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The angular momentum content of the oceans

While energy has microscopic sources and sinks, the angular momentum is a mechanical quantity which is *macroscopically* conserved. Therefore it is especially suited for the description and understanding of large mechanical systems like the Earth with its 3 main constituents: solid Earth, atmosphere and oceans. Table 1 shows the order of magnitude of the 3 contributions to the average rotation of the present Earth.

Note that the total angular momentum of the Earth-Moon-system is $34.0 \cdot 10^{33}$ kg m^2 s^{-1} (i.e., about 80% represented by the lunar orbit).

The contributions are practically proportional to the mass fractions since the angular velocity is the same for all three parts. Only the larger mean distance from the axis enhances the role of the two last shares (about twice as much for a homogeneous spherical surface than for the actual mass distribution of the whole Earth). The dominant role of the solid Earth ensures the high degree of constancy (or regularity) of Earth' rotation; therefore it served as *the* time standard for most of the human history.

Table 1
Average values

	Mass [kg]	Moment of inertia [kg m^2]	Angular momentum [kg m^2 s^{-1}]
Whole Earth			
≈ solid Earth	6.0×10^{24}	8.1×10^{37}	5.9×10^{33}
Oceans	1.4×10^{21}	3.9×10^{34}	2.8×10^{30}
Atmosphere	5.1×10^{18}	1.4×10^{32}	1.0×10^{28}

The main causes of variability, detected by ever refined clocks and angular measurements, are connected with the two outer nonsolid parts despite of their small mass fractions. Moreover, the relative role of oceans and atmosphere is reversed from Table 1 to Table 2 exhibiting the variations.

The key for this lies in the occurence of much greater *relative motions* in the atmosphere as compared with the oceans.

All but the values of the last line in Table 2 mean amplitudes of periodical changes.

In passing by we should mention that the comparison of

Table 2
Variations

	Torque [kg m^2 s^{-2}]	Angular momentum [kg m^2 s^{-1}]	Corresponding change in Universal Time of the solid Earth [ms]
Atmosphere seasonal	5×10^{18}	2.4×10^{25}	21
Oceans			
seasonal	3×10^{17}	1.5×10^{24}	1
tidal (M 2)	3.4×10^{21}	2.4×10^{25}	0.03
Secular retardation of the whole Earth by tidal friction	5×10^{16} (constant)	1.5×10^{24} (loss per year)	3 (change for 1 year)

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rotational effects by 'size' is a matter of the level of time derivatives: (1) rotation angle (Universal Time), (2) angular velocity (length of day, angular momentum), (3) angular acceleration (torque).

Especially in case of periodic torques of period P and amplitude L_0 it is clear that the amplitudes of level (1), (2) are proportional to L_0P^2 and L_0P respectively. This means that any ordering by the *magnitude* of the effects depends on that level as well.

The changes associated to the oceans could be sorted in a variety of manners:

First by the causes (extraterrestrial or not), second by the time scales (from 1/2 day to infinity) and third by the magnitude of an affected (observed) parameter.

Let us follow the first path. *Exchanges of angular momentum between atmosphere, oceans and solid Earth* are an almost *internal* affair of the Earth. The total angular momentum, i.e. the sum over the 3 constituents is then conserved. This, however, is the only alleviation within an otherwise very complex situation. We are dealing with a so far underexposed branch of weather and climate. From Table 2 we see that the seasonal oceanic effects are of the order of 10% of the atmospheric ones – and this size at present enters the area of measurability! Therefore and for the principal reason of understanding the angular momentum balance in weather and climate problems, now it is indispensable to attack the problem of the distribution and transfer of the oceanic share. Our present state of the art is based on very elaborated numerical hydrodynamical models of the oceans currently used at the Max-Planck-Institut für Meteorologie in Hamburg (eleven depth levels, 3°.5 × 3°.5 mesh size of surface discretization). Our and other models show clearly that the exchange of angular momentum between oceans and solid Earth takes place mainly by the so-called side pressure of the oceans acting on their continental boundaries. This is the analogy to the 'mountain torque' between atmosphere and solid Earth. With regard to the exchange between oceans and atmosphere, a purely theoretical approach is not possible, since, as we all know, our present science is not able to predict the behaviour of the atmosphere for more than a few days. Therefore these models use *empirical wind data* as input.

The surface wind stresses are responsible for the transfer between oceans and atmosphere. We have got results for the average saisonal variation of the winds [1]. At present we try to obtain the effects of the actual wind fields of the last 10 years, since those are the ones for which Earth' rotation is measured with high-precision VLBI. In order to achieve this goal it is not sufficient to run the model for the same years. The oceans have a very long 'memory' of their previous states. Theoretically one should start with a 'state' which implies also all necessary time derivatives as well. In practice, one tries to get the same by integrating over a long time interval with *average* saisonal variations before applying the actual winds. This last part is then used for the comparison with Earth rotation data.

The closure condition for the total angular momentum is especially easy and separable for the axial component. The sum of the angular momenta of the solid Earth J_s and the atmosphere J_A today is observed accurate enough to reveal its non-constancy (see e.g. [2]). Despite of relatively large errors of this partial sum, we would expect some signal in it and that should be mirror-like to the oceanic angular momentum J_0 – so that the total sum is constant. Until now we have found correlation coefficients around

$$r(-J_0, J_S + J_A) \approx +0.30 \pm 0.09$$

thus only slightly significant [3], see Fig. 1.

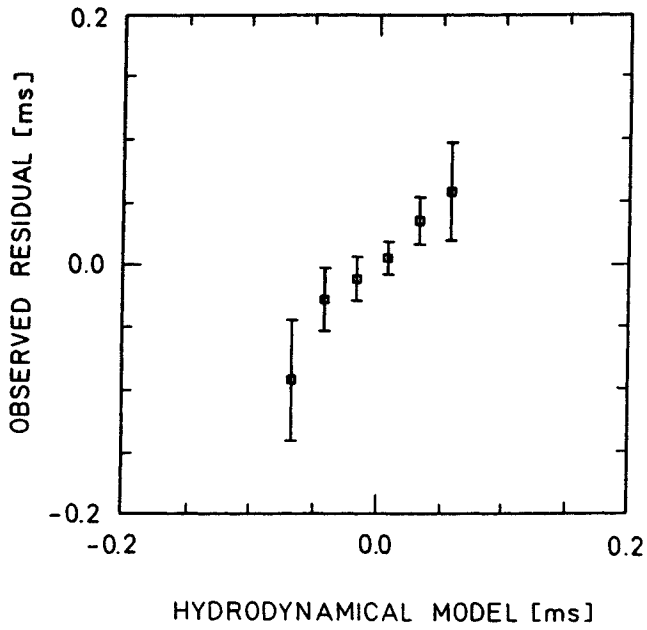

Fig. 1. After subtraction of the mean seasonal variation for the epoch 1981–1989, the residual theoretical values of $-J_0$ are compared with the residual observed $(J_S + J_A)$, both expressed as differences in the length of day and averaged in intervals of $-J_0$.

Since the signature of very important climate events (such as El Nino) has been claimed to be visible in Earth rotation data, it is appealing to look for it in the J_0-results. However, today we cannot say whether the visible peaks in J_0 are only attributable to such an event or to other causes as well.

Let us now turn to a second kind of variations, *the periodic effects of lunar and solar tides*. Although of extraterrestrial origin (and thereby augmenting the number of participating constituents) their treatment is easier for the following reason. Moon and Sun produce forced oscillations in the oceans with the following properties:

(a) The exchange of angular momentum with the disturbing bodies is small; hence the Earth may be seen as a closed system even in this case.
(b) Unlike the surface wind stresses of the foregoing case, the tidal forces are body forces and affect the whole ocean at once. The changes at the surface are small and an exchange with the atmosphere for this reason may be neglected; the exchange with the solid Earth is the only remaining. Practically we are dealing with a two-component system and we expect associated mirror-like changes in oceans and solid Earth.
(c) The periods of the main partial tides are different from other influences so that a separate determination is possible. Especially the solar day (S_1) is not a frequency of our 10 most important tides.
(d) Solid Earth tides do not interfere because the solid Earth is rather symmetric.

The combination of this advantages has allowed us to confirm the existence of these oceanic effects at a 4σ-level despite of their smallness (0.1 ms in UT or a few cm in the position of an equatorial point in space). Fig. 2 compares theory and observations.

Finally, let me briefly mention the secular transfer of angular momentum from the whole Earth into the lunar orbit, better known as *tidal friction*. The physical process is the same as in the foregoing case. Here we are looking at it, however, in the long run. Then the Earth alone is not a closed system

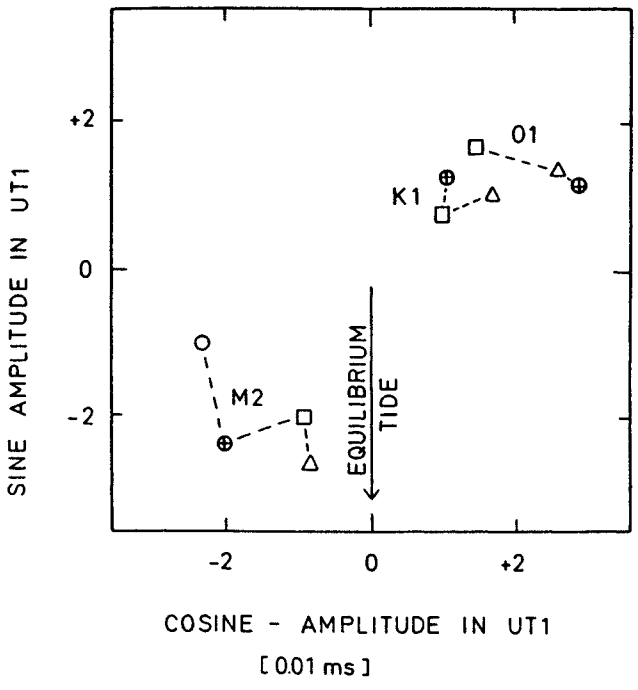

Fig. 2. Periodical effects in UT1 due to oceanic tides: VLBI observations (□ Bonn group [4], Δ HERRING & DONG [5]) versus hydrodynamical models (⊕ 1° mesh size [9] and, for comparison, ○ an older one with 4° mesh size only for M_2 [7])

anymore. Small amounts of angular momentum transferred per tidal period *accumulate* within millenia of human history into UTchanges of hours and change the length of day itself by hours if we go back, e.g. to the Devonian epoch of our Earth' history. It is clear then that these changes must have influenced the Earth' evolution in many ways: the length of day is a primary meteorological parameter, its variation causes concomitant changes in the figure of the Earth and these metamorphoses must have left its signatures in the crust. Ultimately, when going further backwards in time the earliest state of the Earth-Moon-system should be arrived.

For this problem, the oceans have been recognized as the main mediators of the transfer of angular momentum, while the phase lag angle of the solid Earth tides is not large enough for a significant share. The present transfer is shown by historical observations of solar eclipses and lunar laser ranging (LLR). Despite of its smallness, it is quite satisfyingly explained by accurate tidal models (see last line in Table 2).

The large remaining problem is the time dependence of the transfer torque. It is sure that schematic formulae cannot be applied for two reasons: (a) the geometry of the oceans changes through continental drift, (b) the oceans have a very rich eigenvalue spectrum and react very sensitively to such changes and to changes in length of day and tidal periods [8].

The observational evidence from paleontology and, most recent, sedimentology is very appealing. Some tide models for ancient oceans illustrate the potentialities of this kind of research – but its infancy as well [9].

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