CHAPTER 8 OCEAN TIDE MODEL

The dynamical effect of ocean tides is most easily implemented as periodic variations in the normalized geopotential coefficients. The variations can be written as (Eanes, *et al.*, 1983):

$$\Delta \bar{C}_{nm} - i\Delta \bar{S}_{nm} = F_{nm} \sum_{s(n,m)} \sum_{+}^{-} (C^{\pm}_{snm} \mp iS^{\pm}_{snm}) e^{\pm i\theta_s}, \qquad (1)$$

where

$$F_{nm} = \frac{4\pi G\rho_w}{g} \sqrt{\frac{(n+m)!}{(n-m)!(2n+1)(2-\delta_{om})}} \frac{1+k'_n}{2n+1},$$

g = 9.798261 ms^{-2},

G = The universal gravitational constant = 6.673 × $10^{-11} m^3 kg^{-1} s^{-2}$,

ρ_w = density of seawater = 1025 kg m^{-3},

k'_n = load deformation coefficients (k'_2 = −0.3075, k'_3 = −0.195, k'_4 = −0.132, k'_5 = −0.1032, k'_6 = −0.0892),

$C^{\pm}_{snm}$, $S^{\pm}_{snm}$ = ocean tide coefficients in m for the tide constituent s (see Table 8.2),

θ_s = argument of the tide constituent s as defined in the solid tide model (Chapter 7).

The summation, $\sum_{+}^{-}$, implies addition of the expression using the top signs (the prograde waves C^{+}_{snm} and S^{+}_{snm}) to that using the bottom signs (the retrograde waves C^{-}_{snm} and S^{-}_{snm}). The ocean tide coefficients $C^{\pm}_{snm}$ and $S^{\pm}_{snm}$ as used here are related to the Schwiderski (1983) ocean tide amplitude and phase by

$$C^{\pm}_{snm} - iS^{\pm}_{snm} = -i\hat{C}^{\pm}_{snm}\, e^{i(\varepsilon^{\pm}_{snm} + \chi_s)}, \qquad (2)$$

where

$\hat{C}^{\pm}_{snm}$ = ocean tide amplitude for constituent s using the Schwiderski notation,
$\varepsilon^{\pm}_{snm}$ = ocean tide phase for constituent s,

and χ_s is obtained from Table 8.1, with H_s being the Cartwright and Tayler amplitude at frequency s.

Table 8.1. Values of χ_s for long-period, diurnal and semidiurnal tides.

Tidal Band	$H_s>0$	$H_s<0$
Long Period	π	0
Diurnal	$\pi/2$	$-\pi/2$
Semi-diurnal	0	π

For clarity, equation 1 is rewritten in two forms below:

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [(C^+_{snm} + C^-_{snm}) \cos\theta_s + (S^+_{snm} + S^-_{snm}) \sin\theta_s] \quad (3a)$$

or

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^+_{snm} \sin(\theta_s+\epsilon^+_{snm}+\chi_s) + \hat{C}^-_{snm} \sin(\theta_s+\epsilon^-_{snm}+\chi_s)], \quad (3b)$$

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [(S^+_{snm} - S^-_{snm}) \cos\theta_s - (C^+_{snm} - C^-_{snm}) \sin\theta_s] \quad (3c)$$

or

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^+_{snm} \cos(\theta_s+\epsilon^+_{snm}+\chi_s) - \hat{C}^-_{snm} \cos(\theta_s+\epsilon^-_{snm}+\chi_s)]. \quad (3d)$$

The summation over s(n,m) should include all constituents for which Schwiderski has computed a model. Except for cases of near resonance, the retrograde terms do not produce long period (> 1 day) orbit perturbations for the diurnal and semi-diurnal tides. The rms of the along-track perturbations on Lageos due to the combination of all of the retrograde waves is less than 5 cm.

For computing inclination and node perturbations, only the even degree terms are required, but for the eccentricity and periapsis the odd degree terms are not negligible. Long period perturbations are only produced when the degree (n) is greater than 1 and the order (m) is 0 for long period tides, 1 for diurnal tides, and 2 for semi-diurnal tides. Finally, the ocean tide amplitudes and their effect on satellite orbits decrease with increasing degree, so truncation above degree 6 is justified for Lageos.

Thus, for the diurnal tides (Q_1, O_1, P_1, K_1) only the n = 2, 3, 4, 5, 6 and m = 1 terms need be computed. For the semi-diurnal tides (N_2, M_2, S_2, K_2) only n = 2, 3, 4, 5, 6 and m = 2 terms need be computed. For the long period tides (S_{sa}, M_m, M_f) only n = 2, 3, 4, 5, 6 and m = 0 terms need be computed. Table 8.2 gives the values required for each of the constituents for which Schwiderski has computed a model. Note that the units in Table 8.2 are cm and hence must be scaled to m for use with the constants given for use with equation (1).

The n = 2, m = 2 term for the S_2 argument can be modified to account for the atmospheric tide using the results of Chapman and Lindzen (1970). The modified values to be used instead of those in Table 8.2 are:

$C^+_{22} = -0.537$ (cm), $S^+_{22} = 0.321$ (cm).

For the most precise applications, more than the 11 terms listed in Table 8.2 need to be modelled. This can be accomplished by assuming that the ocean tide admittance varies smoothly with frequency and by using the Schwiderski values as a guide to the interpolation to other frequencies.

Table 8.2. Ocean tide coefficients from the Schwiderski model.

ARGUMENT NUMBER		n	m	$\hat{C}^+_{snm}$ (cm)	ε^+_{snm} (deg)	C^+_{snm} (cm)	S^+_{snm} (cm)
057.555	S_{sa}	2	0	.6215	221.672	-.8264	-.9284
057.555	S_{sa}	3	0	.0311	1.735	.0019	.0621
057.555	S_{sa}	4	0	.1624	92.674	.3244	-.0152
057.555	S_{sa}	5	0	.2628	251.737	-.4991	-.1647
057.555	S_{sa}	6	0	.4363	145.744	.4912	-.7213
065.455	M_m	2	0	.5313	258.900	-1.0428	-.2046
065.455	M_m	3	0	.0317	94.298	.0632	-.0047
065.455	M_m	4	0	.0998	69.054	.1863	.0713
065.455	M_m	5	0	.2279	292.291	-.4218	.1729
065.455	M_m	6	0	.0660	39.882	.0847	.1014
075.555	M_f	2	0	.8525	251.956	-1.6211	-.5281
075.555	M_f	3	0	.0951	148.236	.1001	-.1617
075.555	M_f	4	0	.2984	102.723	.5822	-.1315
075.555	M_f	5	0	.2960	223.167	-.4050	-.4318
075.555	M_f	6	0	.0880	107.916	.1675	-.0542
135.655	Q_1	2	1	.5373	313.735	-.3715	-.3882
135.655	Q_1	3	1	.3136	107.346	.0935	.2994
135.655	Q_1	4	1	.2930	288.992	-.0953	-.2770
135.655	Q_1	5	1	.2209	112.383	.0841	.2042
135.655	Q_1	6	1	.0396	287.824	-.0121	-.0377
145.555	O_1	2	1	2.4186	313.716	-1.6715	-1.7481
145.555	O_1	3	1	1.3161	83.599	-.1467	1.3079
145.555	O_1	4	1	1.4301	276.282	-.1565	-1.4215
145.555	O_1	5	1	.9505	109.128	.3115	.8980
145.555	O_1	6	1	.1870	282.623	-.0409	-.1825

Table 8.2 (continued)

ARGUMENT NUMBER	n	m	$\hat{C}^{+}_{snm}$ (cm)	ε^{+}_{snm} (deg)	C^{+}_{snm} (cm)	S^{+}_{snm} (cm)
163.555 P_1	2	1	.9020	313.912	-.6256	-.6498
163.555 P_1	3	1	.2976	39.958	-.2281	.1911
163.555 P_1	4	1	.6346	258.311	.1286	-.6215
163.555 P_1	5	1	.4130	104.438	.1030	.4000
163.555 P_1	6	1	.0583	276.591	-.0067	-.0579
165.555 K_1	2	1	2.8158	315.113	1.9950	1.9872
165.555 K_1	3	1	.8925	33.752	.7421	-.4959
165.555 K_1	4	1	1.9121	254.229	-.5197	1.8401
165.555 K_1	5	1	1.2111	104.672	-.3068	-1.1716
165.555 K_1	6	1	.1645	281.867	.0338	.1610
245.655 N_2	2	2	.6516	321.788	-.4030	.5120
245.655 N_2	3	2	.1084	171.923	.0152	-.1074
245.655 N_2	4	2	.2137	141.779	.1322	-.1679
245.655 N_2	5	2	.0836	5.034	.0073	.0832
245.655 N_2	6	2	.0674	346.544	-.0157	.0656
255.555 M_2	2	2	2.9551	310.553	-2.2453	1.9213
255.555 M_2	3	2	.3610	168.623	.0712	-.3539
255.555 M_2	4	2	1.0066	124.755	.8270	-.5738
255.555 M_2	5	2	.2751	356.561	-.0165	.2746
255.555 M_2	6	2	.4130	329.056	-.2124	.3542
273.555 S_2	2	2	.9291	314.011	-.6682	.6456
273.555 S_2	3	2	.2633	201.968	-.0985	-.2442
273.555 S_2	4	2	.3716	103.027	.3621	-.0838
273.555 S_2	5	2	.1365	3.772	.0090	.1362
273.555 S_2	6	2	.1726	280.381	-.1698	.0311
275.555 K_2	2	2	.2593	315.069	-.1832	.1836
275.555 K_2	3	2	.0943	195.007	-.0244	-.0911
275.555 K_2	4	2	.1059	103.521	.1029	-.0247
275.555 K_2	5	2	.0382	.411	.0003	.0382
275.555 K_2	6	2	.0467	281.357	-.0458	.0092

NOTES:

1. The Doodson variable multipliers ($\bar{n}$) are coded into the argument number (A) after Doodson (*Proc. R. Soc. A.*, **100**, pp. 305-329, 1921) as:

   $A = n_1(n_2+5)(n_3+5) \;.\; (n_4+5)(n_5+5)(n_6+5)$.

2. For the long period tides (m = 0), the value of $\hat{C}^{+}_{snm}$ used to compute C^{+}_{snm} and S^{+}_{snm} was twice that shown to account for the combined effect of the retrograde and prograde waves.

3. The spherical harmonic decomposition of Schwiderski's models was computed by C. Goad of the Ohio State University.

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