have non-thermal radio emission, and most of them belong to the type of spectroscopic close binary systems RSCVn. The extent on the sky of these systems is no more than 2 milliarcseconds and should not limit the VLBI astrometric measurements for Hipparcos. Algol does not belong to the RSCVn type, but it shows rather similar properties. Astrometrically, it is a multiple system formed by a close binary orbiting around a third component with a period of 1.86 years in an orbit of semi-major axis 0.040". The suitability of Algol for the astrometric link is questionable but special treatments of both the VLBI data and the Hipparcos data might make it useful. Cyg XI and LSI 61°303 are close binaries with strong X-ray emission that are distant enough and hence point-like to make them good link objects.

The parameter values chosen for the simulation are

\[
\begin{align*}
A_1 &= +0.030", \\
A_2 &= +0.025", \\
A_3 &= -0.045", \\
\dot{A}_1 &= +0.002"/year, \\
\dot{A}_2 &= -0.003"/year, \\
\dot{A}_3 &= +0.001"/year, \quad t_0 = 1991.0.
\end{align*}
\]

The optical coordinates and proper motions simulated for Hipparcos mission with these parameters are in table 20. We have modified the simulated Hipparcos coordinates as well as the simulated VLBI data by introducing gaussian noise. The uncertainties adopted in this simulation are based on ESA estimates of Hipparcos expected accuracy and on the most recent VLBI phase-referenced observations performed on Algol and \( \sigma \) CrB (Lestrade et al. 1990, Lestrade et al. 1992).
Table 19. Simulated VLBI positions and proper motions of the link stars.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>h m s</td>
<td>° ' &quot;</td>
<td>μα (s/a)</td>
</tr>
<tr>
<td>LSI 61°303</td>
<td>02 40 31.6860</td>
<td>61 13 45.560</td>
<td>1.3</td>
</tr>
<tr>
<td>ALGOL</td>
<td>03 08 10.1308</td>
<td>40 57 20.359</td>
<td>1.3</td>
</tr>
<tr>
<td>UX ARI</td>
<td>03 26 35.3375</td>
<td>28 42 56.026</td>
<td>1.3</td>
</tr>
<tr>
<td>HR 1099</td>
<td>03 36 47.3251</td>
<td>00 35 18.600</td>
<td>1.3</td>
</tr>
<tr>
<td>HD 26337</td>
<td>04 09 40.8640</td>
<td>-07 53 35.640</td>
<td>6.7</td>
</tr>
<tr>
<td>HD 32918</td>
<td>04 55 11.0440</td>
<td>-74 56 16.020</td>
<td>6.7</td>
</tr>
<tr>
<td>AB Dor</td>
<td>05 28 44.5351</td>
<td>-65 27 02.191</td>
<td>6.7</td>
</tr>
<tr>
<td>HD 77137</td>
<td>08 59 42.7650</td>
<td>-27 48 58.110</td>
<td>6.7</td>
</tr>
<tr>
<td>HR 5110</td>
<td>13 34 47.6893</td>
<td>37 10 56.859</td>
<td>1.3</td>
</tr>
<tr>
<td>σ CrB</td>
<td>16 14 41.2011</td>
<td>33 51 32.470</td>
<td>1.3</td>
</tr>
<tr>
<td>Cyg X-1</td>
<td>19 58 21.6804</td>
<td>35 12 05.887</td>
<td>1.3</td>
</tr>
<tr>
<td>AR Lac</td>
<td>22 08 40.8710</td>
<td>45 44 31.510</td>
<td>1.3</td>
</tr>
<tr>
<td>SZ Psc</td>
<td>23 13 23.7645</td>
<td>02 40 31.310</td>
<td>1.3</td>
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<tr>
<td>II Peg</td>
<td>23 52 29.0891</td>
<td>28 21 17.740</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Units: (1) 0.0001 s; (2) 0.001"; (3) 0.0001 s/a; (4) 0.001"/a.
| Star          | Right. Asc. J 2000.0 h m s | Declination J 2000.0 ° ' " | $\sigma_\alpha$ (1) | $\sigma_\delta$ (2) | $\mu_\alpha$ (s/a) | $\mu_\delta$ ("/a) | $\sigma_\mu_\alpha$ (3) | $\sigma_\mu_\delta$ (4) | Epoch -1900 |
|--------------|-----------------------------|-----------------------------|----------------------|----------------------|--------------------|------------------------|------------------------|--------------|
| LSI 61°303   | 02 40 31.685                | 61 13 45.561                | 3.3                  | 5.0                  | +0.0001            | +0.004                | 3.3                    | 5.0          | 91.0        |
| ALGOL        | 03 08 10.131                | 40 57 20.365                | 1.3                  | 2.0                  | +0.0004            | +0.006                | 1.3                    | 2.0          | 91.0        |
| UX ARI       | 03 26 35.341                | 28 42 55.762                | 1.3                  | 2.0                  | +0.0017            | -0.103                | 1.3                    | 1.0          | 91.0        |
| HR 1099      | 03 36 47.323                | 00 35 18.521                | 1.3                  | 2.0                  | -0.0019            | -0.158                | 1.3                    | 2.0          | 91.0        |
| HD 26337     | 04 09 40.862                | -07 53 35.833               | 1.3                  | 2.0                  | +0.0010            | +0.133                | 1.3                    | 2.0          | 91.0        |
| HD 32918     | 04 55 11.033                | -74 56 16.105               | 1.3                  | 2.0                  | +0.0067            | +0.061                | 1.3                    | 2.0          | 91.0        |
| AB Dor       | 05 28 44.522                | -65 27 02.388               | 1.3                  | 2.0                  | +0.0081            | +0.135                | 1.3                    | 2.0          | 91.0        |
| HD 77137     | 08 59 42.770                | -27 48 58.033               | 1.3                  | 2.0                  | -0.0037            | -0.050                | 1.3                    | 2.0          | 91.0        |
| HR 5110      | 13 34 47.699                | 37 10 56.840                | 1.3                  | 2.0                  | +0.0070            | -0.017                | 1.3                    | 2.0          | 91.0        |
| SIG CrB      | 16 14 41.123                | 33 51 32.188                | 1.3                  | 2.0                  | -0.0221            | -0.083                | 1.3                    | 2.0          | 91.0        |
| Cyg X-1      | 19 58 21.681                | 35 12 05.892                | 1.3                  | 2.0                  | -0.0017            | -0.018                | 1.3                    | 2.0          | 91.0        |
| AR Lac       | 22 08 40.866                | 45 44 31.559                | 1.3                  | 2.0                  | -0.0030            | +0.037                | 1.3                    | 2.0          | 91.0        |
| SZ Psc       | 23 13 23.764                | 02 40 31.246                | 1.3                  | 2.0                  | -0.0001            | +0.044                | 1.3                    | 2.0          | 91.0        |
| II Peg       | 23 52 29.023                | 28 21 17.681                | 1.3                  | 2.0                  | +0.0436            | +0.041                | 1.3                    | 2.0          | 91.0        |

Units: (1) 0.0001s; (2) 0.001"; (3) 0.0001s/a; (4) 0.001"/a.
Figure 19. Sky distribution of the 14 radio stars considered in the simulation of the link.

Accuracy of the link

With equations (10) and (11) and the VLBI and Hipparcos simulated coordinates with added noise, we estimate the angles $A_1(t_0)$, $A_2(t_0)$, $A_3(t_0)$ between the axes of the frames, and the components of the angular velocity of rotation $A_1$, $A_2$, $A_3$. We have analysed different cases of link to test the influence of the weighting of data and of the distribution of radio stars. The fit is done by a least squares analysis, with a weighting based on the uncertainties in the VLBI and Hipparcos positions and proper motions. We performed several solutions, assigning different values to the uncertainties in the VLBI data (see table 19).

Tables 21 and 22 present the results obtained by fixing the uncertainties of the positions and proper motions of the northern stars to the values indicated with (wl) in table 19. In case 1 we have adopted the values 0.010" and 0.007"/year for the errors of the coordinates and the proper motions determined with VLBI for the southern stars (values indicated with (w3) in table 19). When these values are reduced to one half (case 2), the precision of the link is 10% better, but the consistency between the adopted and the calculated values of the six link parameters, as well as the correlation coefficients remain unchanged. The parameter R indicates the goodness of the fit normalized by the number of degree of freedom (Bevington, 1969).

In case 3 we have performed the link using only the 10 radio stars in the northern hemisphere. When compared to the solution with the 14 stars, the consistency between the angles $A_i$ and the components of the angular velocity $A_i$, the correlation coefficients do not change. The precision of the adjustment, given by the rms post-fit residuals ($\sigma$ in table 20) is 25% better when only the 10 northern radio stars are included in the link.
Table 21. Results of the simulated VLBI-Hipparcos astrometric link. N is the number of stars used, σ is the standard deviation of the residuals after the fit and R is the goodness of fit. Unit for \( A_1, A_2, A_3, \) and \( σ : 0.001'' \). - Unit for \( \hat{A}_1, \hat{A}_2, \hat{A}_3, : 0.001''/a. \\

<table>
<thead>
<tr>
<th>Case</th>
<th>N</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( \hat{A}_1 )</th>
<th>( \hat{A}_2 )</th>
<th>( \hat{A}_3 )</th>
<th>( σ )</th>
<th>R</th>
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<td>1</td>
<td>14</td>
<td>+31.33±1.7</td>
<td>+22.78±1.54</td>
<td>-44.34±1.15</td>
<td>+2.11±1.13</td>
<td>-3.31±1.01</td>
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<td>+22.96±1.40</td>
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<td>-3.24±0.90</td>
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<td>+22.96±1.40</td>
<td>-44.34±1.01</td>
<td>+2.14±1.01</td>
<td>-3.70±0.95</td>
<td>+1.65±0.67</td>
<td>1.56</td>
<td>1.01</td>
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<td>-44.27±1.07</td>
<td>+1.35±0.83</td>
<td>-3.12±0.75</td>
<td>+1.47±0.70</td>
<td>1.79</td>
<td>1.17</td>
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<td>+22.79±1.03</td>
<td>-43.10±1.10</td>
<td>+1.79±0.77</td>
<td>-3.31±0.67</td>
<td>+1.64±0.70</td>
<td>1.66</td>
<td>0.96</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
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<td>+23.13±1.18</td>
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North and South stars
VLBI uncertainties of South stars:
0.010" and 0.007"/a
VLBI uncertainties of South stars:
0.005" and 0.004"/a
North stars only

Comments:
- LSI 61°303 deleted
- HR 1099 deleted
- ALGOL deleted
- αCrB deleted
- SZ Psc deleted
- AR Lac deleted
- HR 1099 deleted
- ALGOL, αCrB, HR 1099 deleted
- HR 1099, SZ Psc deleted
- ALGOL, HR 1099, HD32918, αCrB, AR Lac deleted
- HR 1099, LSI 61°303 deleted
Table 22. Adjustments to the a priori angles, in relative value with respect to their formal uncertainties.

<table>
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<tr>
<th>Case</th>
<th>ΔA₁</th>
<th>ΔA₂</th>
<th>ΔA₃</th>
<th>ΔÅ₁</th>
<th>ΔÅ₂</th>
<th>ΔÅ₃</th>
<th>Correlation coefficients</th>
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<tr>
<td></td>
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<td>0.6</td>
<td>0.1</td>
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<td>-0.06</td>
</tr>
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<td>0.6</td>
<td>0.0</td>
<td>-0.3</td>
<td>0.5</td>
<td>-0.06</td>
</tr>
<tr>
<td>3</td>
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<td>-1.5</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.7</td>
<td>1.0</td>
<td>-0.01</td>
</tr>
<tr>
<td>4</td>
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<td>-1.5</td>
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<td>-0.8</td>
<td>-0.2</td>
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</tr>
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<td>-2.1</td>
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<td>0.9</td>
<td>+0.02</td>
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<td>-1.3</td>
<td>-0.6</td>
<td>0.9</td>
<td>+0.01</td>
</tr>
<tr>
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<td>-0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>+0.11</td>
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<td>-0.2</td>
<td>-0.5</td>
<td>0.9</td>
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<tr>
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</table>
Seven other cases of link have been analysed including nine radio stars at positive declinations (cases 4 to 10). Here, we intend to test the influence of individual objects on the link. In each case, one object has been eliminated, because its physical and/or astrometric characteristics make it a particular case. The precision of the adjustment given by the rms of the post-fit residuals and by the values of the parameter R indicates that the quality of the link in either of these cases is comparable to that of case 3 (only northern stars).

Table 23 presents the mean adjustment to the a priori angles and rates when the lowest values of the VLBI uncertainties are considered (indicated with \((w2)\) in table 19), and the errors in Hipparcos data are fixed to the values indicated with \((w4)\) in table 19. In this analysis we have considered two cases of link: all 14 radio stars and only the 10 stars at positive declinations. Comparing these results with cases 2 and 3 of table 22 we find that, when the uncertainties of the VLBI data are four times better than the Hipparcos astrometric uncertainties the precision of the link given by the 14 radio stars is more than 50% higher for the angles \(A_i\), and that the error in the components \(\dot{A}_i\) of the relative velocity of rotation is between 10% to 30% better than the Hipparcos astrometric uncertainties. When only the 10 northern radio stars are considered, the best estimates of the VLBI data errors lead to a solution 40%-80% more precise in the angles \(A_i\), and 2%-30% more precise in their time derivatives \(\dot{A}_i\).

Table 23. Accuracy of the simulated link between Hipparcos and VLBI reference frames via radio stars. Units of \(A_k = 0.001"\), units of \(\dot{A}_k = 0.001"/\text{year}\), \(\sigma\) are the post-fit residuals, in units of 0.001", R is the goodness-of-fit parameter.

<table>
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<th>N</th>
<th>14 All stars</th>
<th>10 Northern stars</th>
</tr>
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<tr>
<td>(A1)</td>
<td>+ 1.09 ± 0.91</td>
<td>+ 1.45 ± 0.87</td>
</tr>
<tr>
<td>(A2)</td>
<td>- 1.67 ± 0.91</td>
<td>- 1.73 ± 0.79</td>
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<tr>
<td>(A3)</td>
<td>+ 0.07 ± 0.73</td>
<td>+ 0.35 ± 0.71</td>
</tr>
<tr>
<td>(\dot{A}1)</td>
<td>- 0.51 ± 0.82</td>
<td>- 0.56 ± 0.79</td>
</tr>
<tr>
<td>(\dot{A}2)</td>
<td>+ 0.22 ± 0.77</td>
<td>+ 0.02 ± 0.72</td>
</tr>
<tr>
<td>(\dot{A}3)</td>
<td>+ 0.08 ± 0.67</td>
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<td>(\sigma)</td>
<td>1.92</td>
<td>1.55</td>
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<td>R</td>
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