CHAPTER 3 CONVENTIONAL TERRESTRIAL REFERENCE SYSTEM

Definition

The Terrestrial Reference System adopted for either the analysis of individual data sets by techniques (VLBI, SLR, LLR, GPS, DORIS, PRARE...) or the combination of individual solutions into a unified set of data (station coordinates, Earth orientation parameters, etc...) follows these criteria (Boucher, 1990):

a) It is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere.

b) Its scale is that of a local Earth frame, in the meaning of a relativistic theory of gravitation.

c) Its orientation was initially given by the BIH orientation at 1984.0.

d) Its time evolution in orientation will create no residual global rotation with regards to the crust.

Realization

A Conventional Terrestrial Reference System (CTRS) can be realized through a reference frame i.e. a set of coordinates for a network of stations. Such a realization will be specified by cartesian equatorial coordinates \( X, Y, \) and \( Z, \) by preference. If geographical coordinates are needed, the GRS80 ellipsoid is recommended (\( a=6378137.0 \) m, eccentricity \( ^2 =0.00669438003. \)) The CTRS which is monitored by IERS is called the International Terrestrial Reference System (ITRS) and is specified by the IUGG Resolution no. 2 adopted at the 20th IUGG General Assembly of Vienna in 1991.

Each analysis center should compare its reference frame to a realization of the ITRS. Within IERS, each Terrestrial Reference Frame (TRF) is either directly, or after transformation, expressed as a realization of the ITRS. The position of a point located on the surface of the solid Earth should be expressed by

\[
\vec{X}(t) = \vec{X}_0 + \vec{V}_0(t - t_0) + \sum_i \Delta \vec{X}_i(t),
\]

where \( \Delta \vec{X}_i \) are corrections due to various time changing effects, and \( \vec{X}_0 \) and \( \vec{V}_0 \) are position and velocity at the epoch \( t_0. \) The corrections to be considered are solid Earth tide displacement (full correction including permanent effect, see Chapter 7), ocean loading, post glacial rebound, and atmospheric loading. Further corrections could be added if they are at mm level or greater, and can be computed by a suitable model.

Realizations of the ITRS are produced by IERS under the name International Terrestrial Reference Frames (ITRF), which consist of lists of coordinates (and velocities) for a selection of IERS sites (tracking stations or related ground markers). Currently, ITRF-yy is published annually by the IERS in the Technical Notes (cf. Boucher et al., 1996). The numbers (yy) following the designation “ITRF” specify the last year whose data were used in the formation of the frame. Hence ITRF94 designates the frame of coordinates and velocities constructed in 1995 using all of the IERS data available through 1994. More recently, since 1993, other special realizations have been produced, such as solutions for IGS core stations (ITRF-Py series) (IGS, 1995). It is also anticipated that monthly series may be determined.
Terrestrial reference frames may be defined within systems which account for the effect of the permanent tide differently. In the terminology of Ekman (1989), Rapp (1991) and Poutanen et al. (1996) the permanent or "zero-frequency" tide is retained in the "zero-tide" system. In this system the crust corresponds to the realistic time average of the crust which varies because of the action of the luni-solar tides. In a different system referred to as "tide-free," all of the effects of the permanent tide are removed. This is not realistic since the crust in this case cannot be "observed," nor are the Love Numbers required to describe the permanent tide known adequately. No error is associated with either convention as long as it is clear which system is employed in the analysis of the data. A third "mean-tide" system has been defined in which the crust is equivalent to that of the tide-free system and, in which, the effect of the permanent tide is also removed from the geoid.

Coordinates of the ITRF are given in a conventional frame where the effects of all tides are removed as recommended in IERS Technical Note 13 (McCarthy, 1992). There, it is recommended that equation 6 on p. 57 be used to account for the permanent tide. Note that this is in contradiction to Resolution 16 of the General Assembly of the IAG in 1983. The corresponding equation in this publication is equation 8 of Chapter 7. To place the coordinates in a zero-tide system it is necessary to apply equation 8 of IERS Technical Note 13 (McCarthy, 1992) or the corresponding version, equation 17 of Chapter 7 in this publication.

In data analysis, $\mathbf{X}_0$ and $\mathbf{V}_0$ should be considered as solve-for parameters. In particular, if a non-linear change occurs (earthquake, volcanic event ...), a new $\mathbf{X}_0$ should be adopted. When adjusting parameters, particularly velocities, the IERS orientation should be kept at all epochs, ensuring the alignment at a reference epoch and the time evolution through a no net rotation condition. The way followed by various analysis centers depends on their own view of modelling, and on the techniques themselves. For the origin, only data which can be modelled by dynamical techniques (currently SLR, LLR, GPS or DORIS for IERS) can determine the center of mass. The VLBI system can be referred to a geocentric system by adopting for a station its geocentric position at a reference epoch as provided from external information.

The scale is obtained by appropriate relativistic modelling. Specifically, according to IAU and IUGG resolutions, the scale is consistent with the TCG time coordinate for a geocentric local frame. A detailed treatment can be found in Chapter 11. The orientation is defined by adopting IERS (or BIH) Earth orientation parameters at a reference epoch. In the case of dynamical techniques, an additional constraint in longitude is necessary to remove ill-conditioning.

The unit of length is the meter (SI). The IERS Reference Pole (IRP) and Reference Meridian (IRM) are consistent with the corresponding directions in the BIH Terrestrial System (BTS) within $\pm 0.005$. The BIH reference pole was adjusted to the Conventional International Origin (CIO) in 1967; it was then kept stable independently until 1987. The uncertainty of the tie of the IRP with the CIO is $\pm 0.03$. The time evolution of the orientation will be ensured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth.

**Transformation Parameters to World Coordinate Systems and Various Datums**

The seven-parameter similarity transformation between any two Cartesian systems, e.g., from $(x, y, z)$ to $(xs, ys, zs)$ can be written as

$$\begin{pmatrix} xs \\ ys \\ zs \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix} + \begin{pmatrix} D & -R3 & R2 \\ R3 & D & -R1 \\ -R2 & R1 & D \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

(1)
where $T_1, T_2, T_3$ = coordinates of the origin of the frame $(x_s, y_s, z_s)$ in the frame $(x, y, z)$; $R_1, R_2, R_3 =$ differential rotations (expressed in radians) respectively, around the axes $(x_s, y_s, z_s)$ to establish parallelism with the $(x, y, z)$ frame; and $D =$ differential scale change.

The use of the previous algorithm to transform coordinates from one datum to another must be done with care. At some level of accuracy, each reference system has multiple realizations. In many cases, two such realizations of the same system have nonzero transformation parameters. The usual reason is that such parameters are currently adjusted between two sets of coordinates. In this case, any global systematic error which can be mapped into a scale, shift, or rotation will go into these seven parameters.

Therefore we give here (Table 3.1) only parameters from ITRF94 to previous ITRF series. These numbers are the responsibility of the IERS. For other transformations, either to global systems (such as WGS84) or regional (such as NAD83 or ETRS89), or even local systems, users should contact relevant agencies. For WGS84, the main result is that the most recent realizations of WGS84 based on GPS are consistent with the ITRF realization at the 0.1 meter level (Malys and Slater, 1994). Otherwise, one can refer to IAG bodies, such as Commission X on Global and Regional Geodetic Networks (Geodesist’s Handbook, 1996).

Table 3.1. Transformation parameters from ITRF94 to past ITRFs. “ppb” refers to parts per billion $(10^9)$. Rates must be applied for ITRF93. The units for rate are understood to be “per year.”

<table>
<thead>
<tr>
<th>Coordinate System (datum)</th>
<th>$T_1$ (cm)</th>
<th>$T_2$ (cm)</th>
<th>$T_3$ (cm)</th>
<th>$D$ (ppb)</th>
<th>$R_1$ (mas)</th>
<th>$R_2$ (mas)</th>
<th>$R_3$ (mas)</th>
<th>Epoch</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITRF88</td>
<td>1.8</td>
<td>0.0</td>
<td>-9.2</td>
<td>7.4</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>1988.0</td>
</tr>
<tr>
<td>ITRF89</td>
<td>2.3</td>
<td>3.6</td>
<td>-6.8</td>
<td>4.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1988.0</td>
</tr>
<tr>
<td>ITRF90</td>
<td>1.8</td>
<td>1.2</td>
<td>-3.0</td>
<td>0.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1988.0</td>
</tr>
<tr>
<td>ITRF91</td>
<td>2.0</td>
<td>1.6</td>
<td>-1.4</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1988.0</td>
</tr>
<tr>
<td>ITRF92</td>
<td>0.8</td>
<td>0.2</td>
<td>-0.8</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1988.0</td>
</tr>
<tr>
<td>ITRF93</td>
<td>0.6</td>
<td>-0.5</td>
<td>-1.5</td>
<td>0.4</td>
<td>-0.39</td>
<td>0.80</td>
<td>-0.96</td>
<td>1988.0</td>
</tr>
<tr>
<td>rates</td>
<td>-0.29</td>
<td>0.04</td>
<td>0.08</td>
<td>0.0</td>
<td>-0.11</td>
<td>-0.19</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

Once the Cartesian coordinates $(x, y, z)$ are known, they can be transformed to “datum” or curvilinear geodetic coordinates $(\lambda, \phi, h)$ referred to an ellipsoid of semi-major axis $a$ and flattening $f$, using the following code (Borkowski, 1989). First compute $\lambda = \tan^{-1} \left( \frac{y}{x} \right)$ properly determining the quadrant from $x$ and $y$ $(0 \leq \lambda < 2\pi)$ and $r = \sqrt{x^2 + y^2}$.

```
subroutine GEOD(r,z,fi,h)
  c Program to transform Cartesian to geodetic coordinates
  c based on the exact solution (Borkowski,1989)
  c Input : r, z = equatorial [m] and polar [m] components
  c Output: fi, h = geodetic coord's (latitude [rad], height [m])
  implicit real*8(a-h,o-z)
  c GRS80 ellipsoid: semimajor axis (a) and inverse flattening (fr)
  data a,fr /6378137.0,298.2572221010d0/
    b = dsig(a - a/fr,z)
    E = ((z + b)*b/a - a)/r
    F = ((z - b)*b/a + a)/r

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c Find solution to: \( t^4 + 2Et^3 + 2Ft - 1 = 0 \)

\[
P = (E^2 + 1) \times 4D^2/3D^2
\]

\[
Q = (E^2 - F^2) \times 2D^2
\]

\[
D = P/P + Q/Q
\]

if(D \geq 0) then

\[
s = \text{dsqrt}(D) + Q
\]

\[
s = \text{dsign(dexp(dlog(dabs(s))/3D^2), s)}
\]

\[
v = P/s - s
\]

else

\[
v = 2D^2 \times \text{dsqrt}(-P) \times \text{dcos(dacos(Q/P/\text{dsqrt}(-P))/3D^2)}
\]
endif

\[
G = .5D^2(E + \text{dsqrt}(E^2 + v))
\]

\[
t = \text{dsqrt}(G^2 + (F - vG)/(G + G - E) - G)
\]

\[
fi = \text{datan}((1D^2 - t\times t)a/(2D^2b\times t))
\]

\[
h = (r - a\times t)*\text{dcos}(fi) + (z - b)*\text{dsin}(fi)
\]

return

eend

Plate Motion Model

One of the factors which affect Earth rotation results is the motion of the tectonic plates which make up the Earth’s surface. As the plates move, fixed coordinates for the observing stations become inconsistent with each other. The rates of relative motions for some observing sites are 5 cm per year or larger. The observations of plate motions by modern methods appear to be roughly consistent with the average rates over the last few million years derived from the geological record and other geophysical information. Thus, the NNR-NUVEL1A model for plate motions given by DeMets et al. (1994) is recommended.

If a velocity has not yet been determined in the ITRF for a particular station, the velocity \( \vec{V}_0 \) should be expressed as

\[
\vec{V}_0 = \vec{V}_{\text{plate}} + \vec{V}_r
\]

where \( \vec{V}_{\text{plate}} \) is the horizontal velocity computed from the NNR-NUVEL1A model (DeMets et al., 1994) and \( \vec{V}_r \) a residual velocity.

The Cartesian rotation vector for each of the major plates is given in Table 3.2. A subroutine called ABSMO-NUVEL is also included below. It computes the new site position from the old site position using the recommended plate motion model. Fig. 3.1 shows the plates on a map of the world.
Figure 3.1. Map of the tectonic plates.

Table 3.2. Cartesian rotation vector for each plate using the NNR-NUVEL1A kinematic plate model (no net rotation).

<table>
<thead>
<tr>
<th>Plate Name</th>
<th>$\Omega_x$ rad/My.</th>
<th>$\Omega_y$ rad/My.</th>
<th>$\Omega_z$ rad/My.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pacific</td>
<td>-0.001510</td>
<td>0.004840</td>
<td>-0.009970</td>
</tr>
<tr>
<td>Cocos</td>
<td>-0.010425</td>
<td>-0.021605</td>
<td>0.010925</td>
</tr>
<tr>
<td>Nazca</td>
<td>-0.001532</td>
<td>-0.008577</td>
<td>0.009609</td>
</tr>
<tr>
<td>Caribbean</td>
<td>-0.000178</td>
<td>-0.003385</td>
<td>0.001581</td>
</tr>
<tr>
<td>South America</td>
<td>-0.001038</td>
<td>-0.001515</td>
<td>-0.000870</td>
</tr>
<tr>
<td>Antarctic</td>
<td>-0.000821</td>
<td>-0.001701</td>
<td>0.003706</td>
</tr>
<tr>
<td>India</td>
<td>0.006670</td>
<td>0.000040</td>
<td>0.006790</td>
</tr>
<tr>
<td>Australia</td>
<td>0.007839</td>
<td>0.005124</td>
<td>0.006282</td>
</tr>
<tr>
<td>Africa</td>
<td>0.000891</td>
<td>-0.003099</td>
<td>0.003922</td>
</tr>
<tr>
<td>Arabia</td>
<td>0.006685</td>
<td>-0.000521</td>
<td>0.006760</td>
</tr>
<tr>
<td>Eurasia</td>
<td>-0.000981</td>
<td>-0.002395</td>
<td>0.003153</td>
</tr>
<tr>
<td>North America</td>
<td>0.000258</td>
<td>-0.003599</td>
<td>-0.000153</td>
</tr>
<tr>
<td>Juan de Fuca</td>
<td>0.005200</td>
<td>0.008610</td>
<td>-0.005820</td>
</tr>
<tr>
<td>Philippine</td>
<td>0.010090</td>
<td>-0.007160</td>
<td>-0.009670</td>
</tr>
<tr>
<td>Rivera</td>
<td>-0.009390</td>
<td>-0.030060</td>
<td>0.012050</td>
</tr>
<tr>
<td>Scotia</td>
<td>-0.000410</td>
<td>-0.002660</td>
<td>-0.001270</td>
</tr>
</tbody>
</table>

The NNR-NUVEL1A model should be used as a default, for stations which appear to follow
reasonably its values. For some stations, particularly in the vicinity of plate boundaries, users may benefit by estimating velocities or using specific values not derived from NNR-NUVEL1A. This is also a way to take into account now some non-negligible vertical motions. Published station coordinates should include the epoch associated with the coordinates.

The original subroutine was a coding of the AM0-2 model from J. B. Minster. Changes have been made to represent NNR-NUVEL1A model (DeMets et al., 1994).

SUBROUTINE AM0_NUVEL(PSIT,T0,X0,Y0,Z0,T,X,Y,Z)
C
C AM0_NUVEL takes a site specified by its initial coordinates
C X0,Y0,Z0 at time T0, and computes its updated positions X,Y,Z
C at time T, based on the geological "absolute".
C
C Original author: J.B. Minster, Science Horizons.
C DFA: Revised by Don Argus, Northwestern University
C DFA: uses absolute model NNR-NUVEL1
C
C Transcribed from USNO Circular 167 "Project Merit Standards"
C by Tony Mallama with slight modification to the documentation
C and code.
C
C Times are given in years, e.g. 1988.0 for Jan 1, 1988.
C
C PSIT is the four character abbreviation for the plate name,
C if PSIT is not recognized then the new positions are returned
C as zero.
C
IMPLICIT NONE
CHARACTER*4 PSIT,PNM(16)
REAL*8 OMX(16),OMY(16),OMZ(16)
REAL*8 X0,Y0,Z0
REAL*8 X,Y,Z,T,T0
INTEGER*2 IPSIT,I

DATA (PNM(I), OMX(I), OMY(I), OMZ(I),
 & I = 1,16)
 & '/PCFC', -0.001510, 0.004840, -0.009970,
 & 'AFRC', 0.000891, -0.003099, 0.003922,
 & 'ANT', -0.000821, -0.001701, 0.003706,
 & 'ARAB', 0.006685, -0.000521, 0.006760,
 & 'AUST', 0.007839, 0.005124, 0.006282,
 & 'CARB', -0.000178, -0.003385, 0.001581,
 & 'COCO', 0.007839, -0.000178, 0.007839,
 & 'EURA', 0.000981, -0.002395, 0.003135,
C Initialize things properly
C
IPSIT = -1
X = 0.0D0
Y = 0.0D0
Z = 0.0D0
C Look up the plate in the list.
C
DO 20 I = 1, 16
20 IF (IPSIT .EQ. PNM(I)) IPSIT = I
C If plate name is not recognized return the new plate position as zero.
C
IF (IPSIT .EQ. -1) RETURN
C Compute the new coordinates
C
X = XO + (ORY*Z0 - ORZ*Y0) * (T-T0)/1.0D+6
Y = Y0 + (ORZ*X0 - ORX*Z0) * (T-T0)/1.0D+6
Z = Z0 + (ORX*Y0 - ORY*X0) * (T-T0)/1.0D+6
C Finish up
C
RETURN
END

References


