CHAPTER 3 CONVENTIONAL TERRESTRIAL REFERENCE FRAME

Definition

The Terrestrial Reference System adopted for either the analysis of individual data sets by techniques (VLBI, SLR, LLR, GPS...) or the combination of individual solutions into a unified set of data (station coordinates, Earth orientation parameters, etc...) follows these criteria (Boucher, 1991):

a) It is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere.

b) Its scale is that of a local Earth frame, in the meaning of a relativistic theory of gravitation.

c) Its orientation is given by the BIH orientation at 1984.0.

d) Its time evolution in orientation will create no residual global rotation with regards to the crust.

Realization

When one wants to realize such a conventional terrestrial reference system through a reference frame i.e. a network of station coordinates, it will be specified by Cartesian equatorial coordinates X, Y, and Z, by preference. If geographical coordinates are needed, the GRS80 ellipsoid is recommended (see table 3.2).

Each analysis center compares its reference frame to a realization of the Conventional Terrestrial Reference System (CTRS), as described above. Within IERS, each Terrestrial Reference System (TRF) is either directly, or after transformation, expressed as a realization of the CTRS adopted by IERS as its ITRS. The position of a point located on the surface of the solid Earth should be expressed by

$$\vec{X}(t) = \vec{X}_0 + \vec{V}_0(t-t_0) + \sum_i \Delta\vec{X}_i(t),$$

where $\Delta\vec{X}_i$ are corrections to various time changing effects, and $\vec{X}_0$ and $\vec{V}_0$ are position and velocity at the epoch t_0. The corrections to be considered are solid Earth tide displacement (full correction including permanent effect, so that the extra correction which was originally recommended in order to have zero mean correction is no longer valid), ocean loading, and atmospheric loading.

Further corrections could be added if they are at mm level and can be computed by a suitable model. The velocity $\bar{V}_0$ should be expressed as

$$\vec{V}_0 = \vec{V}_{plate} + \vec{V}_r$$

where $\bar{V}_{plate}$ is the horizontal velocity computed from the NNR-NUVEL-1 model (DeMets, *et al.*, 1990; Argus and Gordon, 1991) and $\bar{V}_r$ a residual velocity.

In data analysis, $\bar{X}_0$ and $\bar{V}_r$ should be considered as solve-for parameters. In particular, if a non linear change occurs (earthquake, volcanic event ...), a new $\bar{X}_0$ parameter should be adopted. When adjusting parameters, particularly velocities, the IERS orientation should be kept at all epochs, ensuring the alignment at a reference epoch and the time evolution through a no net rotation condition. The way followed by various analysis centers depends on their own view of modelling, and on the techniques themselves. For the origin, only data which can be modelled by dynamical techniques (currently SLR, LLR or GPS for IERS) can determine the center of mass. The VLBI system can be referred to a geocentric system by adopting for a station its geocentric position at a reference epoch as provided from external information.

The scale is obtained by appropriate relativistic modelling. This is particularly true for VLBI and LLR which are usually modelled in a barycentric frame. A more detailed treatment can be found in chapter 14. The orientation is defined by adopting IERS (or BIH) Earth orientation parameters at a reference epoch. In the case of SLR, an additional constraint in longitude is necessary.

The unit of length is the meter (SI). The IERS Reference Pole (IRP) and Reference Meridian (IRM) are consistent with the corresponding directions in the BIH Terrestrial System (BTS) within $\pm 0''.005$. The BIH reference pole was adjusted to the Conventional International Origin (CIO) in 1967; it was then kept stable independently until 1987. The uncertainty of the tie of the IRP with the CIO is $\pm 0''.03$. The time evolution of the orientation will be insured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth.

Transformation Parameters of World Coordinate Systems and Datums

The seven-parameter (similarity transformation between any two Cartesian systems, *e. g.*, from (u,v,w) to (x,y,z), or in short (u,v,w) → (x,y,z) can be written (Soler and Hothem, 1989) as

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} + (1 + \delta s) \begin{bmatrix} 1 & \delta\omega & -\delta\psi \\ -\delta\omega & 1 & \delta\epsilon \\ \delta\psi & -\delta\epsilon & 1 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}, \qquad (1)$$

where $\Delta x, \Delta y, \Delta z$ = coordinates of the origin of the frame (u,v,w) in the frame (x,y,z); $\delta\epsilon$, $\delta\psi$, $\delta\omega$ = differential rotations (expressed in radians) respectively, around the axes (u,v,w) to establish parallelism with the (x,y,z) frame (Positive rotations are counterclockwise rotations as viewed looking toward the origin of the right handed coordinate system.); and δs = differential scale change (expressed in ppm $\times 10^{-6}$) (see Table 3.1).

Table 3.1. Transformation parameters from ITRF90 to major global or local datums.

Coordinate System (datum)	Δx T1 (m)	Δy T2 (m)	Δz T3 (m)	δs D (ppm)	$\delta\varepsilon$ -R1 (")	$\delta\phi$ -R2 (")	$\delta\omega$ -R3 (")
WGS84	0.060	-0.517	-0.223	-0.011	-0.0183	0.0003	-0.0070
WGS72	0.060	-0.517	-4.723	-0.231	-0.0183	0.0003	0.5470
NWL9D	0.060	-0.517	-4.723	0.599	-0.0183	0.0003	0.8070
BTS84	-0.058	0.028	-0.036	0.030	-0.0035	-0.0020	-0.0017
BTS85	-0.004	0.049	0.006	0.025	-0.0026	0.0005	0.0014
BTS86	0.027	-0.011	-0.044	0.008	-0.0008	0.0023	0.0072
BTS87	-0.011	-0.008	-0.057	0.006	-0.0004	-0.0002	-0.0003
ITRF0	-0.007	-0.009	-0.055	0.005	-0.0004	-0.0002	-0.0001
ITRF88	0.000	-0.012	-0.062	0.006	-0.0001	0.0000	0.0000
ITRF89	0.005	0.024	-0.038	0.003	0.0000	0.0000	0.0000

Once the Cartesian coordinates (x,y,z) are known, they can be transformed to "datum" or curvilinear geodetic coordinates (λ,ϕ,h) referred to an ellipsoid of semi-major axis a and flattening f (see Table 3.2), using the following noniterative method (Bowring, 1985):

$$\tan \lambda = \frac{y}{x}, \qquad (2)$$

$$\tan \phi = \frac{z(1 - f) + e^2 a \sin^3 \mu}{(1 - f)(p - e^2 a \cos^3 \mu)}, \tag{3}$$

$$h = p \cos \phi + z \sin \phi - a(1 - e^2 \sin^2 \phi)^{1/2}, \tag{4}$$

$$e^2 = 2f - f^2, \tag{5}$$

$$p = (x^2 + y^2)^{1/2}, \tag{6}$$

$$r = (p^2 + z^2)^{1/2}, \tag{7}$$

$$\tan \mu = \frac{z}{p} \left[(1 - f) + \frac{e^2 a}{r} \right]. \tag{8}$$

The preceding equations can be used in conjunction with Tables 3.1 and 3.2 superseding the ones previously presented in Soler and Hothem (1988).

Table 3.2. Parameters of Some Adopted Reference Ellipsoids.

Coordinate system (datum)	Reference ellipsoid used	a (m)	1/f
AGD	AN (or SA-69)	6,378,160	298.25
ED-79	International	6,378,388	297
GEM-8	GEM-8	6,378,145	298.255
GEM-9 (or GEM-10)	GEM-9 or (GEM-10)	6,378,140	298.255
GEM-10B	GEM-10B	6,378,138	298.257
GEM-T1	GEM-T1	6,378,137	298.257
GEM-T3	GEM-T3	6,378,137	298.257
NAD-27	Clarke 1866	6,378,206.4	294.9786982
NAD-83	GRS-80	6,378,137	298.257222101
NWL-9D = NSWC-9Z2	WGS-66	6,378,145	298.25
SA-69	SA-69 (or AN)	6,378,160	298.25
WGS-72	WGS-72	6,378,135	298.26
WGS-84	WGS-84	6,378,137	298.257223563

Note: AGD = Australian geodetic datum; AN = Australian national; ED = European datum; GEM = Goddard Earth model; GRS = geodetic reference system; NAD = North American datum; NSWC = Naval surface warfare center; NWL = Naval Weapons Laboratory; SA = South American; WGS = World geodetic system.

In the case of IERS, these algorithms are used with Table 3.1, where (x,y,z) are coordinates in the specified coordinate system (datum), and (u,v,w,) are identified with ITRF, or a more specific realization (ITRF90 for table 3.1). In the current practice of the IERS/CB and its publications, (u,v,w) are denoted (X,Y,Z) and the coordinates in an individual system by (XS, YS, ZS), while the 7 parameters are identified as (T_1, T_2, T_3) refer to (Δx, Δy, Δz), D to δs and (R_1,R_2,R_3) to ($-\delta\varepsilon$,$-\delta\Psi$,$-\delta\tilde{\omega}$).

Transformations to Current Datums

Table 3.1 gives values recommended to convert ITRF90 coordinates into other datums. Table 3.2 Lists the required two parameters for several adopted reference ellipsoids defining important geodetic datums. Some ellipsoids were introduced by NASA's GSFC to reference their Goddard Earth Models (GEM, series 8,9, and 10). They were primarily used to obtain geoid heights (undulations) and depict global geoid maps.

These transformation formulae should be used with care. There are several ways to determine such parameters, either by direct comparison between two realizations of the datums, or by combining formula through an intermediate datum. This is well illustrated by WGS84 as mentioned below.

The numbers given in Table 3.1 have been determined using the following data:

a) The transformation between BTS87 and WGS84 derived from Boucher, Altamimi, and Willis (1988).

b) Transformation parameters between the Doppler realized frames (i.e., NWL-9D = NSWC-9Z2, WGS-72, WGS-84) have been adopted by the Defense Mapping Agency. Recall that Cartesian coordinates, derived from using the Global Positioning System (GPS), are also referred to the WGS-84 coordinate system. Nevertheless, the Doppler WGS-84 (GPS) frames are not necessarily coincident (Malys, 1988). Similarly, although by definition, the NAD-83 and WGS-84 realized Cartesian frames should coincide, small differences (<0.5 ppm) in shifts, rotations, and scale between the two frames may be discovered. These differences merely reflect small, random regional distortions still present in the NAD-83 horizontal datum, which was primarily established by simultaneously adjusting all archived classical-geodetic observations(Bossler, 1987; Schwarz, 1989).

c) Transformations to old BIH or IERS systems are derived from IERS TN4, 6, and 9.

Transformations to major local datums can be obtained through WGS84.

Plate Motion Model

One of the factors which can affect Earth rotation results is the motion of the tectonic plates which makes up the Earth's surface. As the plates move, fixed coordinates for the observing stations will become inconsistent with each other. The rates of relative motions for some regular observing sites are believed to be 5 cm per year or larger. The observations of plate motions so far by Satellite Laser Ranging and Very Long Baseline Interferometry appear to be roughly consistent with the average rates over the last few million years derived from the geological record and other geophysical information. Thus, in order to reduce inconsistencies in the station coordinates and to make the results from different techniques more directly comparable, a model for plate motions given by DeMets, *et al.* (1990) is recommended.

The Cartesian rotation vector for each of the major plates is given in Table 3.3 A subroutine called ABSMO_NUVEL, provided by J. B. Minster, is also included below. It computes the new site position at time t from the old site position at time t0 using the recommended plate motion model.

Table 3.3. Cartesian rotation vector for each plate using the NUVEL NNR-1 kinematic plate model (no net rotation)

Plate Name	Ω_X deg/My.	Ω_Y deg/My.	Ω_Z deg/My.
Pacific	-0.0907	0.2902	-0.5976
Cocos	-0.6249	-1.2944	0.6544
Nazca	-0.0921	-0.5138	0.5756
Caribbean	-0.0109	-0.2027	0.0945
South America	-0.0624	-0.0906	-0.0523
Antarctica	-0.0494	-0.1018	0.2218
India	0.3995	0.0026	0.4066
Australia	0.4695	0.3072	0.3762
Africa	0.0532	-0.1856	0.2348
Arabia	0.4003	-0.0311	0.4049
Eurasia	-0.0590	-0.1434	0.1887
North America	0.0152	-0.2155	-0.0094
Juan de Fuca	0.2995	0.4805	-0.2936
Philippine	0.5913	-0.4412	-0.5976

The NUVEL model should be used as a default, for stations which appear to follow reasonably its values. For some stations, particularly in the vicinity of plate boundaries, users may benefit by estimating velocities or using specific values not derived from NUVEL. This is also a way to take into account now some non-negligible vertical motions. Published station coordinates should include the epoch associated with the coordinates.

The original subroutine is a coding of the AM0-2 model from J. B. Minster. This was made by modifying the earlier subroutine. The changes were made by Don Argus and verified by Alice Gripp.

```
      SUBROUTINE ABSMO_NUVEL(PSIT,T0,X0,Y0,Z0,T,X,Y,Z)
C
C   ABSMO_NUVEL take a site specified by its initial coordinates
C   X0,Y0,Z0 at time T0, and computes its updated positions X,Y,Z
C   at time T, based on the geological "absolute", (no net
C   rotation) plate motion model AM0-2 (Minster and Jordan,
C   1978).
C
C   Original author: J.B. Minster, Science Horizons.
C   DFA: Revised by Don Argus, Northwestern University
C   DFA: uses absolute model NNR-NUVEL1
C
C   Transcribed from USNO Circular 167 "Project Merit Standards"
C   by Tony Mallama with slight modification to the documentation
C   and code.
C
C   Times are given in years, e.g. 1988.0 for Jan 1, 1988.
C
C   PSIT is the four character abbreviation for the plate name,
C   if PSIT is not recognized then the new positions are returned
C   as zero.
C
      IMPLICIT NONE
      CHARACTER*4         PSIT,PNM(14)
      REAL*8              OMX(14),OMY(14),OMZ(14)
      REAL*8              X0,Y0,Z0
      REAL*8              X,Y,Z,T,T0
      REAL*8              ORX,ORY,ORZ
      INTEGER*2           IPSIT,I
C
C   DFA: NNR-NUVEL1
C
      DATA     (PNM(I),     OMX(I),     OMY(I),     OMZ(I),
     &          I =  1,14)
     &         /'PCFC',    -0.0907,     0.2902,    -0.5976,
     &          'AFRC',     0.0532,    -0.1856,     0.2348,
     &          'ANTA',    -0.0494,    -0.1018,     0.2218,
     &          'ARAB',     0.4003,    -0.0311,     0.4049,
     &          'AUST',     0.4695,     0.3072,     0.3762,
     &          'CARB',    -0.0109,    -0.2027,     0.0945,
     &          'COCO',    -0.6249,    -1.2944,     0.6544,
     &          'EURA',    -0.0590,    -0.1434,     0.1887,
     &          'INDI',     0.3995,     0.0026,     0.4066,
     &          'NAZC',    -0.0921,    -0.5138,     0.5756,
     &          'NOAM',     0.0152,    -0.2155,    -0.0094,
     &          'SOAM',    -0.0624,    -0.0906,    -0.0523,
     &          'JUFU',     0.2995,     0.4805,    -0.2936,
     &          'PHIL',     0.5913,    -0.4412,    -0.5976/
C
C
C    Initialize things properly
C
```

```
      IPSIT = -1
      X = 0.0D0
      Y = 0.0D0
      Z = 0.0D0
C
C   Look up the plate in the list.
C
      DO 20 I = 1,14
   20 IF (PSIT .EQ. PNM(I)) IPSIT = I
C
C   If plate name is not recognized return the new plate position
C   as zero.
C
      IF (IPSIT .EQ. -1) RETURN
C
C   Convert from degree/My to radians/yr.
C
      ORX = OMX(IPSIT) * 1.7453292D-08
      ORY = OMY(IPSIT) * 1.7453292D-08
      ORZ = OMZ(IPSIT) * 1.7453292D-08
C
C   Compute the new coordinates
C
      X = X0 + (ORY*Z0 - ORZ*Y0) * (T-T0)
      Y = Y0 + (ORZ*X0 - ORX*Z0) * (T-T0)
      Z = Z0 + (ORX*Y0 - ORY*X0) * (T-T0)
C
C   Finish up
C
      RETURN
      END
```

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