CHAPTER 11 TROPOSPHERIC MODEL

Satellite Laser Ranging

The formulation of Marini and Murray (1973) is commonly used in laser ranging. The formula has been tested by comparison with ray-tracing radiosonde profiles.

The correction to a one-way range is

$$\Delta R = \frac{f(\lambda)}{f(\phi,H)} \cdot \frac{A+B}{\sin E + \frac{B/(A+B)}{\sin E + 0.01}}, \tag{1}$$

where

$$A = 0.002357P_0 + 0.000141e_0, \tag{2}$$

$$B = (1.084 \times 10^{-8})P_0\ T_0\ K + (4.734 \times 10^{-8})\ \frac{P_0^2}{T_0}\ \frac{2}{(3-1/K)}, \tag{3}$$

$$K = 1.163 - 0.00968 \cos 2\phi - 0.00104\ T_0 + 0.00001435P_0, \tag{4}$$

where

ΔR = range correction (meters),
E = true elevation of satellite,
P_0 = atmospheric pressure at the laser site (in 10^{-1} kPa, equivalent to millibars),
T_0 = atmospheric temperature at the laser site (degrees Kelvin),
e_0 = water vapor pressure at the laser site (10^{-1} kPa, equivalent to millibars),
$f(\lambda)$ = laser frequency parameter (λ = wavelength in micrometers), and
$f(\phi,H)$ = laser site function.

Additional definitions of these parameters are available. The water vapor pressure, e_0, can be calculated from a relative humidity measurement, R_h(%) by

$$e_0 = \frac{R_h}{100} \times 6.11 \times 10^{\frac{7.5(T_0 - 273.15)}{237.3 + (T_0 - 273.15)}}.$$

The laser frequency parameter, $f(\lambda)$, is

$$f(\lambda) \equiv 0.9650 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4}.$$

$f(\lambda) = 1\mu m$ for a ruby laser, [*i. e.* $f(0.6943) = 1\mu m$], while $f(\lambda_G) = 1.02579\mu m$ and $f(\lambda_{IR}) = 0.97966\mu m$ for green and infrared YAG lasers.

The laser site function is

$$f(\phi,H) = 1 - 0.0026 \cos 2\phi - 0.00031\ H,$$

where ϕ is the latitude and H is the geodetic height (km).

Very Long Baseline Interferometry

The most serious problem in practical atmospheric modelling is that of unmeasured atmospheric parameters. The differences between mathematical models are often less than the errors which would be introduced by the character and distribution of the wet component and breakdowns in azimuthal symmetry. For this reason, it is customary in the data reduction to determine the zenith atmospheric delay as a parameter and use models only for the mapping function which is the ratio of delay at a given zenith angle to the zenith delay. Accordingly, the IERS Standard model applies only to the mapping function which is the ratio of delay at a given zenith angle to the zenith delay.

Some standard models are available: CFA2.2 (Davis, *et al.*, 1985), Chao (1974), Saastamoinen (1972), Black (1984), Marini (1972), Hopfield (1969), Yionoulis (1970), Goldfinger (1980), Matsakis, *et al.*, (1986), Baby, *et al.* (1988), and Lanyi (1984). The reader should be aware of typographical errors in the published versions of the last three works cited. The models differ in their allowance for Earth curvature, atmospheric boundary structure, scale heights, and bending. Of these, the model which attempts to address all these aspects, particularly bending, in the most complete manner, is that of Lanyi. Some of the other models can be duplicated by dropping terms from the Lanyi model. Its abundance of adjustable parameters could prove useful for experimental applications. It is recommended that the lapse rate and the wet scale height parameter be adjusted for site dependence and seasonal variation (see Askne and Nordius, 1987). As pointed out by Davis, *et al.* (1985), the mapping function is considerably less sensitive than the zenith delay to the wet component.

There is some discrepancy in the reported literature concerning the numerical values of the refractivity coefficients for the wet delay. Measurements of the refractivity at radio wavelengths at different temperatures have been fit to a linear slope (in 1/T) by Boudouris (1963) and Birnbaum and Chatterjee (1952). Thayer (1974), using the same data, extrapolated values from the optical to derive slightly different values (Table 10.1) that were still within the measurement errors of the previous authors. The are actually a weighted average of their data and those of three other works, going back to 1935. Two works of these are based entirely upon data above 100° C; the coefficients of Boudouris are intermediate between the remaining two measurements. Within the range of atmospheric temperature variations, all three sets of coefficients are consistent with the data (Table 11.1) and the choice has little effect on the mapping function.

Table 11.1. Values of refractivity coefficients K2 and K3 for radio frequencies.

	K2	error	K3	error
Birnbaum and Chatterjee	71.40	5.8	3.747×10^5	0.03
Boudouris	72.00	10.5	3.754×10^5	0.03
Thayer	64.79	0.08	3.776×10^5	0.004

Global Positioning System

For GPS analysis, the model of Lanyi (1984; see also Sovers and Border, 1987) is recommended for mapping the zenith delay to line of sight delay at different elevations. The nominal value of the zenith path delay should include both the wet and dry components. The dry component should be determined from surface pressure measurements. If these are unavailable, a nominal value close to 200 cm should be specified, depending on the altitude of the observing site. Errors in the nominal value of the dry component will be absorbed in the subsequent adjustment for the wet component. The zenith wet delay can be initially between 1-30 cm, depending on *a priori* information (seasonal averages, water vapor radiometer data) available. The estimation strategy described below is recommended:

Random walk stochastic estimation (Bierman 1977; also Lichten 1990) of the zenith wet residual delay should be included in the adjustment procedure. The random walk constraint should be tailored to the observing site. In the absence of such information, a random walk constraint of $2\text{x}10^{-7}$ km/$s^{1/2}$ can be used, which is appropriate for GPS carrier phase data noise of about ± 1 cm over 6 minutes.

Where random walk modeling is not available, alternate approaches are recommended (in order of preference):

1. Estimation of piecewise linear or quadratic zenith troposphere correction to the nominal value, with a new polynomial determined every day.

2. Estimation of a single constant zenith delay correction for each site.

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