CHAPTER 5 TRANSFORMATION BETWEEN THE CELESTIAL AND TERRESTRIAL SYSTEMS

The coordinate transformation to be used from the TRS to the CRS at the date t of the observation can be written as:

$$[CRS] = PN(t) \cdot R(t) \cdot W(t) \ [TRS],$$

where PN(t), R(t) and W(t) are the transformation matrices arising from the motion of the Celestial Ephemeris Pole (CEP) in the CRS, from the rotation of the Earth around the axis of the CEP, and from polar motion respectively.

Two equivalent options can be used giving rise to the forms (1) and (2) of the coordinate transformation. These two options have been shown to be consistent within ±0.05 milliseconds of arc (mas) both theoretically (Capitaine, 1990) and numerically using existing astrometric data (Capitaine and Chollet, 1991) or simulated data over two centuries (Capitaine and Gontier, 1991).

Option (1), corresponding to the classical procedure, makes use of the equinox for realizing the intermediate reference frame of date t. It uses apparent Greenwich Sidereal Time in the transformation matrix R(t) and the classical precession and nutation parameters in the transformation matrix PN(t),

Option (2) makes use of the nonrotating origin (Guinot, 1979) to realize the intermediate reference frame of date t: it uses the stellar angle (from the NRO in the TRS to the NRO in the CRS) in the transformation matrix R(t) and the two coordinates of the Celestial Ephemeris Pole in the CRS (Capitaine, 1990) in the transformation matrix PN(t). This leads to very simple expressions of the partial derivatives of observables with respect to polar coordinates, UT1, and celestial pole offsets.

The following sections give the details of these two options as well as the standard expressions necessary to obtain the numerical values of the relevant parameters at the date of the observation. Subroutines for options 1 and 2 of the coordinate transformation from the TRS to the CRS are available from the Central Bureau on request together with the development of the parameters.

The expressions of the precession and nutation quantities have been developed originally as functions of barycentric dynamical time (TDB) defined by IAU recommendations of 1976 and 1979. In 1991 the IAU adopted definitions of other time scales. See Chapter 13 for the relationships among these time scales. At the level of accuracy needed in the coordinate transformations which are considered here, the parameter t defined by

$$t = (\text{TAI} - 2000 \text{ January 1d 12h TAI}) \text{ in days} / 36\,525$$

can be used in place of

$$[\text{TDB} - \text{J2000.0 (TDB)}] \text{ in days} / 36\,525.$$

In all of the following formulas, t is defined as indicated above. In the following, R_1, R_2 and R_3 denote direct rotations about the axes 1, 2 and 3 of the coordinate frame.

Coordinate Transformation Referred to the Equinox

Option (1) uses the form of the coordinate transformation

$$[\text{CRS}] = PN'(t) \cdot R'(t) \cdot W'(t) \ [\text{TRS}], \qquad (1)$$

in which the three fundamental components are (Mueller, 1969)

$$W'(t) = R_1(y_p) \cdot R_2(x_p),$$

x_p and y_p being the "polar coordinates" of the CEP in the TRS;

$$R'(t) = R_3(-\text{GST}),$$

GST being Greenwich True Sidereal Time at date t, including both the effect of Earth rotation and the accumulated precession and nutation in right ascension; and

$$PN'(t) = [P] \ [N],$$

$$\text{with } [P] = R_3(\zeta_A) \cdot R_2(-\theta_A) \cdot R_3(z_A)$$

for the transformation matrix corresponding to the precession between the reference epoch and the date t,

$$[N] = R_1(-\epsilon_A) \cdot R_3(\Delta\psi) \cdot R_1(\epsilon_A + \Delta\epsilon)$$

for the transformation matrix corresponding to the nutation at date t.

Standard values of the parameters to be used in form (1) of the transformation are explained below.

The standard polar coordinates to be used for the parameters x_p and y_p (if not estimated from the observations) are those published by the IERS.

Apparent Greenwich Sidereal Time GST at the date t of the observation, must be derived from the following expressions:

(i) the relationship between Greenwich Mean Sidereal Time (GMST) and Universal Time as given by Aoki, *et al.* (1982):

$$GMST_{0h\,UT1} = 6^h\ 41^m\ 50^s.548\ 41 + 8\ 640\ 184^s.812\ 866\ T'_u$$
$$+\ 0^s.093\ 104\ T'^2_u - 6^s.2 \times 10^{-6}\ T'^3_u,$$

with $T'_u = d'_u/365\ 25$, d_u being the number of days elapsed since 2000 January 1, 12h UT1, taking on values ±0.5, ±1.5, ...,

(ii) the interval of GMST from 0h UT1 to the hour of the observation in UT1,

$$GMST = GMST_{0h\,UT1} + r[(UT1-UTC)+UTC],$$

where r is the ratio of universal to sidereal time as given by Aoki, *et al.* (1982),

$$r = 1.002\ 737\ 909\ 350\ 795 + 5.900\ 6 \times 10^{-11}\ T'_u - 5.9 \times 10^{-15}\ T'^2_u$$

and the UT1-UTC value to be used (if not estimated from the observations) is the IERS value. The use of (i) and (ii) is equivalent to using the relationship (Sovers and Fanselow, 1987) between GMST and UT1,

$$GMST = (\text{UT1 Julian day fraction}) \times 24h + 18^h41^m50^s.548\ 41$$
$$+\ 8\ 640\ 184^s.812\ 866T_u + 0^s.093\ 104T_u^2 - 6^s.2 \times 10^{-6}\ T_u^3,$$

where T_u = [Julian UT1 date - 2 451 545.0]/36 525).

(iii) accumulated precession and nutation in right ascension (Aoki and Kinoshita, 1983),

$$GST = GMST + \Delta\psi\cos\epsilon_A + 0''.002\ 64\ \sin\Omega + 0''.000\ 063\ \sin 2\Omega,$$

where Ω is the mean longitude of the ascending node of the lunar orbit. The last two terms have not been included in the IERS Standards previously. They should not be included in (iii) until 26 February 1997 when their use will begin. This date is chosen to eliminate any discontinuity in UT1. The effect of these terms on the estimation of UT1 has been described by Gontier and Capitaine (1991).

The numerical expression for the precession quantities ζ_A, θ_A, z_A and ϵ_A have been given by Lieske, *et al.* (1977) as functions of two time parameters t and T (the last parameter representing Julian centuries from J2000.0 to an arbitrary epoch). The simplified expressions when the arbitrary epoch is chosen to be J2000.0 (*i.e.* T = 0) are

$$\zeta_A = 2\ 306''.218\ 1\ t + 0''.301\ 88\ t^2 + 0''.017\ 998\ t^3,$$

$$\theta_A = 2\ 004''.310\ 9\ t - 0''.426\ 65\ t^2 - 0''.041\ 833\ t^3,$$

$$z_A = 2\ 306''.2181\ t + 1''.094\ 68\ t^2 + 0''.018\ 203\ t^3,$$

$$\epsilon_A = 8\ 4381''.448 - 46''.815\ 0\ t - 0''.000\ 59\ t^2 + 0''.001\ 813\ t^3.$$

The nutation quantities $\Delta\psi$ and $\Delta\epsilon$ to be used are the standard nutation angles in longitude and obliquity as derived from the IAU 1980 Theory of Nutation (Seidelmann, 1982; Wahr, 1981) using the fundamental arguments as given below. The constants defining this theory are given in Table 5.1.

For observations requiring values of the nutation angles with an accuracy of ±1 mas, it is necessary to add (if those quantities are not estimated from the observations) the IERS published values (observed or predicted) for the "celestial pole offsets" *i.e.* corrections dpsi and deps).

The IAU 1980 Theory of Nutation

The IAU 1980 Theory of Nutation (Seidelmann, 1982; Wahr, 1981) is based on a modification of a rigid Earth theory published by Kinoshita (1977) and on the geophysical model 1066A of Gilbert and Dziewonski (1975). It therefore includes the effects of a solid inner core and a liquid outer core and a "distribution of elastic parameters inferred from a large set of seismological data."

VLBI and LLR observations have shown that there are deficiencies in the IAU 1976 Precession and in the IAU 1980 Theory of Nutation. However, these models are kept as part of the IERS Standards and the observed differences ($\delta\Delta\psi$ and $\delta\Delta\epsilon$, equivalent to dpsi and deps in the IERS Bulletins) with respect to the conventional celestial pole position defined by the models are monitored and reported by the IERS as "celestial pole offsets". Using these offsets the corrected nutation is given by

$$\Delta\psi = \Delta\psi(\text{IAU 1980}) + \delta\Delta\psi, \text{ and}$$
$$\Delta\epsilon = \Delta\epsilon(\text{IAU 1980}) + \delta\Delta\epsilon.$$

This is practically equivalent to replacing N with the rotation described by Lieske (1991),

$$\tilde{N} = \mathfrak{R} N_{IAU}.$$

where N_{IAU} represents the IAU 1980 Theory of Nutation,

$$\mathfrak{R} = \begin{bmatrix} 1 & -\delta\Delta\psi\cos\epsilon_t & -\delta\Delta\psi\sin\epsilon_t \\ \delta\Delta\psi\cos\epsilon_t & 1 & \delta\Delta\epsilon \\ \delta\Delta\psi\sin\epsilon_t & \delta\Delta\epsilon & 1 \end{bmatrix}$$

and $\epsilon_t = \epsilon_A + \Delta\epsilon$. Mathematical models of the corrections to the IAU 1980 Theory of Nutation derived from observations are available (McCarthy and Luzum, 1991).

Fundamental Arguments of the IAU 1980 Theory of Nutation

The fundamental arguments of the nutation series are given by the following functions of t:

$$\begin{aligned} l &= \text{Mean Anomaly of the Moon} \\ &= 134^\circ\ 57'\ 46''.733 + (1325^r + 198^\circ\ 52'\ 02''.633)\ t \\ &\quad + 31''.310\ t^2 + 0''.064\ t^3, \\ l' &= \text{Mean Anomaly of the Sun} \\ &= 357^\circ\ 31'\ 39''.804 + (99^r + 359^\circ\ 03'\ 01''.224)\ t \\ &\quad - 0''.577\ t^2 - 0''.012\ t^3, \end{aligned}$$

$$\begin{aligned} F &= L - \Omega \\ &= 93^\circ\ 16'\ 18''.877 + (1342^r + 82^\circ\ 01'\ 03''.137)\ t \\ &\quad - 13''.257\ t^2 + 0''.011\ t^3, \end{aligned}$$

$$\begin{aligned} D &= \text{Mean Elongation of the Moon from the Sun} \\ &= 297^\circ\ 51'\ 01''.307 + (1236^r + 307^\circ\ 06'\ 41''.328)\ t \\ &\quad - 6''.891\ t^2 + 0''.019\ t^3, \end{aligned}$$

$$\begin{aligned} \Omega &= \text{Mean Longitude of the Ascending Node of the Moon} \\ &= 125^\circ\ 02'\ 40''.280 - (5^r + 134^\circ\ 08'\ 10''.539)\ t \\ &\quad + 7''.455\ t^2 + 0''.008\ t^3, \end{aligned}$$

L = Mean Longitude of the Moon,

where t is measured in Julian Centuries of 36525 days of 86400 seconds of Dynamical Time since J2000.0 and where $1^r = 360^\circ = 1\ 296\ 000''.0$.

Table 5.1. Series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0.

$$\Delta\psi = \Sigma_{i=1,106} (A_i + A'_i t) \sin(\text{ARGUMENT}),$$

$$\Delta\epsilon = \Sigma_{i=1,106} (B_i + B'_i t) \cos(\text{ARGUMENT}).$$

ARGUMENT					PERIOD	LONGITUDE (0".0001)		OBLIQUITY (0".0001)	
l	l'	F	D	Ω	(days)	A_i	A'_i	B_i	B'_i
0	0	0	0	1	6798.4	-171996	-174.2t	92025	8.9t
0	0	2	-2	2	182.6	-13187	-1.6t	5736	-3.1t
0	0	2	0	2	13.7	-2274	-0.2t	977	-0.5t
0	0	0	0	2	3399.2	2062	0.2t	-895	0.5t
0	1	0	0	0	365.3	1426	-3.4t	54	-0.1t
1	0	0	0	0	27.6	712	0.1t	-7	0.0t
0	1	2	-2	2	121.7	-517	1.2t	224	-0.6t
0	0	2	0	1	13.6	-386	-0.4t	200	0.0t
1	0	2	0	2	9.1	-301	0.0t	129	-0.1t
0	-1	2	-2	2	365.2	217	-0.5t	-95	0.3t
1	0	0	-2	0	31.8	-158	0.0t	-1	0.0t
0	0	2	-2	1	177.8	129	0.1t	-70	0.0t
-1	0	2	0	2	27.1	123	0.0t	-53	0.0t
1	0	0	0	1	27.7	63	0.1t	-33	0.0t
0	0	0	2	0	14.8	63	0.0t	-2	0.0t
-1	0	2	2	2	9.6	-59	0.0t	26	0.0t
-1	0	0	0	1	27.4	-58	-0.1t	32	0.0t
1	0	2	0	1	9.1	-51	0.0t	27	0.0t
2	0	0	-2	0	205.9	48	0.0t	1	0.0t
-2	0	2	0	1	1305.5	46	0.0t	-24	0.0t
0	0	2	2	2	7.1	-38	0.0t	16	0.0t
2	0	2	0	2	6.9	-31	0.0t	13	0.0t
2	0	0	0	0	13.8	29	0.0t	-1	0.0t
1	0	2	-2	2	23.9	29	0.0t	-12	0.0t
0	0	2	0	0	13.6	26	0.0t	-1	0.0t
0	0	2	-2	0	173.3	-22	0.0t	0	0.0t
-1	0	2	0	1	27.0	21	0.0t	-10	0.0t
0	2	0	0	0	182.6	17	-0.1t	0	0.0t
0	2	2	-2	2	91.3	-16	0.1t	7	0.0t
-1	0	0	2	1	32.0	16	0.0t	-8	0.0t
0	1	0	0	1	386.0	-15	0.0t	9	0.0t
1	0	0	-2	1	31.7	-13	0.0t	7	0.0t
0	-1	0	0	1	346.6	-12	0.0t	6	0.0t
2	0	-2	0	0	1095.2	11	0.0t	0	0.0t
-1	0	2	2	1	9.5	-10	0.0t	5	0.0t
1	0	2	2	2	5.6	-8	0.0t	3	0.0t
0	-1	2	0	2	14.2	-7	0.0t	3	0.0t
0	0	2	2	1	7.1	-7	0.0t	3	0.0t
1	1	0	-2	0	34.8	-7	0.0t	0	0.0t
0	1	2	0	2	13.2	7	0.0t	-3	0.0t
-2	0	0	2	1	199.8	-6	0.0t	3	0.0t
0	0	0	2	1	14.8	-6	0.0t	3	0.0t
2	0	2	-2	2	12.8	6	0.0t	-3	0.0t
1	0	0	2	0	9.6	6	0.0t	0	0.0t
1	0	2	-2	1	23.9	6	0.0t	-3	0.0t
0	0	0	-2	1	14.7	-5	0.0t	3	0.0t
0	-1	2	-2	1	346.6	-5	0.0t	3	0.0t
2	0	2	0	1	6.9	-5	0.0t	3	0.0t

Table 5.1 (continued)

ARGUMENT					PERIOD	LONGITUDE (0".0001)		OBLIQUITY (0".0001)	
l	l'	F	D	Ω	(days)	A_i	A'_i	B_i	B'_i
1	-1	0	0	0	29.8	5	0.0t	0	0.0t
1	0	0	-1	0	411.8	-4	0.0t	0	0.0t
0	0	0	1	0	29.5	-4	0.0t	0	0.0t
0	1	0	-2	0	15.4	-4	0.0t	0	0.0t
1	0	-2	0	0	26.9	4	0.0t	0	0.0t
2	0	0	-2	1	212.3	4	0.0t	-2	0.0t
0	1	2	-2	1	119.6	4	0.0t	-2	0.0t
1	1	0	0	0	25.6	-3	0.0t	0	0.0t
1	-1	0	-1	0	3232.9	-3	0.0t	0	0.0t
-1	-1	2	2	2	9.8	-3	0.0t	1	0.0t
0	-1	2	2	2	7.2	-3	0.0t	1	0.0t
1	-1	2	0	2	9.4	-3	0.0t	1	0.0t
3	0	2	0	2	5.5	-3	0.0t	1	0.0t
-2	0	2	0	2	1615.7	-3	0.0t	1	0.0t
1	0	2	0	0	9.1	3	0.0t	0	0.0t
-1	0	2	4	2	5.8	-2	0.0t	1	0.0t
1	0	0	0	2	27.8	-2	0.0t	1	0.0t
-1	0	2	-2	1	32.6	-2	0.0t	1	0.0t
0	-2	2	-2	1	6786.3	-2	0.0t	1	0.0t
-2	0	0	0	1	13.7	-2	0.0t	1	0.0t
2	0	0	0	1	13.8	2	0.0t	-1	0.0t
3	0	0	0	0	9.2	2	0.0t	0	0.0t
1	1	2	0	2	8.9	2	0.0t	-1	0.0t
0	0	2	1	2	9.3	2	0.0t	-1	0.0t
1	0	0	2	1	9.6	-1	0.0t	0	0.0t
1	0	2	2	1	5.6	-1	0.0t	1	0.0t
1	1	0	-2	1	34.7	-1	0.0t	0	0.0t
0	1	0	2	0	14.2	-1	0.0t	0	0.0t
0	1	2	-2	0	117.5	-1	0.0t	0	0.0t
0	1	-2	2	0	329.8	-1	0.0t	0	0.0t
1	0	-2	2	0	23.8	-1	0.0t	0	0.0t
1	0	-2	-2	0	9.5	-1	0.0t	0	0.0t
1	0	2	-2	0	32.8	-1	0.0t	0	0.0t
1	0	0	-4	0	10.1	-1	0.0t	0	0.0t
2	0	0	-4	0	15.9	-1	0.0t	0	0.0t
0	0	2	4	2	4.8	-1	0.0t	0	0.0t
0	0	2	-1	2	25.4	-1	0.0t	0	0.0t
-2	0	2	4	2	7.3	-1	0.0t	1	0.0t
2	0	2	2	2	4.7	-1	0.0t	0	0.0t
0	-1	2	0	1	14.2	-1	0.0t	0	0.0t
0	0	-2	0	1	13.6	-1	0.0t	0	0.0t
0	0	4	-2	2	12.7	1	0.0t	0	0.0t
0	1	0	0	2	409.2	1	0.0t	0	0.0t
1	1	2	-2	2	22.5	1	0.0t	-1	0.0t
3	0	2	-2	2	8.7	1	0.0t	0	0.0t
-2	0	2	2	2	14.6	1	0.0t	-1	0.0t
-1	0	0	0	2	27.3	1	0.0t	-1	0.0t
0	0	-2	2	1	169.0	1	0.0t	0	0.0t
0	1	2	0	1	13.1	1	0.0t	0	0.0t
-1	0	4	0	2	9.1	1	0.0t	0	0.0t
2	1	0	-2	0	131.7	1	0.0t	0	0.0t
2	0	0	2	0	7.1	1	0.0t	0	0.0t
2	0	2	-2	1	12.8	1	0.0t	-1	0.0t
2	0	-2	0	1	943.2	1	0.0t	0	0.0t
1	-1	0	-2	0	29.3	1	0.0t	0	0.0t
-1	0	0	1	1	388.3	1	0.0t	0	0.0t
-1	-1	0	2	1	35.0	1	0.0t	0	0.0t

Table 5.1 (continued)

ARGUMENT					PERIOD	LONGITUDE (0"0001)		OBLIQUITY (0"0001)	
l	l'	F	D	Ω	(days)	A_i	A'_i	B_i	B'_i
0	1	0	1	0	27.3	1	0.0t	0	0.0t

ϵ_0 = 23° 26′ 21"448
sin ϵ_0 = 0.39777716

Coordinate Transformation Referred to the Nonrotating Origin

Option (2) uses form (2) of the coordinate transformation from the TRS to the CRS

$$[CRS] = PN''(t) \cdot R''(t) \cdot W''(t) \quad [TRS], \tag{2}$$

where the three fundamental components of (2) are given below (Capitaine, 1990)

$$W''(t) = R_3(-s') \cdot R_1(y_p) \cdot R_2(x_p),$$

x_p and y_p being the "polar coordinates" of the CEP in the TRS and s' the accumulated displacement of the terrestrial NRO on the true equator due to polar motion. The use of the quantity s' (which is neglected in the classical form (1)) provides an exact realization of the "instantaneous prime meridian".

$$R''(t) = R_3(-\theta),$$

θ being the stellar angle at date t due to the Earth's angle of rotation,

$$PN''(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E) \cdot R_3(S),$$

E and d being such that the coordinates of the CEP in the CRS are $X = \sin d \cos E$, $Y = \sin d \sin E$, $Z = \cos d$ and S being the accumulated rotation (between the epoch and the date t) of the celestial NRO on the true equator due to the celestial motion of the CEP. PN"(t) can be given in an equivalent form involving directly X and Y (to which all the observations of a celestial object from the Earth are actually sensitive) as:

$$PN''(t) = Q_1 = \begin{bmatrix} 1-aX^2 & -aXY & X \\ -aXY & 1-aY^2 & Y \\ -X & -Y & 1-a(X^2+Y^2) \end{bmatrix} \cdot R_3(s),$$

with $a = 1/(1+\cos d)$, which can also be written, with sufficient accuracy as $a = 1/2 + 1/8\ (X^2+Y^2)$.

The standard values of the parameters to be used in the form (2) of the transformation are detailed below.

The standard pole coordinates to be used for the parameters x_p and y_p (if not estimated from the observations) are those published by the IERS. The quantity s′ (of the order of 0.1 mas/c) is:

$$s' = 0.0015(a_c^2/1.2 + a_a^2)\ t,$$

a_c and a_a being the average amplitudes (in arc seconds) of the Chandlerian and annual wobbles, respectively in the period considered (Capitaine, *et al.*, 1986).

The stellar angle is obtained by the use of the conventional relationship between the stellar angle θ, the hour angle of the nonrotating origin of Guinot (1979) and UT1 as given by Capitaine, *et al.*, (1986),

$$\theta(T_u) = 2\pi\ (0.779\ 057\ 273\ 264 + 1.002\ 737\ 811\ 911\ 354\ 48\ T_u \times 365\ 25),$$

where T_u = (Julian UT1 date - 2 451 545.0)/36 525, and

UT1 = UTC + (UT1-UTC), or equivalently

$$\theta(T_u) = 2\pi\ (\text{UT1 Julian day number elapsed since } 2451545.0 + 0.779\ 057\ 273\ 264 + 0.002\ 737\ 811\ 911\ 354\ 48\ T_u \times 36\ 525),$$

the quantity UT1-UTC to be used (if not estimated from the observations) being the IERS value.

The celestial coordinates X and Y of the CEP to be used are the standard values as derived from the series are in Table 5.2 (with the same fundamental arguments and similar coefficients as in Table 5.1). These developments of the celestial polar coordinates have been derived (Capitaine, 1990) from the previous standard expressions for precession and nutation with a consistency of 5×10^{-5}" after a century; such consistency has been numerically checked over two centuries (Gontier, 1990). For observations requiring values of the nutation angles with a milliarcsecond accuracy, it is necessary to add (if those quantities are not estimated from the observations) the IERS published values (observed or predicted) for the "celestial pole offsets" (*i.e.* corrections $dX = d\psi\ \sin\epsilon_0$ and $dY = d\epsilon$).

The standard value of s to be used can be derived with an accuracy of 5×10^{-5}" after a century (Capitaine, 1990) from the following numerical development and the numerical values of X and Y (Table 5.2),

$$s = -XY/2 + 0\overset{''}{.}003\ 85\ t - 0\overset{''}{.}072\ 59\ t^3 - 0\overset{''}{.}002\ 65 \sin \Omega$$
$$- 0.000\ 06 \sin 2\Omega + 0\overset{''}{.}000\ 74\ t^2 \sin \Omega + 0\overset{''}{.}000\ 06\ t^2 \sin 2(F-D+\Omega)$$

Table 5.2 Series for the celestial coordinates X and Y of the CEP referred to the mean equator and equinox of epoch J2000.0, with t measured in Julian centuries from epoch J2000.0. The terms between the two lines are identical in Tables 5.1 and 5.2

$$X = 2004\overset{''}{.}310\ 9\ t - 0\overset{''}{.}426\ 65\ t^2 - 0\overset{''}{.}198\ 656\ t^3 + 0\overset{''}{.}000\ 014\ 0\ t^4$$
$$+ 0\overset{''}{.}000\ 06\ t^2 \cos \Omega$$
$$+ \sin\epsilon_0 \{\Sigma_{i=1,106} [(A_i + A'_i t) \sin (\text{ARGUMENT}) + A''_i t \cos (\text{ARGUMENT})]\}$$
$$+ 0\overset{''}{.}002\ 04\ t^2 \sin \Omega + 0\overset{''}{.}000\ 16\ t^2 \sin 2(F - D + \Omega),$$

$$Y = - 0\overset{''}{.}000\ 13 - 22\overset{''}{.}409\ 92\ t^2 + 0\overset{''}{.}001\ 836\ t^3 + 0\overset{''}{.}001\ 113\ 0\ t^4$$
$$+ \Sigma_{i=1,106} [(B_i + B'_i t) \cos (\text{ARGUMENT}) + B''_i t \sin (\text{ARGUMENT})]$$
$$- 0\overset{''}{.}002\ 31\ t^2 \cos \Omega - 0\overset{''}{.}000\ 14\ t^2 \cos 2(F - D + \Omega)$$

ARGUMENT					PERIOD	LONGITUDE (0.0001")			OBLIQUITY (0.0001")		
l	l'	F	D	Ω	(days)	A_i	$A'_i t$	$A''_i t$	B_i	$B'_i t$	$B''_i t$
0	0	0	0	1	6798.4	-171996	-84.2t	5173.2t	92025	8.9t	1529.9t
0	0	2	-2	2	182.6	-13187	5.3t	322.2t	5736	-3.1t	117.3t
0	0	2	0	2	13.7	-2274	1.0t	54.8t	977	-0.5t	20.2t
0	0	0	0	2	3399.2	2053.2	-1.0t	-50.5t	-893.7	0.5t	-18.3t
0	1	0	0	0	365.3	1426	-4.3t	3.0t	54	-0.1t	-12.7t
1	0	0	0	0	27.6	712	0.1t	0.0t	-7	0.0t	-6.3t
0	1	2	-2	2	121.7	-517	1.5t	12.6t	224	-0.6t	4.6t
0	0	2	0	1	13.6	-386	-0.4t	11.3t	200	0.0t	3.4t
1	0	2	0	2	9.1	-301	0.0t	7.3t	129	-0.1t	2.7t
0	-1	2	-2	2	365.2	217	-0.5t	-5.3t	-95	0.3t	-1.9t
1	0	0	-2	0	31.8	-158	0.0t	0.0t	-1	0.0t	1.4t
0	0	2	-2	1	177.8	129	0.1t	-4.0t	-70	0.0t	-1.2t
-1	0	2	0	2	27.1	123	0.0t	-3.0t	-53	0.0t	-1.1t
1	0	0	0	1	27.7	63	0.1t	-1.8t	-33	0.0t	-0.6t
0	0	0	2	0	14.8	63	0.0t	0.0t	-2	0.0t	-0.6t
-1	0	2	2	2	9.6	-59	0.0t	1.5t	26	0.0t	0.5t
-1	0	0	0	1	27.4	-58	-0.1t	1.8t	32	0.0t	0.5t
1	0	2	0	1	9.1	-51	0.0t	1.5t	27	0.0t	0.5t
2	0	0	-2	0	205.9	48	0.0t	0.0t	1	0.0t	0.0t
-2	0	2	0	1	1305.5	46	0.0t	-1.3t	-24	0.0t	0.0t
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
0	0	2	2	2	7.1	-38	0.0t		16	0.0t	
2	0	2	0	2	6.9	-31	0.0t		13	0.0t	
2	0	0	0	0	13.8	29	0.0t		-1	0.0t	
1	0	2	-2	2	23.9	29	0.0t		-12	0.0t	
0	0	2	0	0	13.6	26	0.0t		-1	0.0t	
0	0	2	-2	0	173.3	-22	0.0t		0	0.0t	
-1	0	2	0	1	27.0	21	0.0t		-10	0.0t	
0	2	0	0	0	182.6	17	-0.1t		0	0.0t	
0	2	2	-2	2	91.3	-16	0.1t		7	0.0t	
1	0	0	2	1	32.0	16	0.0t		-8	0.0t	
0	1	0	0	1	386.0	-15	0.0t		9	0.0t	
1	0	0	-2	1	31.7	-13	0.0t		7	0.0t	
0	-1	0	0	1	346.6	-12	0.0t		6	0.0t	
2	0	-2	0	0	1095.2	11	0.0t		0	0.0t	
-1	0	2	2	1	9.5	-10	0.0t		5	0.0t	

1	0	2	2	2	5.6	-8	0.0t	3	0.0t
0	-1	2	0	2	14.2	-7	0.0t	3	0.0t
0	0	2	2	1	7.1	-7	0.0t	3	0.0t
1	1	0	-2	0	34.8	-7	0.0t	0	0.0t
0	1	2	0	2	13.2	7	0.0t	-3	0.0t
-2	0	0	2	1	199.8	-6	0.0t	3	0.0t
0	0	0	2	1	14.8	-6	0.0t	3	0.0t
2	0	2	-2	2	12.8	6	0.0t	-3	0.0t
1	0	0	2	0	9.6	6	0.0t	0	0.0t
1	0	2	-2	1	23.9	6	0.0t	-3	0.0t
0	0	0	-2	1	14.7	-5	0.0t	3	0.0t
0	-1	2	-2	1	346.6	-5	0.0t	3	0.0t
2	0	2	0	1	6.9	-5	0.0t	3	0.0t
1	-1	0	0	0	29.8	5	0.0t	0	0.0t
1	0	0	-1	0	411.8	-4	0.0t	0	0.0t
0	0	0	1	0	29.5	-4	0.0t	0	0.0t
0	1	0	-2	0	15.4	-4	0.0t	0	0.0t
1	0	-2	0	0	26.9	4	0.0t	0	0.0t
2	0	0	-2	1	212.3	4	0.0t	-2	0.0t
0	1	2	-2	1	119.6	4	0.0t	-2	0.0t
1	1	0	0	0	25.6	-3	0.0t	0	0.0t
1	-1	0	-1	0	3232.9	-3	0.0t	0	0.0t
-1	-1	2	2	2	9.8	-3	0.0t	1	0.0t
0	-1	2	2	2	7.2	-3	0.0t	1	0.0t
1	-1	2	0	2	9.4	-3	0.0t	1	0.0t
3	0	2	0	2	5.5	-3	0.0t	1	0.0t
-2	0	2	0	2	1615.7	-3	0.0t	1	0.0t
1	0	2	0	0	9.1	3	0.0t	0	0.0t
-1	0	2	4	2	5.8	-2	0.0t	1	0.0t
1	0	0	0	2	27.8	-2	0.0t	1	0.0t
-1	0	2	-2	1	32.6	-2	0.0t	1	0.0t
0	-2	2	-2	1	6786.3	-2	0.0t	1	0.0t
-2	0	0	0	1	13.7	-2	0.0t	1	0.0t
2	0	0	0	1	13.8	2	0.0t	-1	0.0t
3	0	0	0	0	9.2	2	0.0t	0	0.0t
1	1	2	0	2	8.9	2	0.0t	-1	0.0t
0	0	2	1	2	9.3	2	0.0t	-1	0.0t
1	0	0	2	1	9.6	-1	0.0t	0	0.0t
1	0	2	2	1	5.6	-1	0.0t	1	0.0t
1	1	0	-2	1	34.7	-1	0.0t	0	0.0t
0	1	0	2	0	14.2	-1	0.0t	0	0.0t
0	1	2	-2	0	117.5	-1	0.0t	0	0.0t
0	1	-2	2	0	329.8	-1	0.0t	0	0.0t
1	0	-2	2	0	32.8	-1	0.0t	0	0.0t
1	0	-2	-2	0	9.5	-1	0.0t	0	0.0t
1	0	2	-2	0	32.8	-1	0.0t	0	0.0t
1	0	0	-4	0	10.1	-1	0.0t	0	0.0t
2	0	0	-4	0	15.9	-1	0.0t	0	0.0t
0	0	2	4	2	4.8	-1	0.0t	0	0.0t
0	0	2	-1	2	25.4	-1	0.0t	0	0.0t
-2	0	2	4	2	7.3	-1	0.0t	1	0.0t
2	0	2	2	2	4.7	-1	0.0t	0	0.0t
0	-1	2	0	1	14.2	-1	0.0t	0	0.0t
0	0	-2	0	1	13.6	-1	0.0t	0	0.0t
0	0	4	-2	2	12.7	1	0.0t	0	0.0t
0	1	0	0	2	409.2	1	0.0t	0	0.0t
1	1	2	-2	2	22.5	1	0.0t	-1	0.0t
3	0	2	-2	2	8.7	1	0.0t	0	0.0t
-2	0	2	2	2	14.6	1	0.0t	-1	0.0t
-1	0	0	0	2	27.3	1	0.0t	-1	0.0t
0	0	-2	2	1	169.0	1	0.0t	0	0.0t
0	1	2	0	1	13.1	1	0.0t	0	0.0t

Table 5.2 (continued)

ARGUMENT					PERIOD	LONGITUDE (0.0001")			OBLIQUITY (0.0001")		
l	l'	F	D	Ω	(days)	A_i	$A'_i t$	$A''_i t$	B_i	$B'_i t$	$B''_i t$
-1	0	4	0	2	9.1	1	0.0t		0	0.0t	
2	1	0	-2	0	131.7	1	0.0t		0	0.0t	
2	0	0	2	0	7.1	1	0.0t		0	0.0t	
2	0	2	-2	1	12.8	1	0.0t		-1	0.0t	
2	0	-2	0	1	943.2	1	0.0t		0	0.0t	
1	-1	0	-2	0	29.3	1	0.0t		0	0.0t	
-1	0	0	1	1	388.3	1	0.0t		0	0.0t	
-1	-1	0	2	1	35.0	1	0.0t		0	0.0t	
0	1	0	1	0	27.3	1	0.0t		0	0.0t	
0	0	2	-2	3	177.8	-1.2	0.0t		0	0.0t	

Geodesic Nutation

Fukushima (1990) has pointed out that, if extreme precision is required, the effect of geodesic nutation must be taken into account. For Option (1) this would require a correction in longitude of

$$\Delta\psi_g = -0''.000\,153 \sin l' - 0''.000\,002 \sin 2l',$$

where l' is the mean anomaly of the Sun. For Option (2) it would require a correction to X of

$$\Delta X_g = (-0''.000\,060\,9 \sin l' - 0''.000\,000\,8 \sin 2l') \sin \epsilon_0.$$

In both cases the correction would be added to the uncorrected determination of ψ or X.

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