CHAPTER 5 TRANSFORMATION BETWEEN THE CELESTIAL AND TERRESTRIAL SYSTEMS

The coordinate transformation to be used from the TRS to the CRS at the date t of the observation can be written as:

$$[\mathrm{CRS}] = PN(t)R(t)W(t)\ [\mathrm{TRS}],$$

where $PN(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the Celestial Ephemeris Pole (CEP) in the CRS, from the rotation of the Earth around the axis of the CEP, and from polar motion respectively.

Two equivalent options can be used. These two options have been shown to be consistent within ± 0.05 milliseconds of arc (mas) both theoretically (Capitaine, 1990) and numerically using existing astrometric data (Capitaine and Chollet, 1991) or simulated data over two centuries (Capitaine and Gontier, 1991).

Option 1, corresponding to the classical procedure, makes use of the equinox for realizing the intermediate reference frame of date t. It uses apparent Greenwich Sidereal Time in the transformation matrix $R(t)$ and the classical precession and nutation parameters in the transformation matrix $PN(t)$,

Option 2 makes use of the Conventional Ephemeris Origin (CEO) originally referred to as the nonrotating origin (Guinot, 1979) to realize the intermediate reference frame of date t: it uses the stellar angle (from the origin in the TRS to the CEO in the CRS) in the transformation matrix $R(t)$ and the two coordinates of the Celestial Ephemeris Pole in the CRS (Capitaine, 1990) in the transformation matrix $PN(t)$. This leads to very simple expressions of the partial derivatives of observables with respect to polar coordinates, UT1, and celestial pole offsets.

The following sections give the details of these two options as well as the standard expressions necessary to obtain the numerical values of the relevant parameters at the date of the observation. Subroutines for options 1 and 2 of the coordinate transformation from the TRS to the CRS are available from the Central Bureau on request together with the development of the parameters.

The expressions of the precession and nutation quantities have been developed originally as functions of barycentric dynamical time (TDB) defined by IAU recommendations of 1976 and 1979. In 1991 the IAU adopted definitions of other time scales. See Chapter 11 for the relationships among these time scales. The parameter t is defined by

$$t = (\mathrm{TT} - 2000\ \text{January 1d 12h TT})\ \text{in days}/36525.$$

This definition is consistent with Resolution C7 passed at the 1994 Hague General Assembly of the IAU which recommends that J2000.0 be defined at the geocenter and at the date 2000.0 January 1.5 TT = Julian Date 2451545.0 TT. In the following, R_1, R_2 and R_3 denote direct rotations about the axes 1, 2 and 3 of the coordinate frame.

Coordinate Transformation Referred to the Equinox

Option 1 uses the form of the coordinate transformation

$$[\mathrm{CRS}] = PN'(t)R'(t)W'(t)\ [\mathrm{TRS}], \tag{1}$$

in which the three fundamental components are (Mueller, 1969)

$$W'(t) = R_1(y_p) \cdot R_2(x_p),$$

x_p and y_p being the "polar coordinates" of the CEP in the TRS;

$$R'(t) = R_3(-\mathrm{GST}),$$

GST being Greenwich True Sidereal Time at date t, including both the effect of Earth rotation and the accumulated precession and nutation in right ascension; and

$$PN'(t) = [P][N],$$

$$\text{with } [P] = R_3(\zeta_A) \cdot R_2(-\theta_A) \cdot R_3(z_A)$$

for the transformation matrix corresponding to the precession between the reference epoch and the date t,

$$[N] = R_1(-\epsilon_A) \cdot R_3(\Delta\psi) \cdot R_1(\epsilon_A + \Delta\epsilon)$$

for the transformation matrix corresponding to the nutation at date t.

Standard values of the parameters to be used in form (1) of the transformation are explained below.

The standard polar coordinates to be used for the parameters x_p and y_p (if not estimated from the observations) are those published by the IERS. Apparent Greenwich Sidereal Time GST at the date t of the observation, must be derived from the following expressions:

(i) the relationship between Greenwich Mean Sidereal Time (GMST) and Universal Time as given by Aoki *et al.* (1982):

$$\mathrm{GMST}_{0\mathrm{h\ UT1}} = 6^h\ 41^m\ 50\overset{s}{.}54841 + 8640184\overset{s}{.}812866 T'_u + 0\overset{s}{.}093104 T'^2_u - 6\overset{s}{.}2 \times 10^{-6} T'^3_u,$$

with $T'_u = d'_u/36525$, d'_u being the number of days elapsed since 2000 January 1, 12h UT1, taking on values ± 0.5, ± 1.5, ...,

(ii) the interval of GMST from 0h UT1 to the hour of the observation in UT1,

$$\mathrm{GMST} = \mathrm{GMST}_{0\mathrm{h\ UT1}} + r[(\mathrm{UT1} - \mathrm{UTC}) + \mathrm{UTC}],$$

where r is the ratio of universal to sidereal time as given by Aoki *et al.* (1982),

$$r = 1.002737909350795 + 5.9006 \times 10^{-11} T'_u - 5.9 \times 10^{-15} T'^2_u$$

and the UT1-UTC value to be used (if not estimated from the observations) is the IERS value.

(iii) accumulated precession and nutation in right ascension (Aoki and Kinoshita, 1983),

$$\mathrm{GST} = \mathrm{GMST} + \Delta\psi \cos\epsilon_A + 0\overset{''}{.}00264 \sin\Omega + 0\overset{''}{.}000063 \sin 2\Omega,$$

where Ω is the mean longitude of the ascending node of the lunar orbit. The last two terms have not been included in the IERS Standards previously. They should not be included in (iii) until 1 January

1997 when their use will begin. This date is chosen to minimize any discontinuity in UT1. The effect of these terms on the estimation of UT1 has been described by Capitaine and Gontier (1993).

The numerical expression for the precession quantities ζ_A, θ_A, z_A and ϵ_A have been given by Lieske *et al.* (1977) as functions of two time parameters t and T (the last parameter representing Julian centuries from J2000.0 to an arbitrary epoch). The simplified expressions when the arbitrary epoch is chosen to be J2000.0 (*i.e.* $T = 0$) are

$$\zeta_A = 2306''.2181t + 0''.30188t^2 + 0''.017998t^3,$$

$$\theta_A = 2004''.3109t - 0''.42665t^2 - 0''.041833t^3,$$

$$z_A = 2306''.2181t + 1''.09468t^2 + 0''.018203t^3,$$

$$\epsilon_A = 84381''.448 - 46''.8150t - 0''.00059t^2 + 0''.001813t^3.$$

The nutation quantities $\Delta\psi$ and $\Delta\epsilon$ to be used are the nutation angles in longitude and obliquity.

For observations requiring values of the nutation angles with an accuracy of ± 1 mas, it is necessary to add (if those quantities are not estimated from the observations) the IERS published values (observed or predicted) for the "celestial pole offsets" (*i.e.* corrections dpsi and deps).

The IAU 1980 Theory of Nutation

The IAU 1980 Theory of Nutation (Seidelmann, 1982; Wahr, 1981) is based on a modification of a rigid Earth theory published by Kinoshita (1977) and on the geophysical model 1066A of Gilbert and Dziewonski (1975). It therefore includes the effects of a solid inner core and a liquid outer core and a "distribution of elastic parameters inferred from a large set of seismological data." The constants defining the theory are given in Table 5.1.

VLBI and LLR observations have shown that there are deficiencies in the IAU 1976 Precession and in the IAU 1980 Theory of Nutation. However, these models are kept as part of the IERS Standards and the observed differences ($\delta\Delta\psi$ and $\delta\Delta\epsilon$, equivalent to dpsi and deps in the IERS Bulletins) with respect to the conventional celestial pole position defined by the models are monitored and reported by the IERS as "celestial pole offsets." Using these offsets the corrected nutation is given by

$$\Delta\psi = \Delta\psi(\text{IAU 1980}) + \delta\Delta\psi,$$

and

$$\Delta\epsilon = \Delta\epsilon(\text{IAU1980}) + \delta\Delta\epsilon.$$

This is practically equivalent to replacing N with the rotation described by Lieske (1991),

$$\tilde{N} = \mathcal{R}N_{\text{IAU}}.$$

where

$$\mathcal{R} = \begin{pmatrix} 1 & -\delta\Delta\psi\cos\epsilon_t & -\delta\Delta\psi\sin\epsilon_t \\ \delta\Delta\psi\cos\epsilon_t & 1 & \delta\Delta\epsilon \\ \delta\Delta\psi\sin\epsilon_t & \delta\Delta\epsilon & 1 \end{pmatrix},$$

$\epsilon_t = \epsilon_A + \Delta\epsilon$, and $N_{\rm IAU}$ represents the IAU 1980 Theory of Nutation according to which

$$\Delta\psi = \sum_{i=1}^{106}(A_i + A_i't)\sin(\text{ARGUMENT}), \quad \Delta\epsilon = \sum_{i=1}^{106}(B_i + B_i't)\cos(\text{ARGUMENT}).$$

Here

$$\text{ARGUMENT} = \sum N_i F_i,$$

the $N_i, (i = 1, \cdots, 5)$ being integers multiplying the Fundamental Arguments F_i of nutation theory, namely,

$$\begin{aligned}
F_1 \equiv l \quad &= \text{Mean Anomaly of the Moon} \\
&= 134^\circ\!.96340251 + 1717915923''\!.2178t + 31''\!.8792t^2 + 0''\!.051635t^3 - 0''\!.00024470t^4,
\end{aligned}$$

$$\begin{aligned}
F_2 \equiv l' \quad &= \text{Mean Anomaly of the Sun} \\
&= 357^\circ\!.52910918 + 129596581''\!.0481t - 0''\!.5532t^2 - 0''\!.000136t^3 - 0''\!.00001149t^4,
\end{aligned}$$

$$\begin{aligned}
F_3 \equiv F \quad &= L - \Omega \\
&= 93^\circ\!.27209062 + 1739527262''\!.8478t - 12''\!.7512t^2 - 0''\!.001037t^3 + 0''\!.00000417t^4,
\end{aligned}$$

$$\begin{aligned}
F_4 \equiv D \quad &= \text{Mean Elongation of the Moon from the Sun} \\
&= 297^\circ\!.85019547 + 1602961601''\!.2090t - 6''\!.3706t^2 + 0''\!.006593t^3 - 0''\!.00003169t^4,
\end{aligned}$$

$$\begin{aligned}
F_5 \equiv \Omega \quad &= \text{Mean Longitude of the Ascending Node of the Moon} \\
&= 125^\circ\!.04455501 - 6962890''\!.2665t + 7''\!.4722t^2 + 0''\!.007702t^3 - 0''\!.00005939t^4,
\end{aligned}$$

where t is measured in Julian Centuries of 36525 days of 86400 seconds of Dynamical Time since J2000.0 and where $1^r = 360^\circ = 1296000''\!.0$ (Simon *et al.*, 1994). L is the Mean Longitude of the Moon.

The reader is cautioned that there may be an ambiguity in the assignment of the set of five multipliers N_j to any particular term of the nutation series. This ambiguity is illustrated, and a convention for unique assignment of multipliers for prograde and retrograde circular nutations is presented, in a note at the end of this chapter.

Table 5.1. IAU 1980 Theory of Nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with t measured in Julian centuries from epoch J2000.0. The signs of the fundamental arguments, periods, and coefficients may differ from the original publication. These have been changed to be consistent with other portions of this chapter.

$$\Delta\psi = \sum_{i=1}^{106}(A_i + A_i't)\sin(\text{ARGUMENT}), \quad \Delta\epsilon = \sum_{i=1}^{106}(B_i + B_i't)\cos(\text{ARGUMENT}).$$

MULTIPLIERS OF					PERIOD	LONGITUDE (0″.0001)		OBLIQUITY (0″.0001)	
l	l'	F	D	Ω	(days)	A_i	A'_i	B_i	B'_i
0	0	0	0	1	-6798.4	-171996	-174.2	92025	8.9
0	0	2	-2	2	182.6	-13187	-1.6	5736	-3.1
0	0	2	0	2	13.7	-2274	-0.2	977	-0.5
0	0	0	0	2	-3399.2	2062	0.2	-895	0.5
0	-1	0	0	0	-365.3	-1426	3.4	54	-0.1
1	0	0	0	0	27.6	712	0.1	-7	0.0
0	1	2	-2	2	121.7	-517	1.2	224	-0.6
0	0	2	0	1	13.6	-386	-0.4	200	0.0
1	0	2	0	2	9.1	-301	0.0	129	-0.1
0	-1	2	-2	2	365.2	217	-0.5	-95	0.3
-1	0	0	2	0	31.8	158	0.0	-1	0.0
0	0	2	-2	1	177.8	129	0.1	-70	0.0
-1	0	2	0	2	27.1	123	0.0	-53	0.0
1	0	0	0	1	27.7	63	0.1	-33	0.0
0	0	0	2	0	14.8	63	0.0	-2	0.0
-1	0	2	2	2	9.6	-59	0.0	26	0.0
-1	0	0	0	1	-27.4	-58	-0.1	32	0.0
1	0	2	0	1	9.1	-51	0.0	27	0.0
-2	0	0	2	0	-205.9	-48	0.0	1	0.0
-2	0	2	0	1	1305.5	46	0.0	-24	0.0
0	0	2	2	2	7.1	-38	0.0	16	0.0
2	0	2	0	2	6.9	-31	0.0	13	0.0
2	0	0	0	0	13.8	29	0.0	-1	0.0
1	0	2	-2	2	23.9	29	0.0	-12	0.0
0	0	2	0	0	13.6	26	0.0	-1	0.0
0	0	2	-2	0	173.3	-22	0.0	0	0.0
-1	0	2	0	1	27.0	21	0.0	-10	0.0
0	2	0	0	0	182.6	17	-0.1	0	0.0
0	2	2	-2	2	91.3	-16	0.1	7	0.0
-1	0	0	2	1	32.0	16	0.0	-8	0.0
0	1	0	0	1	386.0	-15	0.0	9	0.0
1	0	0	-2	1	-31.7	-13	0.0	7	0.0
0	-1	0	0	1	-346.6	-12	0.0	6	0.0
2	0	-2	0	0	-1095.2	11	0.0	0	0.0
-1	0	2	2	1	9.5	-10	0.0	5	0.0
1	0	2	2	2	5.6	-8	0.0	3	0.0
0	-1	2	0	2	14.2	-7	0.0	3	0.0
0	0	2	2	1	7.1	-7	0.0	3	0.0
1	1	0	-2	0	-34.8	-7	0.0	0	0.0
0	1	2	0	2	13.2	7	0.0	-3	0.0
-2	0	0	2	1	-199.8	-6	0.0	3	0.0
0	0	0	2	1	14.8	-6	0.0	3	0.0
2	0	2	-2	2	12.8	6	0.0	-3	0.0
1	0	0	2	0	9.6	6	0.0	0	0.0
1	0	2	-2	1	23.9	6	0.0	-3	0.0
0	0	0	-2	1	-14.7	-5	0.0	3	0.0
0	-1	2	-2	1	346.6	-5	0.0	3	0.0
2	0	2	0	1	6.9	-5	0.0	3	0.0
1	-1	0	0	0	29.8	5	0.0	0	0.0
1	0	0	-1	0	411.8	-4	0.0	0	0.0
0	0	0	1	0	29.5	-4	0.0	0	0.0
0	1	0	-2	0	-15.4	-4	0.0	0	0.0
1	0	-2	0	0	-26.9	4	0.0	0	0.0
2	0	0	-2	1	212.3	4	0.0	-2	0.0
0	1	2	-2	1	119.6	4	0.0	-2	0.0
1	1	0	0	0	25.6	-3	0.0	0	0.0
1	-1	0	-1	0	-3232.9	-3	0.0	0	0.0
-1	-1	2	2	2	9.8	-3	0.0	1	0.0
0	-1	2	2	2	7.2	-3	0.0	1	0.0
1	-1	2	0	2	9.4	-3	0.0	1	0.0
3	0	2	0	2	5.5	-3	0.0	1	0.0
-2	0	2	0	2	1615.7	-3	0.0	1	0.0
1	0	2	0	0	9.1	3	0.0	0	0.0
-1	0	2	4	2	5.8	-2	0.0	1	0.0
1	0	0	0	2	27.8	-2	0.0	1	0.0
-1	0	2	-2	1	-32.6	-2	0.0	1	0.0

0	-2	2	-2	1	6786.3	-2	0.0	1	0.0
-2	0	0	0	1	-13.7	-2	0.0	1	0.0
2	0	0	0	1	13.8	2	0.0	-1	0.0
3	0	0	0	0	9.2	2	0.0	0	0.0
1	1	2	0	2	8.9	2	0.0	-1	0.0
0	0	2	1	2	9.3	2	0.0	-1	0.0
1	0	0	2	1	9.6	-1	0.0	0	0.0
1	0	2	2	1	5.6	-1	0.0	1	0.0
1	1	0	-2	1	-34.7	-1	0.0	0	0.0
0	1	0	2	0	14.2	-1	0.0	0	0.0
0	1	2	-2	0	117.5	-1	0.0	0	0.0
0	1	-2	2	0	-329.8	-1	0.0	0	0.0
1	0	-2	2	0	23.8	-1	0.0	0	0.0
1	0	-2	-2	0	-9.5	-1	0.0	0	0.0
1	0	2	-2	0	32.8	-1	0.0	0	0.0
1	0	0	-4	0	-10.1	-1	0.0	0	0.0
2	0	0	-4	0	-15.9	-1	0.0	0	0.0
0	0	2	4	2	4.8	-1	0.0	0	0.0
0	0	2	-1	2	25.4	-1	0.0	0	0.0
-2	0	2	4	2	7.3	-1	0.0	1	0.0
2	0	2	2	2	4.7	-1	0.0	0	0.0
0	-1	2	0	1	14.2	-1	0.0	0	0.0
0	0	-2	0	1	-13.6	-1	0.0	0	0.0
0	0	4	-2	2	12.7	1	0.0	0	0.0
0	1	0	0	2	409.2	1	0.0	0	0.0
1	1	2	-2	2	22.5	1	0.0	-1	0.0
3	0	2	-2	2	8.7	1	0.0	0	0.0
-2	0	2	2	2	14.6	1	0.0	-1	0.0
-1	0	0	0	2	-27.3	1	0.0	-1	0.0
0	0	-2	2	1	-169.0	1	0.0	0	0.0
0	1	2	0	1	13.1	1	0.0	0	0.0
-1	0	4	0	2	9.1	1	0.0	0	0.0
2	1	0	-2	0	131.7	1	0.0	0	0.0
2	0	0	2	0	7.1	1	0.0	0	0.0
2	0	2	-2	1	12.8	1	0.0	-1	0.0
2	0	-2	0	1	-943.2	1	0.0	0	0.0
1	-1	0	-2	0	-29.3	1	0.0	0	0.0
-1	0	0	1	1	-388.3	1	0.0	0	0.0
-1	-1	0	2	1	35.0	1	0.0	0	0.0
0	1	0	1	0	27.3	1	0.0	0	0.0

$$\epsilon_0 = 23°26'21''.448$$
$$\sin \epsilon_0 = 0.39777716$$

The IERS 1996 Theory of Precession/Nutation

At the request of the IERS, T. Herring (1996) has analyzed the most recent VLBI and LLR data for geophysical parameters required to adjust the rigid Earth theory of nutation to the non-rigid theory. The non-rigid Earth theory coefficients resulting from this procedure are shown in Table 5.2. Table 5.3 contains the planetary nutation terms and the necessary planetary arguments. These coefficients are meant to be used for prediction purposes and by those requiring accurate *a priori* estimates of nutation. They are not meant to replace the IAU Theory. The IERS will continue to publish observed estimates of the corrections to the IAU 1976 Precession and the IAU 1980 Theory of Nutation in its publications.

The use of the IERS 1996 Theory of nutation must also be associated with the use of improved numerical values for the precession rate of the equator in longitude and obliquity:

$$\delta\psi_A = -0.299''/\mathrm{c} \text{ and } \delta\omega_A = -0.024''/\mathrm{c}$$

Table 5.2. IERS 1996 series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with t measured in Julian centuries from epoch J2000.0. The signs of the fundamental arguments, periods, and coefficients may differ from the original publication. These have been changed to be consistent with other portions of this chapter.

$$\Delta\psi = \sum_{i=1}^{263}(A_i + A_i' t)\sin(\text{ARGUMENT}) + A_i'' \cos(\text{ARGUMENT}),$$

$$\Delta\epsilon = \sum_{i=1}^{263}(B_i + B_i' t)\cos(\text{ARGUMENT}) + B_i'' \sin(\text{ARGUMENT}).$$

MULTIPLIERS OF					PERIOD	LONGITUDE (0.001mas)		OBLIQUITY (0.001mas)			
l	l'	F	D	Ω	(days)	A_i	A_i'	B_i	B_i'	A_i''	B_i''
0	0	0	0	1	-6798.35	-17206277	-17419	9205356	886	3645	1553
0	0	2	-2	2	182.62	-1317014	-156	573058	-306	-1400	-464
0	0	2	0	2	13.66	-227720	-23	97864	-48	269	136
0	0	0	0	2	-3399.18	207429	21	-89747	47	-71	-29
0	-1	0	0	0	-365.26	-147538	364	7388	-19	1121	198
0	1	2	-2	2	121.75	-51687	123	22440	-68	-54	-18
1	0	0	0	0	27.55	71118	7	-687	0	-94	39
0	0	2	0	1	13.63	-38752	-37	20076	2	34	32
1	0	2	0	2	9.13	-30137	-4	12896	-6	77	35
0	-1	2	-2	2	365.22	21583	-49	-9591	30	6	12
0	0	2	-2	1	177.84	12820	14	-6897	-1	18	4
-1	0	2	0	2	27.09	12353	1	-5334	3	2	0
-1	0	0	2	0	31.81	15699	1	-127	0	-18	9
1	0	0	0	1	27.67	6314	6	-3323	0	3	-1
-1	0	0	0	1	-27.44	-5797	-6	3141	0	-19	-8
-1	0	2	2	2	9.56	-5965	-1	2554	-1	14	7
1	0	2	0	1	9.12	-5163	-4	2635	0	12	8
-2	0	2	0	1	1305.48	4590	5	-2424	-1	1	1
0	0	0	2	0	14.77	6336	1	-125	0	-15	3
0	0	2	2	2	7.10	-3854	0	1643	0	15	6
-2	0	0	2	0	-205.89	-4774	0	48	0	-2	-3
2	0	2	0	2	6.86	-3102	0	1323	-1	12	5
1	0	2	-2	2	23.94	2863	0	-1235	1	0	0
-1	0	2	0	1	26.98	2044	2	-1076	0	1	0
2	0	0	0	0	13.78	2923	0	-62	0	-8	1
0	0	2	0	0	13.61	2585	0	-56	0	-7	1
0	1	0	0	1	386.00	-1406	-3	857	0	8	-4
-1	0	0	2	1	31.96	1517	1	-801	0	1	0
0	2	2	-2	2	91.31	-1578	7	685	-4	-2	-1
0	0	-2	2	0	-173.31	2178	0	-15	0	1	1
1	0	0	-2	1	-31.66	-1286	-1	694	0	-4	-2
0	-1	0	0	1	-346.64	-1269	1	642	1	6	2
-1	0	2	2	1	9.54	-1022	-1	522	0	2	1
0	-2	0	0	0	-182.63	-1671	8	14	0	-1	-1
1	0	2	2	2	5.64	-768	0	325	0	4	2
-2	0	2	0	0	1095.18	-1102	0	10	0	-1	0
0	1	2	0	2	13.17	757	-2	-326	-2	-1	0
0	0	2	2	1	7.09	-664	-1	335	-1	2	1
0	-1	2	0	2	14.19	-714	2	307	2	1	0
0	0	0	2	1	14.80	-631	-1	327	0	0	0
1	0	2	-2	1	23.86	580	1	-307	0	0	0
2	0	2	-2	2	12.81	643	0	-277	0	-1	0
-2	0	0	2	1	-199.84	-579	-1	304	0	-1	0
2	0	2	0	1	6.85	-533	0	269	0	2	1
0	-1	2	-2	1	346.60	-477	-1	271	-1	0	0
0	0	0	-2	1	-14.73	-493	-1	272	0	-2	-1
-1	-1	0	2	0	34.85	735	0	-5	0	-1	0
2	0	0	-2	1	212.32	405	0	-220	0	1	0
1	0	0	2	0	9.61	657	0	-20	0	-2	0
0	1	2	-2	1	119.61	361	0	-194	0	1	0

1	-1	0	0	0	29.80	471	0	-4	0	-1	0
-2	0	2	0	2	1615.76	-311	0	131	0	0	0
3	0	2	0	2	5.49	-289	0	124	0	2	1
0	-1	0	2	0	15.39	435	0	-9	0	-1	0
1	-1	2	0	2	9.37	-287	0	123	0	1	0
-1	-1	2	2	2	9.81	-282	0	122	0	1	0
0	0	0	1	0	29.53	-422	0	3	0	1	0
-1	0	2	0	0	26.88	-404	0	4	0	1	0
0	-1	2	2	2	7.24	-264	0	114	0	1	0
-2	0	0	0	1	-13.75	-228	0	126	0	-1	0
1	1	2	0	2	8.91	246	0	-106	0	-1	0
2	0	0	0	1	13.81	218	0	-114	0	0	0
-1	1	0	1	0	3232.87	327	0	-1	0	0	0
1	1	0	0	0	25.62	-338	0	4	0	0	0
1	0	2	0	0	9.11	334	0	-11	0	-1	0
-1	0	2	-2	1	-32.61	-199	0	107	0	-1	0
1	0	0	0	2	27.78	-197	0	85	0	0	0
-1	0	0	1	0	-411.78	405	0	-55	0	-35	-14
0	0	2	1	2	9.34	165	0	-72	0	0	0
-1	0	2	4	2	5.80	-151	0	66	0	1	0
0	-2	2	-2	1	6786.31	-130	0	69	0	0	0
-1	1	0	1	1	6164.17	132	0	-68	0	0	0
1	0	2	2	1	5.64	-133	0	66	0	1	0
-2	0	2	2	2	14.63	139	0	-60	0	0	0
-1	0	0	0	2	-27.33	139	0	-60	0	0	0
1	1	2	-2	2	22.47	128	0	-55	0	0	0
-2	0	2	4	2	7.35	-121	0	52	0	0	0
-1	0	4	0	2	9.06	115	0	-49	0	0	0
2	0	2	-2	1	12.79	101	0	-54	0	0	0
2	0	2	2	2	4.68	-108	0	47	0	1	0
1	0	0	2	1	9.63	-95	0	49	0	0	0
3	0	0	0	0	9.18	157	0	-5	0	-1	0
3	0	2	-2	2	8.75	94	0	-40	0	0	0
0	0	4	-2	2	12.66	91	0	-39	0	0	0
0	0	-2	2	1	-169.00	87	0	-44	0	0	0
0	1	2	0	1	13.14	81	0	-42	0	0	0
0	0	2	-2	3	187.66	123	0	-20	0	0	0
-1	0	0	4	0	10.08	133	0	-4	0	0	0
2	0	-2	0	1	-943.23	71	0	-38	0	0	0
2	0	0	-4	0	-15.91	-128	0	1	0	0	0
-1	-1	0	2	1	35.03	75	0	-39	0	0	0
-2	-1	0	2	0	-131.67	-115	0	1	0	0	0
0	-1	2	0	1	14.16	-66	0	35	0	0	0
-1	0	0	1	1	-388.27	101	0	-49	0	-3	-1
0	0	-2	0	1	-13.58	-68	0	36	0	0	0
0	1	0	0	2	409.23	69	0	-33	0	-1	0
0	0	2	-1	2	25.42	-74	0	31	0	0	0
0	0	2	4	2	4.79	-69	0	29	0	0	0
1	1	0	-2	1	-34.67	-61	0	32	0	0	0
-1	1	0	2	0	29.26	-94	0	0	0	0	0
1	-1	2	2	2	5.73	-59	0	25	0	0	0
1	-1	0	0	1	29.93	51	0	-27	0	0	0
0	1	-2	2	0	-329.79	-90	0	3	0	0	0
3	0	2	0	1	5.49	-50	0	25	0	0	0
-1	1	2	2	2	9.31	56	0	-24	0	0	0
0	1	2	2	2	6.96	54	0	-22	0	0	0
-1	0	0	-2	1	-9.60	-50	0	27	0	0	0
-1	1	0	1	2	66079.30	-52	0	23	0	0	0
0	-1	2	2	1	7.23	-44	0	24	0	0	0
1	0	2	-4	1	-38.74	-47	0	24	0	0	0
-1	0	-2	2	0	-23.77	77	0	0	0	0	0
-1	-1	2	2	1	9.80	-46	0	24	0	0	0
0	-1	0	0	2	-329.82	59	0	-25	0	0	0
2	-1	2	0	2	6.99	-48	0	21	0	0	0
1	-1	2	0	1	9.35	-42	0	22	0	0	0
0	0	0	2	2	14.83	-46	0	20	0	0	0
0	1	0	2	0	14.19	-67	0	0	0	0	0
-1	1	2	0	2	25.22	47	0	-20	0	0	0
0	3	2	-2	2	73.05	-44	0	19	0	0	0
0	-1	-2	2	0	-117.54	66	0	0	0	0	0

0	0	0	1	1	29.66	-37	0	20	0	0	0
1	0	-2	-2	0	-9.53	-64	0	1	0	0	0
1	1	2	0	1	8.90	36	0	-18	0	0	0
2	1	2	0	2	6.73	40	0	-17	0	0	0
0	1	0	1	0	27.32	57	0	0	0	0	0
1	0	-2	2	0	32.76	-58	0	0	0	0	0
1	1	0	0	1	25.72	-34	0	19	0	0	0
-2	0	0	-2	0	-7.13	-59	0	1	0	0	0
-1	0	0	2	2	32.11	-38	0	17	0	0	0
0	0	0	-1	1	-29.40	33	0	-18	0	0	0
0	1	0	-2	1	-15.35	-33	0	18	0	0	0
-1	0	2	-2	2	-32.45	36	0	-16	0	0	0
-1	1	0	0	1	-29.67	-31	0	17	0	0	0
1	0	2	1	2	6.98	33	0	-14	0	0	0
0	0	0	-4	0	-7.38	-48	0	1	0	0	0
0	0	2	1	1	9.33	27	0	-14	0	0	0
1	0	0	-2	2	-31.52	32	0	-14	0	0	0
1	0	2	-1	2	13.22	-33	0	13	0	0	0
1	-1	0	2	0	9.87	48	0	0	0	0	0
-1	0	2	4	1	5.80	-26	0	13	0	0	0
0	0	-2	-2	0	-7.08	-41	0	1	0	0	0
1	0	-2	0	1	-26.77	27	0	-14	0	0	0
-1	0	2	-1	1	313.04	-23	0	14	0	0	0
1	1	2	-2	1	22.40	23	0	-12	0	0	0
4	0	2	0	2	4.58	-26	0	11	0	0	0
0	1	2	1	2	9.11	-24	0	10	0	0	0
-2	0	-2	0	0	-6.85	-36	0	1	0	0	0
2	1	2	-2	2	12.38	25	0	-10	0	0	0
2	-1	0	0	0	14.32	38	0	0	0	0	0
-1	-1	0	0	1	-25.53	21	0	-12	0	0	0
-2	0	2	2	1	14.60	22	0	-11	0	0	0
0	0	0	0	3	-2266.12	-22	0	10	0	0	0
1	0	4	-2	2	8.68	23	0	-9	0	0	0
2	0	2	2	1	4.68	-19	0	10	0	0	0
-2	0	2	4	1	7.34	-20	0	10	0	0	0
0	1	0	2	1	14.22	18	0	-9	0	0	0
1	0	0	1	0	14.25	-33	0	0	0	0	0
-1	0	0	4	1	10.10	-18	0	9	0	0	0
-1	0	4	0	1	9.05	19	0	-9	0	0	0
0	0	2	-3	2	-35.23	-20	0	8	0	0	0
0	0	4	0	2	6.82	19	0	-8	0	0	0
2	1	0	0	0	13.28	-28	0	0	0	0	0
0	0	2	-4	1	-16.10	-16	0	9	0	0	0
-1	-1	2	4	2	5.90	-17	0	7	0	0	0
-1	-2	0	2	0	38.52	27	0	0	0	0	0
0	0	0	4	1	7.39	-16	0	7	0	0	0
0	-1	0	2	1	15.42	-14	0	7	0	0	0
1	0	2	4	2	4.08	-16	0	7	0	0	0
-2	0	0	2	2	-194.13	18	0	-8	0	0	0
-2	2	0	2	0	1616.44	-22	0	0	0	0	0
-2	-1	2	0	1	-507.16	9	0	-5	0	0	0
-3	0	0	0	1	-9.17	-14	0	7	0	0	0
0	0	2	0	3	13.69	20	0	0	0	0	0
0	0	2	4	1	4.79	-12	0	6	0	0	0
0	0	4	-2	1	12.64	12	0	-7	0	0	0
0	-2	0	2	0	16.06	21	0	0	0	0	0
1	0	0	-1	1	438.33	17	0	-5	0	-3	1
1	1	2	2	2	5.56	15	0	-6	0	0	0
3	0	2	-2	1	8.73	12	0	-7	0	0	0
-1	-1	2	0	2	29.26	-16	0	6	0	0	0
-2	-1	0	2	1	-129.17	-13	0	7	0	0	0
0	0	0	-2	2	-14.70	13	0	-5	0	0	0
0	-2	2	2	2	7.38	-13	0	5	0	0	0
1	0	0	-4	1	-10.07	-12	0	6	0	0	0
-1	1	0	2	1	29.39	-10	0	6	0	0	0
-2	0	0	4	1	15.94	11	0	-6	0	0	0
0	0	2	-1	1	25.33	-10	0	5	0	0	0
0	2	0	0	1	187.67	-9	0	5	0	0	0
0	2	2	-2	1	90.10	8	0	-5	0	0	0
2	0	0	2	1	7.13	-9	0	5	0	0	0

2	0	0	-4	1	-15.87	-11	0	5	0	0	0
2	0	2	-4	1	95.42	10	0	-5	0	0	0
-1	0	-2	0	1	-9.10	-10	0	5	0	0	0
-1	1	2	0	1	25.13	9	0	-5	0	0	0
-1	1	2	-2	1	-35.80	-11	0	5	0	0	0
-1	-1	0	4	0	10.37	15	0	0	0	0	0
-3	0	0	4	0	37.63	16	0	0	0	0	0
3	0	2	2	2	4.00	-14	0	0	0	0	0
2	-1	0	-2	0	471.89	-9	0	1	0	-1	0
0	2	-2	2	0	-3396.16	-9	0	0	0	0	0
0	-1	2	4	2	4.86	-9	0	0	0	0	0
0	-1	2	-1	2	27.32	9	0	0	0	0	0
1	1	0	2	0	9.37	-10	0	0	0	0	0
2	0	0	-2	2	219.17	-11	0	0	0	0	0
2	-1	2	2	2	4.74	-9	0	0	0	0	0
4	0	0	0	0	6.89	9	0	0	0	0	0
4	0	2	-2	2	6.64	12	0	0	0	0	0
-1	0	0	3	0	15.31	-10	0	0	0	0	0
-1	0	4	-2	2	23.43	-9	0	0	0	0	0
-1	-2	2	2	2	10.08	-9	0	0	0	0	0
-2	-1	0	4	0	16.63	12	0	0	0	0	0
-2	-1	2	4	2	7.50	-12	0	0	0	0	0
0	1	2	2	1	6.95	7	0	0	0	0	0
0	2	2	0	2	12.71	7	0	0	0	0	0
0	-2	2	0	2	14.77	-8	0	0	0	0	0
1	0	0	4	0	5.82	8	0	0	0	0	0
1	0	2	2	0	5.63	8	0	0	0	0	0
1	0	2	-4	2	-38.52	7	0	0	0	0	0
1	-1	2	2	1	5.73	-8	0	0	0	0	0
1	-1	2	-2	2	25.62	-7	0	0	0	0	0
1	-2	0	0	0	32.45	8	0	0	0	0	0
2	0	0	0	2	13.83	-8	0	0	0	0	0
2	1	0	-2	1	134.27	8	0	0	0	0	0
3	0	0	0	1	9.20	7	0	0	0	0	0
-1	0	2	1	2	14.13	8	0	0	0	0	0
-1	0	2	3	2	7.22	8	0	0	0	0	0
-1	0	-2	4	0	38.96	-7	0	0	0	0	0
-1	1	2	2	1	9.30	7	0	0	0	0	0
-1	2	0	2	0	27.09	-8	0	0	0	0	0
-1	-1	2	-1	1	2189.73	7	0	0	0	0	0
-2	0	2	-2	1	-14.93	-8	0	0	0	0	0
-2	0	4	0	2	13.49	-7	0	0	0	0	0
-2	0	-2	2	0	-12.76	8	0	0	0	0	0
-2	1	2	0	1	285.41	9	0	0	0	0	0
-3	0	2	0	1	-28.15	-8	0	0	0	0	0
0	1	0	1	1	27.43	5	0	0	0	0	0
0	-1	0	4	0	7.53	6	0	0	0	0	0
0	-1	0	-2	1	-14.16	5	0	0	0	0	0
0	-2	0	0	1	-177.85	-6	0	0	0	0	0
1	0	2	1	1	6.97	5	0	0	0	0	0
1	0	2	-3	2	126.51	-6	0	0	0	0	0
1	0	-2	1	0	-299.26	-7	0	0	0	0	0
1	1	0	1	0	13.72	5	0	0	0	0	0
1	-1	0	-2	1	-29.14	6	0	0	0	0	0
2	0	2	-1	2	8.93	-6	0	0	0	0	0
2	1	2	0	1	6.73	5	0	0	0	0	0
2	-1	2	0	1	6.98	-6	0	0	0	0	0
2	-1	2	-2	2	13.28	5	0	0	0	0	0
3	0	0	2	0	5.66	5	0	0	0	0	0
3	-1	2	0	2	5.58	-5	0	0	0	0	0
-1	-1	2	0	1	29.14	-6	0	0	0	0	0
-2	0	0	0	2	-13.72	6	0	0	0	0	0
-2	0	0	3	0	34.48	-5	0	0	0	0	0
-2	0	0	-2	1	-7.12	-5	0	0	0	0	0
-2	0	2	2	0	14.57	-6	0	0	0	0	0
-2	-1	2	0	0	-548.04	-5	0	0	0	0	0
-2	-1	2	2	2	15.24	6	0	0	0	0	0
0	0	1	0	0	27.21	0	0	0	0	8	0
0	0	1	0	1	27.32	0	0	0	0	-16	-14
-1	0	1	0	0	2190.35	0	0	0	0	33	0

-1	0	1	0	1	3231.51	0	0	0	0	-105	-89
-1	0	1	0	2	6159.22	0	0	0	0	36	18
-1	0	1	0	3	65514.10	0	0	0	0	-6	0

Table 5.3. IERS 1996 Series for planetary nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with t measured in Julian centuries from epoch J2000.0. The signs of the fundamental arguments, periods, and coefficients may differ from the original publication. These have been changed to be consistent with other portions of this chapter.

$$\Delta\psi = \sum_{i=1}^{118}(A_i + A_i^{\prime}t)\sin(\text{ARGUMENT}), \quad \Delta\epsilon = \sum_{i=1}^{118}(B_i + B_i^{\prime}t)\cos(\text{ARGUMENT}).$$

MULTIPLIERS OF										PERIOD	LONGITUDE (0.001mas)		OBLIQUITY (0.001mas)	
l_{Ve}	l_E	l_{Ma}	l_J	l_{Sa}	p_a	D	F	l	Ω	(days)	A_i	$A_i^{\prime}$	B_i	$B_i^{\prime}$
0	2	0	-2	0	0	2	0	-2	1	100171.17	28	0	0	-15
18	-16	0	0	0	0	0	0	-1	0	99728.92	23	10	0	0
8	-12	0	0	0	0	1	-1	0	-1	88074.16	-120	-60	32	-64
0	0	2	0	0	0	1	-1	0	0	38036.19	27	-8	0	0
0	1	-2	0	0	0	0	0	0	-1	-37884.54	46	43	-23	25
3	-4	0	0	0	0	1	0	-1	0	-34988.76	0	13	0	0
5	-6	0	0	0	0	2	-2	0	0	18185.76	0	23	0	0
6	-8	0	0	0	0	2	0	-2	0	-17494.38	8	-2	0	0
0	8	-15	0	0	0	0	0	0	0	14765.98	5	-2	0	0
0	-2	0	3	0	0	-2	0	2	1	-13630.86	9	-2	0	-5
0	2	0	-3	0	0	2	0	-2	0	-13562.72	-35	-6	0	0
0	1	0	-1	0	0	1	0	-1	0	12732.58	-5	0	0	0
0	1	0	1	0	0	1	-1	0	0	11960.41	-2	-8	0	0
0	0	0	1	0	0	0	0	0	1	11945.37	2	-7	0	0
0	1	0	0	1	0	1	-1	0	-1	10771.42	-17	-8	4	-9
0	0	0	0	1	0	0	0	0	0	10759.23	1	6	0	0
0	0	0	0	1	1	0	0	0	0	10746.94	5	0	0	0
0	-1	0	0	1	0	-1	1	0	1	10747.06	-7	-1	0	0
8	-13	0	0	0	0	0	0	0	1	-7372.72	5	7	0	0
18	-16	0	0	0	0	0	0	-1	1	-7295.69	-7	-3	0	0
0	0	0	-2	5	0	0	0	0	1	-6944.70	8	2	0	0
0	4	-8	3	0	0	0	0	0	-1	6870.05	-8	-30	16	-4
0	4	-8	3	0	0	0	0	0	1	-6728.13	-8	29	16	4
0	0	0	2	-5	0	0	0	0	1	-6658.05	-7	2	0	0
0	2	0	-2	0	0	2	0	-2	0	6366.29	-44	0	0	0
-18	16	0	0	0	0	0	0	1	1	-6364.50	6	-3	0	0
-8	13	0	0	0	0	0	0	0	1	-6307.01	-4	6	0	0
0	0	2	0	0	0	1	-1	0	-1	5767.51	-27	-8	4	-15
0	1	-2	0	0	0	0	0	0	0	-5764.01	-46	44	0	0
0	-2	2	0	0	0	-1	1	0	1	5760.51	0	-5	0	0
0	0	0	0	-2	-1	0	0	0	0	-5376.54	-5	-10	6	-3
0	-1	0	0	2	0	-1	1	0	1	5376.57	-5	-11	-6	3
0	0	0	0	2	2	0	0	0	0	5373.47	-12	0	0	5
5	-6	0	0	0	0	2	-2	0	-1	4948.47	2	44	-23	0
0	2	0	-3	0	0	2	0	-2	1	-4528.45	-5	0	0	0
0	-1	0	-1	0	0	-1	1	0	1	-4334.57	-5	19	10	3
0	0	0	1	0	-1	0	0	0	0	4334.58	2	6	0	0
0	0	0	1	0	0	0	0	0	0	4332.59	-8	25	0	0
0	0	0	1	0	1	0	0	0	0	4330.60	0	5	0	0
0	-1	0	1	0	0	-1	1	0	1	4330.61	0	-5	0	0
3	-3	0	0	0	0	2	0	-2	0	3561.67	-14	0	0	0
0	-2	0	2	0	0	-2	0	2	1	-3287.62	5	0	0	0
3	-5	0	0	0	0	0	0	0	0	-2959.21	-22	7	0	0
3	-5	0	0	0	-1	0	0	0	0	-2958.28	-1	-7	3	-1
3	-5	0	0	0	-2	0	0	0	0	-2957.35	211	0	0	96
0	2	-4	0	0	0	0	0	0	0	-2882.00	-8	14	0	0
0	2	-4	0	0	-2	0	0	0	0	-2880.24	5	0	0	0
5	-8	0	0	0	-2	0	0	0	0	2863.89	0	-26	13	0
5	-7	0	0	0	0	1	-1	0	-1	2863.01	-14	-3	1	-7

0	0	0	-2	0	-1	0	0	0	0	-2165.80	-4	-27	12	-2
0	1	0	-2	0	0	1	-1	0	-1	-2165.80	3	14	-8	1
0	0	0	2	0	2	0	0	0	0	2165.30	-116	0	0	51
-3	3	0	0	0	0	-2	2	0	1	2060.87	12	0	0	-6
0	-2	0	2	0	0	-2	2	0	1	1642.24	5	0	0	0
2	-3	0	0	0	0	0	0	0	0	1454.94	0	63	0	0
0	0	0	3	0	2	0	0	0	0	1443.75	-12	0	0	5
1	-2	0	0	0	0	0	0	0	0	-975.38	0	-9	0	0
0	2	-3	0	0	0	0	0	0	0	901.99	-8	5	0	0
0	1	-1	0	0	0	0	0	0	0	779.94	-6	0	0	0
4	-7	0	0	0	-2	0	0	0	0	-733.47	0	6	0	0
4	-6	0	0	0	-2	0	0	0	0	727.58	-52	0	0	-23
4	-6	0	0	0	-1	0	0	0	0	727.52	0	9	-5	1
1	-1	0	0	0	0	0	0	0	0	583.92	153	0	0	0
1	-1	0	0	0	1	0	0	0	0	583.89	0	-6	-5	-1
0	1	0	-3	0	-2	0	0	0	0	488.96	-11	0	0	-5
2	-4	0	0	0	-1	0	0	0	0	-487.66	0	-8	0	0
2	-4	0	0	0	-2	0	0	0	0	-487.64	47	0	0	20
0	1	0	-2	0	0	0	0	0	0	439.33	-18	27	0	0
0	3	-4	0	0	0	0	0	0	0	418.27	-8	5	0	0
3	-4	0	0	0	0	0	0	0	0	416.69	0	29	0	0
0	1	0	-1	0	0	0	0	0	0	398.88	-123	-3	0	0
0	2	-2	0	0	0	0	0	0	0	389.97	-38	0	0	0
0	1	0	0	-1	0	0	0	0	0	378.09	-8	0	0	0
0	0	2	0	0	2	0	0	0	0	343.46	-7	0	0	0
0	1	0	1	0	2	0	0	0	0	336.83	-25	0	0	11
3	-6	0	0	0	-2	0	0	0	0	-325.10	0	-5	0	0
5	-7	0	0	0	-2	0	0	0	0	323.94	-21	0	0	-9
0	1	0	2	0	2	0	0	0	0	312.54	-3	-5	0	0
2	-2	0	0	0	-1	0	0	0	0	291.97	0	5	0	0
2	-2	0	0	0	0	0	0	0	0	291.96	-60	0	0	0
1	-3	0	0	0	-2	0	0	0	0	-265.73	-11	0	0	-5
1	-3	0	0	0	-1	0	0	0	0	-265.73	0	-5	0	0
0	2	0	-3	0	0	0	0	0	0	209.07	8	2	0	0
2	-5	0	0	0	-2	0	0	0	0	-208.83	0	-13	6	0
6	-8	0	0	0	-2	0	0	0	0	208.35	-12	0	0	-5
0	2	0	-2	0	0	0	0	0	0	199.44	39	0	0	0
3	-3	0	0	0	0	0	0	0	0	194.64	10	0	0	0
3	-3	0	0	0	2	0	0	0	0	194.63	5	-2	0	0
0	-2	0	1	0	-2	0	0	0	0	-190.66	15	3	-1	7
0	3	-2	0	0	2	0	0	0	0	188.60	8	-7	0	0
8	-15	0	0	0	-2	0	0	0	0	-183.00	-6	-10	4	-3
0	6	-8	3	0	2	0	0	0	0	182.67	12	-42	-18	-5
0	2	0	0	0	2	0	0	0	0	182.62	-9	0	0	0
0	-2	8	-3	0	2	0	0	0	0	182.57	-12	42	18	5
-8	11	0	0	0	-2	0	0	0	0	-182.24	6	10	-4	13
0	1	2	0	0	2	0	0	0	0	177.01	-8	-7	0	0
0	2	0	1	0	2	0	0	0	0	175.23	17	-1	0	-7
3	-7	0	0	0	-2	0	0	0	0	-172.01	7	-2	0	0
2	-1	0	0	0	2	0	0	0	0	162.26	0	-17	-8	0
7	-9	0	0	0	-2	0	0	0	0	153.56	-7	0	0	0
4	-4	0	0	0	0	0	0	0	0	145.98	11	0	0	0
1	1	0	0	0	2	0	0	0	0	139.11	-30	0	0	13
0	3	0	-2	0	2	0	0	0	0	129.00	7	-9	-4	-3
3	-2	0	0	0	2	0	0	0	0	126.97	0	-11	-5	0
0	3	0	-1	0	2	0	0	0	0	125.27	52	2	0	-22
0	4	-2	0	0	2	0	0	0	0	124.38	14	0	0	-6
8	-10	0	0	0	-2	0	0	0	0	121.59	-5	0	0	0
5	-5	0	0	0	0	0	0	0	0	116.78	7	0	0	0
2	0	0	0	0	2	0	0	0	0	112.35	39	0	0	-17
0	4	0	-2	0	2	0	0	0	0	95.33	-18	0	0	8
-18	16	0	0	0	0	0	-2	1	-2	-13.66	13	6	-3	5
-18	16	0	0	0	0	0	2	1	2	13.66	13	-6	-3	-5

$$l_{Ve} = 181^{\circ}\!.979800853 + 58517^{\circ}\!.8156748 \times t$$

$$l_E = 100^{\circ}\!.466448494 + 35999^{\circ}\!.3728521 \times t$$

$$l_{Ma} = 355^\circ\!.433274605 + 19140^\circ\!.299314 \times t$$

$$l_{Ju} = 34^\circ\!.351483900 + 3034^\circ\!.90567464 \times t$$

$$l_{Sa} = 50^\circ\!.0774713998 + 1222^\circ\!.11379404 \times t$$

$$p_a = 1^\circ\!.39697137214 \times t + 0^\circ\!.0003086 \times t^2$$

The multipliers of the fundamental arguments of nutation theory

Each of the lunisolar terms in the nutation series is characterized by a set of five integers N_j which determines the ARGUMENT for the term as a linear combination of the five Fundamental Arguments F_j, namely the Delaunay variables $(\ell, \ell', F, D, \Omega)$: ARGUMENT $= \sum_{j=1}^{5} N_j F_j \equiv \vec{N} \cdot \vec{F}$, where $\vec{N}$ is the five-vector composed of the values $(N_1, \cdots, N_5)$ which characterize the term, and $\vec{F}$ is the five-vector $(F_1, \cdots, F_5)$. The F_j are functions of time, and the angular frequency of the nutation described by the term is given by

$$\omega \equiv \frac{d(\text{ARGUMENT})}{dt}.$$

The frequency thus defined is positive for most terms, and negative for some. Planetary nutation terms differ from the above only in that ARGUMENT $= \sum_{j=1}^{10} N_j' F_j'$ as noted in Table 5.3.

Over time scales involved in nutation studies, the frequency ω is effectively time-independent, and one may write, for the sth term in the nutation series,

$$\text{ARGUMENT} = \omega_s t + \alpha_s.$$

Different tables of nutations in longitude and obliquity do not necessarily assign the same set of multipliers N_j to a particular term in the nutation series. Compare, for instance, the 31.8 day term in Table 5.1 wherein $\vec{N} = (-1, 0, 0, 2, 0)$ with the same term in Seidelmann (1982) where the assignment is $\vec{N} = (1, 0, 0, -2, 0)$. The differences in the assignments arises from the fact that the replacement $\vec{N}_j \rightarrow -\vec{N}_j$ accompanied by reversal of the sign of the coefficient of sin(ARGUMENT$_j$) in the series for $\Delta\psi$ and $\Delta\epsilon$ leaves these series unchanged. This freedom has led to some confusion in the assignment of multipliers of the Delaunay arguments for amplitudes of retrograde and prograde circular nutations.

The nutations in longitude and obliquity at a given frequency ω_s may be viewed as the superposition of a pair of prograde and retrograde circular nutations whose frequencies have the same absolute value, $|\omega_s|$, but with opposite signs. These circular nutation terms may be written as

$$A_s^{pro} e^{-iq_s(\omega_s t + \alpha_s)} \quad \text{and} \quad A_s^{ret} e^{iq_s(\omega_s t + \alpha_s)},$$

where q_s is the sign of ω_s:

$$q_s \omega_s = |\omega_s|.$$

With the convention that the exponential factor involving the frequency ν is to be written as $e^{-i\nu t}$, one sees from the above expressions that the frequency of the prograde nutation, $\omega_s^{pro} = (q_s \omega_s) \equiv |\omega_s|$, is positive, and that of the retrograde nutation is negative ($\omega_s^{ret} = -|\omega_s|$).

Now, given the vector $\vec{N}$ of multipliers of the fundamental arguments for the nutations in longitude and obliquity of frequency ω_s, one can choose multipliers $\vec{N}^{(pro)}$ and $\vec{N}^{(ret)}$ for the corresponding prograde and retrograde nutations in such a manner as to ensure that

$$\omega_s^{pro} \equiv \vec{N}^{pro} \cdot \frac{d\vec{F}}{dt} = |\omega_s|, \quad \omega_s^{ret} \equiv \vec{N}^{ret} \cdot \frac{d\vec{F}}{dt} = -|\omega_s|.$$

This is accomplished by making the assignments

$$\vec{N}^{pro} = q_s \vec{N}; \quad \vec{N}^{ret} = -q_s \vec{N},$$

a practice that will be adhered to in this work. Note that $\vec{N}^{(pro)}$ and $\vec{N}^{(ret)}$ are unaffected by whether $\vec{N}$ or $-\vec{N}$ is used to label the nutations in longitude and obliquity, because on switching from one to the other, the sign of ω_s also is switched simultaneously.

Conversion to Prograde and Retrograde Nutation Amplitudes

With the convention for the time dependence of the circular nutation amplitudes as stated above, the amplitudes of prograde and retrograde circular nutations are written as

$$A_s^{pro} = A_s^{pro\ ip} - iA_s^{pro\ op},$$

$$A_s^{ret} = A_s^{ret\ ip} - iA_s^{ret\ op},$$

where $A_s^{pro\ ip}$ and $A_s^{pro\ op}$ are the in-phase and out-of-phase parts of the prograde nutation with angular frequency $|\omega_s|$ as defined in the foregoing section, and $A_s^{ret\ ip}$ and $A_s^{ret\ op}$ are the in-phase and out-of-phase parts of the retrograde nutation with angular frequency $(-|\omega_s|)$. These amplitudes are related to the coefficients in longitude and obliquity through the following equations (see Defraigne *et al.*, 1995):

$$A_s^{pro\ ip} = -\frac{1}{2}\left(\Delta\epsilon_s^{ip} - q_s \Delta\psi_s^{ip} \sin\epsilon_0\right),$$

$$A_s^{ret\ ip} = -\frac{1}{2}\left(\Delta\epsilon_s^{ip} + q_s \Delta\psi_s^{ip} \sin\epsilon_0\right),$$

$$A_s^{pro\ op} = \frac{1}{2}\left(q_s \Delta\epsilon_s^{op} + \Delta\psi_s^{op} \sin\epsilon_0\right),$$

$$A_s^{ret\ op} = -\frac{1}{2}\left(q_s \Delta\epsilon_s^{op} - \Delta\psi_s^{op} \sin\epsilon_0\right).$$

$\Delta\epsilon_s^{ip}$ and $\Delta\epsilon_s$ are the in-phase (cosine) and out-of-phase (sine) parts of the nutation in obliquity for a nutation of frequency ω_s; $\Delta\psi_s^{ip}$ and $\Delta\psi_s^{op}$ are the in-phase (sine) and out-of-phase (cosine) parts of the nutation in longitude for the same nutation.

The relations stated above are valid when the convention regarding the time dependence as stated in the previous section is followed, *i.e.*, when the time dependence of the prograde amplitude is $e^{-i|\omega_s|t}$ and that of the retrograde amplitude is $e^{i|\omega_s|t}$. However the opposite convention is often used in the literature. The amplitudes to be employed in such a case are the complex conjugates of those given above, *i.e.*, $(A_s^{pro\ ip} + iA_s^{pro\ op})$ goes together with $e^{i|\omega_s|t}$ for the prograde nutation and $(A_s^{ret\ ip} + iA_s^{ret\ op})$ with $e^{-i|\omega_s|t}$ for the retrograde nutation. Since the real parts are to be eventually taken, switching between mutually complex conjugates versions does not affect the results.

Coordinate Transformation Referred to the Nonrotating Origin

Option 2 uses form (2) of the coordinate transformation from the TRS to the CRS

$$[\mathrm{CRS}] = PN''(t)R''(t)W''(t)[\mathrm{TRS}], \tag{2}$$

where the three fundamental components of (2) are given below (Capitaine, 1990)

$$W''(t) = R_3(-s') \cdot R_1(y_p) \cdot R_2(x_p),$$

x_p and y_p being the "polar coordinates" of the CEP in the TRS and s' the accumulated displacement of the terrestrial origin on the true equator due to polar motion. The use of the quantity s' (which is neglected in the classical form (1)) provides an exact realization of the "instantaneous prime meridian."

$$R''(t) = R_3(-\theta),$$

θ being the stellar angle at date t due to the Earth's angle of rotation,

$$PN''(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E) \cdot R_3(S),$$

E and d being such that the coordinates of the CEP in the CRS are $X = \sin d \cos E$, $Y = \sin d \sin E$, $Z = \cos d$ and S being the accumulated rotation (between the epoch and the date t) of the celestial CEO on the true equator due to the celestial motion of the CEP. $PN''(t)$ can be given in an equivalent form involving directly X and Y (to which all the observations of a celestial object from the Earth are actually sensitive) as:

$$PN''(t) = Q_1 = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \cdot R_3(s),$$

with $a = 1/(1 + \cos d)$, which can also be written, with sufficient accuracy as $a = 1/2 + 1/8(X^2 + Y^2)$.

The standard values of the parameters to be used in the form (2) of the transformation are detailed below.

The standard pole coordinates to be used for the parameters x_p and y_p (if not estimated from the observations) are those published by the IERS. The quantity s' (of the order of 0.1 mas/cy) is:

$$s' = 0.0015(a_c^2/1.2 + a_a^2)t,$$

a_c and a_a being the average amplitudes (in arc seconds) of the Chandlerian and annual wobbles, respectively in the period considered (Capitaine *et al.*, 1986).

The stellar angle is obtained by the use of the conventional relationship between the stellar angle θ, the hour angle of the CEO and UT1 as given by Capitaine *et al.*, (1986),

$$\theta(T_u) = 2\pi(0.779057273264 + 1.00273781191135448 T_u \times 36525),$$

where T_u = (Julian UT1 date - 2451545.0)/36525, and

UT1 = UTC + (UT1-UTC), or equivalently

$$\theta(T_u) = 2\pi(\text{UT1 Julian day number elapsed since } 2451545.0 + 0.779057273264$$

$$+0.00273781191135448 T_u \times 36525),$$

the quantity UT1-UTC to be used (if not estimated from the observations) being the IERS value.

The celestial coordinates X and Y of the CEP to be used for consistency with the IAU 1980 theory of nutation are the standard values as derived from the series given in Table 5.4. These developments of the celestial polar coordinates have been derived (Capitaine, 1990) from the previous standard expressions for precession and nutation with a consistency of 5×10^{-5} sec. of arc after a century; such consistency has been numerically checked over two centuries (Gontier, 1990). For observations requiring values of the nutation angles with a milliarcsecond accuracy, it is necessary to add (if those quantities are not estimated from the observations) the IERS published values (observed or predicted) for the "celestial pole offsets" (*i.e.* corrections $dX = \Delta\psi \sin\epsilon_0$ and $dY = \Delta\epsilon$).

For consistency with the IERS 1995 series for nutation as given by Tables 5.2 and 5.3, it is necessary to use the following expressions for X and Y (Capitaine, 1990):

$$X = \sin\omega \sin\psi,$$

$$Y = -\sin\epsilon_0 \cos\omega + \cos\epsilon_0 \sin\omega \cos\psi,$$

where ϵ_0 is the obliquity of the ecliptic at J2000, ω is the inclination of the true equator of date on the fixed ecliptic of epoch and ψ is the longitude, on the ecliptic of epoch of the node of the true equator of date on the fixed ecliptic of epoch; these quantities are such that:

$$\omega = \omega_A + \Delta\epsilon_1, \psi = \psi_A + \Delta\psi_1,$$

where ψ_A and ω_A are the precession quantities in longitude and obliquity (Lieske *et al.* 1977) referred to the ecliptic of epoch and $\Delta\psi_1$, $\Delta\epsilon_1$ are the nutation angles in longitude and obliquity referred to the ecliptic of epoch. $\Delta\psi_1$, $\Delta\epsilon_1$ can be obtained with an accuracy better than one micorarcsecond after one century from the nutation angles $\Delta\psi$, $\Delta\epsilon$ in longitude and obliquity referred to the ecliptic of date, following Aoki and Kinoshita (1983) by:

$$\Delta\psi_1 = \frac{(\Delta\psi \sin\epsilon_A \cos\chi_A - \Delta\epsilon \sin\chi_A)}{\sin\omega_A},$$

$$\Delta\epsilon_1 = \Delta\psi \sin\epsilon_A \sin\chi_A + \Delta\epsilon \cos\chi_A,$$

ω_A and ϵ_A being the precession quantities in obliquity referred to ecliptic of epoch and ecliptic of date respectively and χ_A the precession quantity for planetary precession along the equator (Lieske *et al.* 1977).

The standard value of s compatible with the IAU 1980 Theory of Nutation and the Lieske *et al.* (1977) precession can be derived with an accuracy of $5 \times 10^{-5\prime\prime}$ after a century (Capitaine, 1990) from the following numerical development and the numerical values of X and Y (Table 5.3),

$$s = -XY/2 + 0.''00385t - 0.''07259t^3 - 0.''00264 \sin\Omega - 0.''00006 \sin 2\Omega$$

$$+0.''00074t^2 \sin\Omega + 0.''00006t^2 \sin 2(F - D + \Omega)$$

Table 5.4 Series for the celestial coordinates X and Y of the CEP referred to the mean equator and equinox of epoch J2000.0, with t measured in Julian centuries from epoch J2000.0. The terms following the dotted line are identical in Tables 5.1 and 5.4. The signs of the fundamental arguments, periods, and coefficients may differ from the original publication. These have been changed to be consistent with other portions of this chapter.

$$
\begin{aligned}
X =\ & 2004''.3109t - 0''.42665t^2 - 0''.198656t^3 + 0''.0000140t^4 \\
& +0''.00006t^2\cos\Omega + \sin\epsilon_0\{\textstyle\sum_{i=1,106}[(A_i + A_i't)\sin(\mathrm{ARGUMENT}) + A_i''t\cos(\mathrm{ARGUMENT})]\} \\
& +0''.00204t^2\sin\Omega + 0''.00016t^2\sin 2(F - D + \Omega),
\end{aligned}
$$

$$
\begin{aligned}
Y =\ & -0''.00013 - 22''.40992t^2 + 0''.001836t^3 + 0''.0011130t^4 \\
& + \textstyle\sum_{i=1,106}[(B_i + B_i't)\cos(\mathrm{ARGUMENT}) + B_i''t\sin(\mathrm{ARGUMENT})] \\
& -0''.00231t^2\cos\Omega - 0''.00014t^2\cos 2(F - D + \Omega)
\end{aligned}
$$

MULTIPLIERS OF					PERIOD	LONGITUDE (0″.0001)			OBLIQUITY (0″.0001)		
l	l'	F	D	Ω	(days)	A_i	A_i'	A_i''	B_i	B_i'	B_i''
0	0	0	0	1	-6798.4	-171996	-84.2	5173.2	92025	8.9	1529.9
0	0	2	-2	2	182.6	-13187	5.3	322.2	5736	-3.1	117.3
0	0	2	0	2	13.7	-2274	1.0	54.8	977	-0.5	20.2
0	0	0	0	2	-3399.2	2053.2	-1.0	-50.5	-893.7	0.5	-18.3
0	-1	0	0	0	-365.3	-1426	4.3	3.0	54	-0.1	-12.7
1	0	0	0	0	27.6	712	0.1	0.0	-7	0.0	-6.3
0	1	2	-2	2	121.7	-517	1.5	12.6	224	-0.6	4.6
0	0	2	0	1	13.6	-386	-0.4	11.3	200	0.0	3.4
1	0	2	0	2	9.1	-301	0.0	7.3	129	-0.1	2.7
0	-1	2	-2	2	365.2	217	-0.5	-5.3	-95	0.3	-1.9
-1	0	0	2	0	31.8	158	0.0	0.0	-1	0.0	1.4
0	0	2	-2	1	177.8	129	0.1	-4.0	-70	0.0	-1.2
-1	0	2	0	2	27.1	123	0.0	-3.0	-53	0.0	-1.1
1	0	0	0	1	27.7	63	0.1	-1.8	-33	0.0	-0.6
0	0	0	2	0	14.8	63	0.0	0.0	-2	0.0	-0.6
-1	0	2	2	2	9.6	-59	0.0	1.5	26	0.0	0.5
-1	0	0	0	1	-27.4	-58	-0.1	1.8	32	0.0	0.5
1	0	2	0	1	9.1	-51	0.0	1.5	27	0.0	0.5
-2	0	0	2	0	-205.9	-48	0.0	0.0	1	0.0	0.0
-2	0	2	0	1	1305.5	46	0.0	-1.3	-24	0.0	0.0
....											
0	0	2	2	2	7.1	-38	0.0		16	0.0	
2	0	2	0	2	6.9	-31	0.0		13	0.0	
2	0	0	0	0	13.8	29	0.0		-1	0.0	
1	0	2	-2	2	23.9	29	0.0		-12	0.0	
0	0	2	0	0	13.6	26	0.0		-1	0.0	
0	0	2	-2	0	173.3	-22	0.0		0	0.0	
-1	0	2	0	1	27.0	21	0.0		-10	0.0	
0	2	0	0	0	182.6	17	-0.1		0	0.0	
0	2	2	-2	2	91.3	-16	0.1		7	0.0	
1	0	0	2	1	32.0	16	0.0		-8	0.0	
0	1	0	0	1	386.0	-15	0.0		9	0.0	
1	0	0	-2	1	-31.7	-13	0.0		7	0.0	
0	-1	0	0	1	-346.6	-12	0.0		6	0.0	
2	0	-2	0	0	-1095.2	11	0.0		0	0.0	
-1	0	2	2	1	9.5	-10	0.0		5	0.0	
1	0	2	2	2	5.6	-8	0.0		3	0.0	
0	-1	2	0	2	14.2	-7	0.0		3	0.0	
0	0	2	2	1	7.1	-7	0.0		3	0.0	
1	1	0	-2	0	-34.8	-7	0.0		0	0.0	
0	1	2	0	2	13.2	7	0.0		-3	0.0	
-2	0	0	2	1	-199.8	-6	0.0		3	0.0	
0	0	0	2	1	14.8	-6	0.0		3	0.0	
2	0	2	-2	2	12.8	6	0.0		-3	0.0	
1	0	0	2	0	9.6	6	0.0		0	0.0	
1	0	2	-2	1	23.9	6	0.0		-3	0.0	
0	0	0	-2	1	-14.7	-5	0.0		3	0.0	
0	-1	2	-2	1	346.6	-5	0.0		3	0.0	

2	0	2	0	1	6.9	-5	0.0	3	0.0
1	-1	0	0	0	29.8	5	0.0	0	0.0
1	0	0	-1	0	411.8	-4	0.0	0	0.0
0	0	0	1	0	29.5	-4	0.0	0	0.0
0	1	0	-2	0	-15.4	-4	0.0	0	0.0
1	0	-2	0	0	-26.9	4	0.0	0	0.0
2	0	0	-2	1	212.3	4	0.0	-2	0.0
0	1	2	-2	1	119.6	4	0.0	-2	0.0
1	1	0	0	0	25.6	-3	0.0	0	0.0
1	-1	0	-1	0	-3232.9	-3	0.0	0	0.0
-1	-1	2	2	2	9.8	-3	0.0	1	0.0
0	-1	2	2	2	7.2	-3	0.0	1	0.0
1	-1	2	0	2	9.4	-3	0.0	1	0.0
3	0	2	0	2	5.5	-3	0.0	1	0.0
-2	0	2	0	2	1615.7	-3	0.0	1	0.0
1	0	2	0	0	9.1	3	0.0	0	0.0
-1	0	2	4	2	5.8	-2	0.0	1	0.0
1	0	0	0	2	27.8	-2	0.0	1	0.0
-1	0	2	-2	1	-32.6	-2	0.0	1	0.0
0	-2	2	-2	1	6786.3	-2	0.0	1	0.0
-2	0	0	0	1	-13.7	-2	0.0	1	0.0
2	0	0	0	1	13.8	2	0.0	-1	0.0
3	0	0	0	0	9.2	2	0.0	0	0.0
1	1	2	0	2	8.9	2	0.0	-1	0.0
0	0	2	1	2	9.3	2	0.0	-1	0.0
1	0	0	2	1	9.6	-1	0.0	0	0.0
1	0	2	2	1	5.6	-1	0.0	1	0.0
1	1	0	-2	1	-34.7	-1	0.0	0	0.0
0	1	0	2	0	14.2	-1	0.0	0	0.0
0	1	2	-2	0	117.5	-1	0.0	0	0.0
0	1	-2	2	0	-329.8	-1	0.0	0	0.0
1	0	-2	2	0	32.8	-1	0.0	0	0.0
1	0	-2	-2	0	-9.5	-1	0.0	0	0.0
1	0	2	-2	0	32.8	-1	0.0	0	0.0
1	0	0	-4	0	-10.1	-1	0.0	0	0.0
2	0	0	-4	0	-15.9	-1	0.0	0	0.0
0	0	2	4	2	4.8	-1	0.0	0	0.0
0	0	2	-1	2	25.4	-1	0.0	0	0.0
-2	0	2	4	2	7.3	-1	0.0	1	0.0
2	0	2	2	2	4.7	-1	0.0	0	0.0
0	-1	2	0	1	14.2	-1	0.0	0	0.0
0	0	-2	0	1	-13.6	-1	0.0	0	0.0
0	0	4	-2	2	12.7	1	0.0	0	0.0
0	1	0	0	2	409.2	1	0.0	0	0.0
1	1	2	-2	2	22.5	1	0.0	-1	0.0
3	0	2	-2	2	8.7	1	0.0	0	0.0
-2	0	2	2	2	14.6	1	0.0	-1	0.0
-1	0	0	0	2	-27.3	1	0.0	-1	0.0
0	0	-2	2	1	-169.0	1	0.0	0	0.0
0	1	2	0	1	13.1	1	0.0	0	0.0
-1	0	4	0	2	9.1	1	0.0	0	0.0
2	1	0	-2	0	131.7	1	0.0	0	0.0
2	0	0	2	0	7.1	1	0.0	0	0.0
2	0	2	-2	1	12.8	1	0.0	-1	0.0
2	0	-2	0	1	-943.2	1	0.0	0	0.0
1	-1	0	-2	0	-29.3	1	0.0	0	0.0
-1	0	0	1	1	-388.3	1	0.0	0	0.0
-1	-1	0	2	1	35.0	1	0.0	0	0.0
0	1	0	1	0	27.3	1	0.0	0	0.0
0	0	2	-2	3	177.8	-1.2	0.0	0	0.0

Geodesic Nutation

Fukushima (1991) has pointed out that, if extreme precision is required, the effect of geodesic nutation must be taken into account. For Option (1) this would require a correction in longitude of

$$\Delta\psi_g = -0''.000153 \sin l' - 0''.000002 \sin 2l',$$

where l' is the mean anomaly of the Sun. For Option (2) it would require a correction to X of

$$\Delta X_g = (-0''.0000609 \sin l' - 0''.0000008 \sin 2l') \sin \epsilon_0 .$$

In both cases, the correction would be added to the uncorrected determination of ψ or X.

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