CHAPTER 13 GENERAL RELATIVISTIC MODELS FOR TIME, COORDINATES AND EQUATIONS OF MOTION

The relativistic treatment of the near-Earth satellite orbit determination problem includes correction to the equations of motion, the time transformations, and the measurement model. The two coordinate systems generally used when including relativity in near-Earth orbit determination solutions are the solar system barycentric frame of reference and the geocentric or Earth-centered frame of reference.

Ashby and Bertotti (1986) constructed a locally inertial E-frame in the neighborhood of the gravitating Earth and demonstrated that the gravitational effects of the Sun, Moon, and other planets are basically reduced to their tidal forces, with very small relativistic corrections. Thus the main relativistic effects on a near-Earth satellite are those described by the Schwarzschild field of the Earth itself. This result makes the geocentric frame more suitable for describing the motion of a near-Earth satellite (Ries, *et al.*, 1988).

The time coordinate in the inertial E-frame is Terrestrial Time (designated TT) (Guinot, 1991) which can be considered to be equivalent to the previously defined Terrestrial Dynamical Time (TDT). This time coordinate (TT) is realized in practice by International Atomic Time (TAI), whose rate is defined by the atomic second in the International System of Units (SI). Terrestrial Time adopted by the International Astronomical Union in 1991 differs from Geocentric Coordinate Time (TCG) by a scaling factor:

$$\mathrm{TCG-TT} = 6.9693 \times 10^{-10} \times (\mathrm{MJD}-43144.0) \times 86400 \text{ seconds},$$

where MJD refers to the modified Julian date. Figure 13.1 shows graphically the relationships between the time scales.

Equations of Motion for an Artificial Earth Satellite

The correction to the acceleration of an artificial Earth satellite $\Delta\vec{a}$ is

$$\Delta\vec{a} = -\frac{GM_\oplus}{c^2r^3}\left[[2(\beta+\gamma)\frac{GM_\oplus}{r} - \gamma v^2]\vec{r} + [2(1+\gamma)(\vec{r}\cdot\vec{v})\vec{v}]\right], \qquad (1)$$

where c = speed of light,

β, γ = PPN parameters equal to 1 in General Relativity,

$\vec{r}, \vec{v}, \vec{a}$ = geocentric satellite position, velocity, and acceleration, respectively,

$GM_{\oplus}$ = gravitational parameter of the Earth.

The effects of Lense-Thirring precession (frame-dragging), geodesic (de Sitter) precession, and the relativistic effects of the Earth's oblateness have been neglected.

Equations of Motion in the Barycentric Frame

The n-body equations of motion for the solar system frame of reference (the isotropic Parameterized Post-Newtonian system with Barycentric Coordinate Time (TCB) as the time coordinate) are required to describe the dynamics of the solar system and artificial probes moving about the solar system (for example, see Moyer, 1971). These are the equations applied to the Moon's motion for Lunar Laser Ranging (Newhall, Williams, and Dickey, 1987). In addition, relativistic corrections to the laser range measurement, the data timing, and the station coordinates are required (see Chapter 14).

Scale Effect and Choice of Time Coordinate

The previous IAU definition of the time coordinate in the barycentric frame required that only periodic differences exist between Barycentric Dynamical Time (TDB) and Terrestrial Dynamical Time (TDT) (Kaplan, 1981). As a consequence, the spatial coordinates in the barycentric frame had to be rescaled to keep the speed of light unchanged between the barycentric and the geocentric frames (Misner, 1982; Hellings, 1986). Thus, when barycentric (or TDB) units of length were compared to geocentric (or TDT) units of length, a scale difference, L, appeared. This is no longer required with the use of the TCG time scale.

The difference between TCB and TDB is given in seconds by Fukushima *et al.* (1986) as

$$\mathrm{TCB-TDB} = 1.550505\times10^{-8}(\pm1\times10^{-14})\times(\mathrm{MJD}-43144.0)\times 86400.$$

The difference between Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG) involves a four-dimensional transformation,

$$\mathrm{TCB-TCG} = c^{-2}\{\int_{t_0}^{t}[\vec{v}\cdot\vec{v}/2 + U_{ext}(\vec{x}_e)]dt + \vec{v}_e\cdot(\vec{x}-\vec{x}_e)\},$$

where $\vec{x}_e$ and $\vec{v}_e$ denote the barycentric position and velocity of the Earth's center of mass and $\vec{x}$ is the barycentric position of the observer. U_{ext} is the Newtonian potential of all of the solar system bodies apart from the Earth evaluated at the geocenter. t_0 is chosen to be consistent with 1977 January 1, $0^h\ 0^m\ 0^s$ TAI and t is

TCB. An approximation is given in seconds by Fukushima *et al.* (1986) as

$$(TCB-TCG) = 1.480813\times10^{-8}(\pm1\times10^{-14})\times(MJD-43144.0)\times86400 + c^{-2}\vec{v}_e\cdot(\vec{x}-\vec{x}_e) + P.$$

with MJD measured in TAI. For observers on the Earth's surface, diurnal periodic differences denoted by P with a maximum amplitude of 2.1 μs also remain. These can be evaluated from positions and motions of solar system bodies using expressions of Hirayama *et al.* (1987).

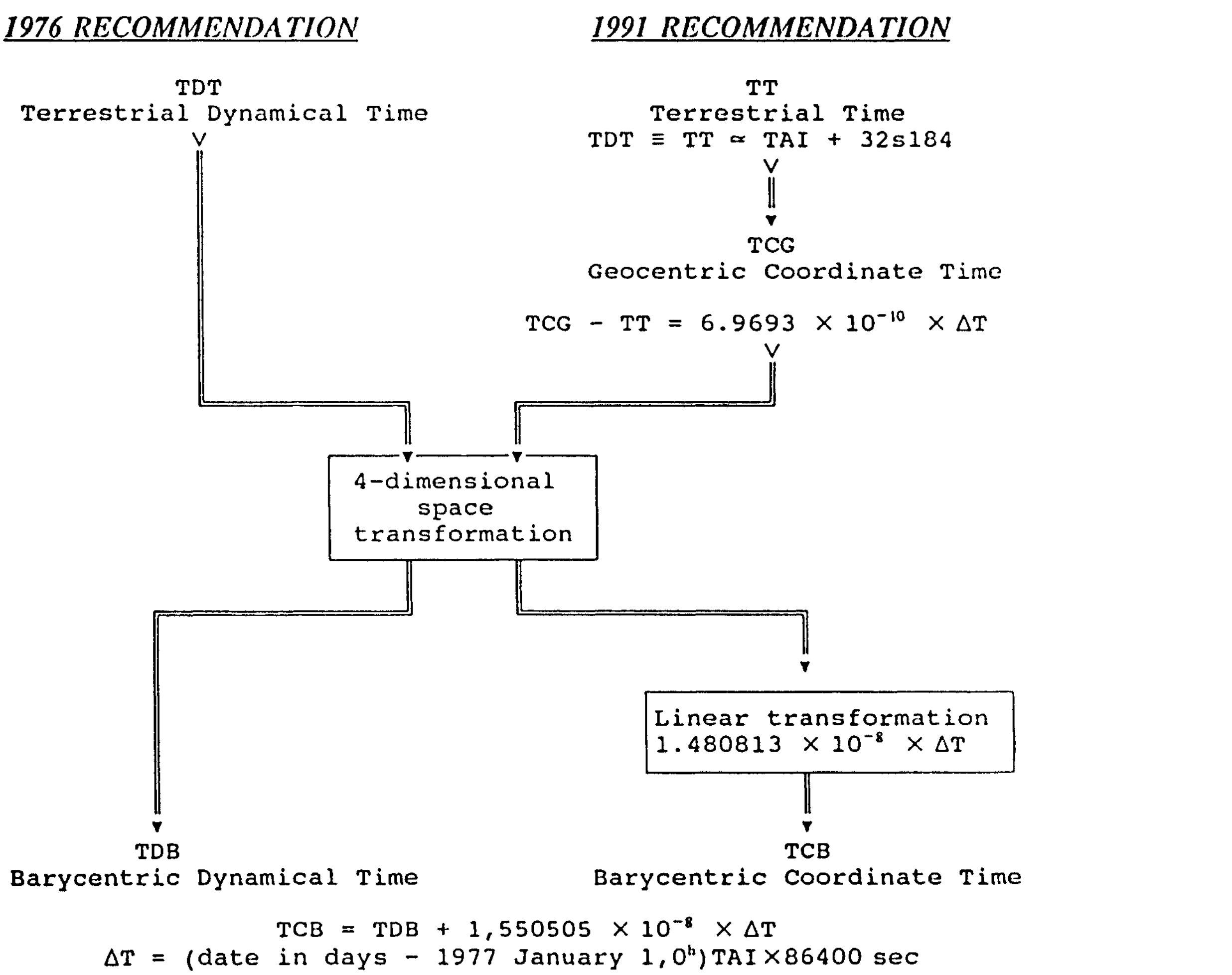

Fig 13.1 Relations between time scales.

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