CHAPTER 6 GEOPOTENTIAL

The recommended geopotential field is the JGM-3 model (Tapley *et al.*, 1995). The $GM_\oplus$ and a_e values reported with JGM-3 ($398600.4415\,\mathrm{km}^3/\mathrm{s}^2$ and 6378136.3 m) should be used as scale parameters with the geopotential coefficients. The recommended $GM_\oplus = 398600.4418$ should be used with the two-body term when working with SI units (398600.4415 or 398600.4356 should be used by those still working with TDT or TDB units, respectively). Although the JGM-3 model is given with terms through degree and order 70, only terms through degree and order twenty are required for Lageos.

Values for the C_{21} and S_{21} coefficients are included in the JGM-3 model. The C_{21} and S_{21} coefficients describe the position of the Earth's figure axis. When averaged over many years, the figure axis should closely coincide with the observed position of the rotation pole averaged over the same time period. Any differences between the mean figure and mean rotation pole averaged would be due to long-period fluid motions in the atmosphere, oceans, or Earth's fluid core (Wahr, 1987, 1990). At present, there is no independent evidence that such motions are important. The JGM-3 values for C_{21} and S_{21} give a mean figure axis that corresponds to the mean pole position recommended in Chapter 3 Terrestrial Reference Frame.

This choice for C_{21} and S_{21} is realized as follows. First, to use the geopotential coefficients to solve for a satellite orbit, it is necessary to rotate from the Earth-fixed frame, where the coefficients are pertinent, to an inertial frame, where the satellite motion is computed. This transformation between frames should include polar motion. We assume the polar motion parameters used are relative to the IERS Reference Pole. If $\bar{x}$ and $\bar{y}$ are the angular displacements of the Terrestrial Reference Frame described in Chapter 3 relative to the IERS Reference Pole, then the values

$$\bar{C}_{21} = \sqrt{3}\bar{x}\bar{C}_{20},$$

$$\bar{S}_{21} = -\sqrt{3}\bar{y}\bar{C}_{20},$$

where $\bar{x} = 0.223 \times 10^{-6}$ radians (equivalent to 0.046 arcsec) and $\bar{y} = 1.425 \times 10^{-6}$ radians (equivalent to 0.294 arcsec) (Nerem *et al.*, 1994) are those used in the geopotential model, so that the mean figure axis coincides with the pole described in Chapter 3. This gives normalized coefficients of

$$\bar{C}_{21}(\mathrm{IERS}) = -0.187 \times 10^{-9},$$

$$\bar{S}_{21}(\mathrm{IERS}) = 1.195 \times 10^{-9}.$$

JGM-3 is available via ftp at ftp.csr.utexas.edu on the directory pub/grav in file JGM3.GEO.Z. It can also be accessed by World Wide Web at http://www.csr.utexas.edu by clicking the "library of data files" selection.

Effect of Solid Earth Tides

The changes induced by the solid Earth tides in the free space potential are most conveniently modeled as variations in the standard geopotential coefficients C_{nm} and S_{nm} (Eanes *et al.*, 1983). The contributions ΔC_{nm} and ΔS_{nm} from the tides are expressible in terms of the k Love number. The effects of ellipticity and rotation of the Earth on tidal deformations necessitates the use, in general, of three k parameters, $k_{nm}^{(0)}$ and $k_{nm}^{(\pm)}$, to characterize the changes produced in the free space potential by tides of spherical harmonic degree and order (nm) (Wahr, 1981). Within the diurnal tidal band, for $(mn) = (21)$, these parameters have a strong frequency dependence due to the Nearly Diurnal Free Wobble resonance. Anelasticity of the mantle causes $k_{nm}^{(0)}$ and $k_{nm}^{(\pm)}$ to acquire small imaginary parts (reflecting a phase lag in the deformational response of the Earth to tidal forces), and also gives rise to a further variation with frequency which is particularly pronounced within the long period band. Though modeling of anelasticity at the periods relevant to tidal phenomena (8 hours to 18.6 years) is not yet definitive, it is clear that the magnitudes of the contributions from anelasticity cannot be ignored (see below) Consequently the anelastic Earth model is recommended for use in precise data analysis.

The degree 2 tides produce time dependent changes in C_{2m} and S_{2m}, through $k_{2m}^{(0)}$, which can exceed 10^{-8} in magnitude. They also produce changes exceeding a cutoff of 3×10^{-12} in C_{4m} and S_{4m} through $k_{2m}^{(+)}$. (The direct contributions of the degree 4 tidal potential to these coefficients are negligible.) The only other changes exceeding this cutoff are in C_{3m} and S_{3m}, produced by the degree 3 part of the tide generating potential.

The computation of the tidal contributions to the geopotential coefficients is most efficiently done by a two-step procedure. In Step 1, the $(2m)$ part of the tidal potential is evaluated in the time domain for each m using lunar and solar ephemerides, and the corresponding changes ΔC_{2m} and ΔS_{2m} are computed using frequency independent nominal values k_{2m} for the respective $k_{2m}^{(0)}$. The contributions of the degree 3 tides to C_{3m} and S_{3m} through $k_{3m}^{(0)}$ and also those of the degree 2 tides to C_{4m} and S_{4m} through $k_{2m}^{(+)}$ may be computed by a similar procedure; they are at the level of 10^{-11}.

Step 2 corrects for the deviations of the $k_{21}^{(0)}$ of several of the constituent tides of the diurnal band from the constant nominal value k_{21} assumed for this band in the first step. Similar corrections need to be applied to a few of the constituents of the other two bands also.

With frequency-independent values k_{nm} (**Step 1**), changes induced by the (nm) part of the tide generating potential in the normalized geopotential coefficients having the same (nm) are given in the time domain by

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\bar{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j} \tag{1}$$

(with $S_{n0} = 0$), where

k_{nm} = nominal degree Love number for degree n and order m,

R_e = equatorial radius of the Earth,

$GM_\oplus$ = gravitational parameter for the Earth,

GM_j = gravitational parameter for the Moon ($j = 2$) and Sun ($j = 3$),

r_j = distance from geocenter to Moon or Sun,

Φ_j = body fixed geocentric latitude of Moon or Sun,

λ_j = body fixed east longitude (from Greenwich) of Moon or Sun,

and $\bar{P}_{nm}$ is the normalized associated Legendre function related to the classical (unnormalized) one by

$$\bar{P}_{nm} = N_{nm} P_{nm}, \tag{2a}$$

where

$$N_{nm} = \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{om})}{(n+m)!}}. \tag{2b}$$

Correspondingly, the normalized geopotential coefficients $(\bar{C}_{nm}, \bar{S}_{nm})$ are related to the unnormalized coefficients (C_{nm}, S_{nm}) by

$$C_{nm} = N_{nm}\bar{C}_{nm}, \quad S_{nm} = N_{nm}\bar{S}_{nm}. \tag{3}$$

Equation (1) yields $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ for both $n = 2$ and $n = 3$ for all m, apart from the corrections for frequency dependence to be evaluated in Step 2. (The particular case $(nm) = (20)$ needs special consideration, however, because it includes a time-independent part which will be discussed below in the section on the permanent tide.)

One further computation to be done in Step 1 is that of the changes in the degree 4 coefficients produced by the degree 2 tides. They are given by

$$\Delta\bar{C}_{4m} - i\Delta\bar{S}_{4m} = \frac{k_{2m}^{(+)}}{5} \sum_{j=2}^{3} \frac{GM_j}{GM_\oplus} \left(\frac{R_e}{r_j}\right)^3 \bar{P}_{2m}(\sin\Phi_j) e^{-im\lambda_j}, \quad (m = 0, 1, 2), \tag{4}$$

which has the same form as Equation (1) for $n = 2$ except for the replacement of k_{2m} by $k_{2m}^{(+)}$.

The parameter values for the computations of Step 1 are given in Table 6.1. The choice of these nominal values (which are complex for $m = 1$ and $m = 2$ in the anelastic case) has been made so as to minimize the number of terms for which corrections will have to be applied in Step 2. The nominal value for $m = 0$ has to be chosen real because the contribution to $\bar{C}_{20}$ from the imaginary part of $k_{20}^{(0)}$. The frequency dependent values for use in Step 2 are taken from the results of computations by Mathews and Buffett (private communication) using the PREM elastic Earth model with the ocean layer replaced by solid, and for the evaluation of anelasticity effects, the Widmer *et al.* (1991) model of mantle Q. As in Wahr and Bergen (1986), a power law was assumed for the frequency dependence of Q with 200 s as the reference period; the value $\alpha = 0.15$ was used for the power law index. The anelasticity contribution (out of phase and in phase) to the tidal changes in the geopotential coefficients is at the level of one to two percent in-phase, and half to one percent out-of-phase, *i.e.*, of the order of 10^{-10}.

Table 6.1. Nominal values of solid Earth tide external potential Love numbers.

		Elastic Earth		Anelastic Earth		
n	m	k_{nm}	k^{+}_{nm}	Re k_{nm}	Im k_{nm}	k^{+}_{nm}
2	0	0.29525	−0.00087	0.30190	−0.00000	−0.00089
2	1	0.29470	−0.00079	0.29830	−0.00144	−0.00080
2	2	0.29801	−0.00057	0.30102	−0.00130	−0.00057
3	0	0.093	...			
3	1	0.093	...			
3	2	0.093	...			
3	3	0.094	...			

The frequency dependence corrections to the $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ values obtained from Step 1 are computed in **Step 2** as the sum of contributions from a number of tidal constituents belonging to the respective bands. The contribution to $\Delta\bar{C}_{20}$ from the long period tidal constituents of various frequencies f is

$$\mathrm{Re} \sum_{f(2,0)} (A_0 \delta k_f H_f e^{i\theta_f}) = \sum_{f(2,0)} (A_0 H_f (\delta k_f^R \cos\theta_f - \delta k_f^i \sin\theta_f), \qquad (5a)$$

while the contribution to $(\Delta\bar{C}_{21} - i\Delta\bar{S}_{21})$ from the diurnal tidal constituents and to $\Delta\bar{C}_{22} - i\Delta\bar{S}_{22}$ from the semidiurnals are given by

$$\Delta\bar{C}_{2m} - i\Delta\bar{S}_{2m} = \eta_m \sum_{f(2,m)} (A_m \delta k_f H_f) e^{i\theta_f}, \quad (m = 1, 2), \qquad (5b)$$

where

$$A_0 = \frac{1}{R_e\sqrt{4\pi}} = 4.4228 \times 10^{-8} \text{ m}^{-1}, \qquad (5c)$$

$$A_m = \frac{(-1)^m}{R_e\sqrt{8\pi}} = (-1)^m (3.1274 \times 10^{-8}) \text{ m}^{-1}, \qquad (m \neq 0), \qquad (5d)$$

$$\eta_1 = -i, \eta_2 = 1, \qquad (5e)$$

δk_f = difference between $k_f \equiv k_{2m}^{(0)}$ at frequency f and the nominal value k_{2m}, in the sense $k_f - k_{2m}$,

δk_f^R = real part, and δk_f^I = imaginary part, of δk_f,

H_f = amplitude (m) of the term at frequency f from the harmonic expansion of the tide generating potential, defined according to the convention of Cartwright and Tayler (1971), and

$\theta_f = \bar{n} \cdot \bar{\beta} = \sum_{i=1}^{6} n_i \beta_i$, or $\theta_f = m(\theta_g + \pi) - \bar{N} \cdot \bar{F} = m(\theta_g + \pi) - \sum_{j=1}^{5} N_j F_j$,

where

$\bar{\beta}$ = six-vector of Doodson's fundamental arguments β_i, (τ, s, h, p, N', p_s),

$\bar{n}$ = six-vector of multipliers n_i (for the term at frequency f) of the fundamental arguments,

$\bar{F}$ = five-vector of fundamental arguments F_j (the Delaunay variables l, l', F, D, Ω) of nutation theory,

$\bar{N}$ = five-vector of multipliers N_i of the Delaunay variables for the nutation of frequency $-f + d\theta_g/dt$,

and θ_g is the Greenwich Mean Sidereal Time expressed in angle units (*i.e.* $24^h = 360°$; see Chapter 5).

(π in $(\theta_g + \pi)$ is now to be replaced by 180.)

For the fundamental arguments (l, l', F, D, Ω) of nutation theory and the convention followed here in choosing their multipliers N_j, see Chapter 5. For conversion of tidal amplitudes defined according to different conventions to the amplitude H_f corresponding to the Cartwright-Tayler convention, use Table 6.4 given at the end of this Chapter.

The correction due to the K_1 constituent, for example, is obtained as follows, given that $A_m = A_1 = -3.1274 \times 10^{-8}$, $H_f = 0.36871$, and $\theta_f = (\theta_g + \pi)$ for this tide. If anelasticity is ignored, $(k_{21}^{(0)})_{K_1} = 0.25377$, and the nominal value chosen is 0.29470. Hence δk_f is $0.25377 - 0.29470 = -0.04093$, and $A_m(\delta k)_f H_f$ reduces to 472.0×10^{-12}. The corrections to the (21) coefficients then become

$$(\Delta \bar{C}_{21})_{K_1} = 472.0 \times 10^{-12} \sin(\theta_g + \pi),$$
$$(\Delta \bar{S}_{21})_{K_1} = 472.0 \times 10^{-12} \cos(\theta_g + \pi).$$

With anelasticity included, $(k_{21}^{(0)})_{K_1} = 0.25745 - i\,0.00148$, and on choosing the nominal value as $(0.29830 - i\,0.00144)$ one obtains the corrections to the coefficients by replacing δk_f in the above calculation by $(-0.04085 - i\,0.00004)$.

In general, if $\delta k_f = \delta k_f^R + i \delta k_f^I$,

$$(\Delta \bar{C}_{2m})_{K_1} = A_m H_f \; (\delta k_f^R \sin\theta_f + \delta k_f^I \cos\theta_f),$$
$$(\Delta \bar{S}_{2m})_{K_1} = A_m H_f \; (\delta k_f^R \cos\theta_f - \delta k_f^I \sin\theta_f).$$

Table 6.2a lists the results for all tidal terms which contribute 10^{-13} or more, after round-off, to the $(nm) = (21)$ geopotential coefficient. A cutoff at this level is used for the individual terms in order that accuracy at the level of 3×10^{-12} be not affected by the accumulated contributions from the numerous smaller terms that are disregarded. The imaginary parts of the contributions are below cutoff and are not listed. Results relating to the (20) and (22) coefficients are presented in Tables (6.2b) and (6.2c), respectively.

Table 6.2a. Amplitudes $(A_1 \delta k_f H_f)$ of the corrections for frequency dependence of $k_{21}^{(0)}$, taking the nominal value k_{21} for the diurnal tides as 0.29470 for the elastic case, and $(0.29830 - i\,0.00144)$ for the anelastic case. Units: 10^{-12}. Multipliers of the Doodson arguments identifying the tidal terms are given, as also those of the Delaunay variables.

Name	deg/hr	Doodson No.	τ	s	h	p	N'	p_s	ℓ	ℓ'	F	D	Ω	δk_f^{el}	Amp. elas.	δk_f^{anel}	Amp. anel.
	13.39645	135,645	1	-2	0	1	-1	0	1	0	2	0	1	-0.00044	-0.1	-0.00045	-0.1
Q_1	13.39866	135,655	1	-2	0	1	0	0	1	0	2	0	2	-0.00044	-0.7	-0.00046	-0.7
ρ_1	13.47151	137,455	1	-2	2	-1	0	0	-1	0	2	2	2	-0.00047	-0.1	-0.00049	-0.1
	13.94083	145,545	1	-1	0	0	-1	0	0	0	2	0	1	-0.00081	-1.2	-0.00082	-1.3
O_1	13.94303	145,555	1	-1	0	0	0	0	0	0	2	0	2	-0.00081	-6.6	-0.00082	-6.7
$N\tau_1$	14.41456	153,655	1	0	-2	1	0	0	1	0	2	-2	2	-0.00167	0.1	-0.00168	0.1
LK_1	14.48741	155,455	1	0	0	-1	0	0	-1	0	2	0	2	-0.00193	0.4	-0.00193	0.4
NO_1	14.49669	155,655	1	0	0	1	0	0	1	0	0	0	0	-0.00196	1.3	-0.00197	1.3
	14.49890	155,665	1	0	0	1	1	0	1	0	0	0	1	-0.00197	0.2	-0.00198	0.3
χ_1	14.56955	157,455	1	0	2	-1	0	0	-1	0	0	2	0	-0.00231	0.3	-0.00231	0.3
π_1	14.91787	162,556	1	1	-3	0	0	1	0	1	2	-2	2	-0.00834	-1.9	-0.00832	-1.9
	14.95673	163,545	1	1	-2	0	-1	0	0	0	2	-2	1	-0.01114	0.5	-0.01111	0.5
P_1	14.95893	163,555	1	1	-2	0	0	0	0	0	2	-2	2	-0.01135	-43.3	-0.01132	-43.2
S_1	15.00000	164,556	1	1	-1	0	0	1	0	1	0	0	0	-0.01650	2.0	-0.01642	2.0
	15.03886	165,545	1	1	0	0	-1	0	0	0	0	0	-1	-0.03854	-8.8	-0.03846	-8.8
K_1	15.04107	165,555	1	1	0	0	0	0	0	0	0	0	0	-0.04093	472.0	-0.04085	471.0
	15.04328	165,565	1	1	0	0	1	0	0	0	0	0	1	-0.04365	68.3	-0.04357	68.2
	15.04548	165,575	1	1	0	0	2	0	0	0	0	0	2	-0.04678	-1.6	-0.04670	-1.6
ψ_1	15.08214	166,554	1	1	1	0	0	-1	0	-1	0	0	0	0.23083	-20.8	0.22609	-20.4
ϕ_1	15.12321	167,555	1	1	2	0	0	0	0	0	-2	2	-2	0.03051	-5.0	0.03027	-5.0
θ_1	15.51259	173,655	1	2	-2	1	0	0	1	0	0	-2	0	0.00374	-0.5	0.00371	-0.5
J_1	15.58545	175,455	1	2	0	-1	0	0	-1	0	0	0	0	0.00329	-2.1	0.00325	-2.1
	15.58765	175,465	1	2	0	-1	1	0	-1	0	0	0	1	0.00327	-0.4	0.00324	-0.4
SO_1	16.05697	183,555	1	3	-2	0	0	0	0	0	0	-2	0	0.00198	-0.2	0.00195	-0.2
OO_1	16.13911	185,555	1	3	0	0	0	0	0	0	-2	0	-2	0.00187	-0.7	0.00184	-0.6
	16.14131	185,565	1	3	0	0	1	0	0	0	-2	0	-1	0.00187	-0.4	0.00184	-0.4

Table 6.2b. Corrections for frequency dependence of $k_{20}^{(0)}$ of the zonal tides due to anelasticity. Units: 10^{-12}. The nominal value k_{20} for the zonal tides is taken as 0.30190. The real and imaginary parts δk_f^R and δk_f^I of δk_f are listed, along with the corresponding in phase (ip) amplitude ($A_0 H_f \delta k_f^R$) and out of phase (op) amplitude ($-A_0 H_f \delta k_f^I$) to be used in equation (5a). In the elastic case, $k_{20}^{(0)} = 0.29525$ for all the zonal tides, and no second step corrections are needed.

Name	Doodson No.	deg/hr	τ	s	h	p	N'	p_s	ℓ	ℓ'	F	D	Ω	δk_f^R	Amp. (ip)	δk_f^I	Amp. (op)
	55,565	0.00221	0	0	0	0	1	0	0	0	0	0	1	0.01347	16.6	-0.00541	-6.7
	55,575	0.00441	0	0	0	0	2	0	0	0	0	0	2	0.01124	-0.1	-0.00488	0.1
S_a	56,554	0.04107	0	0	1	0	0	-1	0	-1	0	0	0	0.00547	-1.2	-0.00349	0.8
S_{sa}	57,555	0.08214	0	0	2	0	0	0	0	0	-2	2	-2	0.00403	-5.5	-0.00315	4.3
	57,565	0.08434	0	0	2	0	1	0	0	0	-2	2	-1	0.00398	0.1	-0.00313	-0.1
	58,554	0.12320	0	0	3	0	0	-1	0	-1	-2	2	-2	0.00326	-0.3	-0.00296	0.2
M_{sm}	63,655	0.47152	0	1	-2	1	0	0	1	0	0	-2	0	0.00101	-0.3	-0.00242	0.7
	65,445	0.54217	0	1	0	-1	-1	0	-1	0	0	0	-1	0.00080	0.1	-0.00237	-0.2

M_m	65,455	0.54438	0 1 0 -1 0 0	-1 0 0 0 0	0.00080	-1.2	-0.00237	3.7
	65,465	0.54658	0 1 0 -1 1 0	-1 0 0 0 1	0.00079	0.1	-0.00237	-0.2
	65,655	0.55366	0 1 0 1 0 0	1 0 -2 0 -2	0.00077	0.1	-0.00236	-0.2
M_{sf}	73,555	1.01590	0 2 -2 0 0 0	0 0 0 -2 0	-0.00009	0.0	-0.00216	0.6
	75,355	1.08875	0 2 0 -2 0 0	-2 0 0 0 0	-0.00018	0.0	-0.00213	0.3
M_f	75,555	1.09804	0 2 0 0 0 0	0 0 -2 0 -2	-0.00019	0.6	-0.00213	6.3
	75,565	1.10024	0 2 0 0 1 0	0 0 -2 0 -1	-0.00019	0.2	-0.00213	2.6
	75,575	1.10245	0 2 0 0 2 0	0 0 -2 0 0	-0.00019	0.0	-0.00213	0.2
M_{stm}	83,655	1.56956	0 3 -2 1 0 0	1 0 -2 -2 -2	-0.00065	0.1	-0.00202	0.2
M_{tm}	85,455	1.64241	0 3 0 -1 0 0	-1 0 -2 0 -2	-0.00071	0.4	-0.00201	1.1
	85,465	1.64462	0 3 0 -1 1 0	-1 0 -2 0 -1	-0.00071	0.2	-0.00201	0.5
M_{sqm}	93,555	2.11394	0 4 -2 0 0 0	0 0 -2 -2 -2	-0.00102	0.1	-0.00193	0.2
M_{qm}	95,355	2.18679	0 4 0 -2 0 0	-2 0 -2 0 -2	-0.00106	0.1	-0.00192	0.1

Table 6.2c. Amplitudes $(A_2 \delta k_f H_f)$ of the corrections for frequency dependence of $k_{22}^{(0)}$, taking the nominal value k_{22} for the sectorial tides as 0.29801 for the elastic case, and $(0.30102 - i\,0.00130)$ for the anelastic case. Units: 10^{-12}. The corrections are only to the real part, and are the same in both the elastic and the anelastic cases.

Name	Doodson No.	deg/hr	$\tau\ s\ h\ p\ N'\ p_s$	$\ell\ \ell'\ F\ D\ \Omega$	δk_f^R	Amp.
N_2	245,655	28.43973	2 -1 0 1 0 0	1 0 2 0 2	0.00006	-0.3
M_2	255,555	28.98410	2 0 0 0 0 0	0 0 2 0 2	0.00004	-1.2

The total variation in geopotential coefficient $\bar{C}_{20}$ is obtained by adding to the result of Step 1 the sum of the contributions from the tidal constituents listed in Table 6.2b computed using equation (5a). The tidal variations in $\bar{C}_{2m}$ and $\bar{S}_{2m}$ for the other m are computed similarly, except that equation (5b) is to be used together with Table 6.2a for $m = 1$ and Table 6.2c for $m = 2$.

Solid Earth Pole Tide

The pole tide is generated by the centrifugal effect of polar motion, characterized by the potential

$$\Delta V = -(\Omega^2 R_e^2/2)\sin 2\theta(x_p \cos\lambda - y_p \sin\lambda).$$

(See the section on Deformation due to Polar Motion in Chapter 7 for further details). The deformation which constitutes this tide produces a perturbation $k_2 \Delta V$ in the external potential which is equivalent to changes in the geopotential coefficients C_{21} and S_{21}. Using for k_2 the elastic Earth value 0.2977 appropriate to the polar tide yields

$$\Delta\bar{C}_{21} = -1.290 \times 10^{-9}(x_p),$$

$$\Delta\bar{S}_{21} = 1.290 \times 10^{-9}(y_p),$$

where x_p and y_p are in seconds of arc as defined in Chapter 7. For the anelastic Earth, k_2 has real and imaginary parts $k_2^R = 0.3111$ and $k_2^I = -0.0035$, leading to

$$\Delta\bar{C}_{21} = -1.348 \times 10^{-9}(x_p + 0.0112 y_p),$$

$$\Delta\bar{S}_{21} = 1.348 \times 10^{-9}(y_p - 0.0112 x_p).$$

Treatment of the Permanent Tide

The degree 2 zonal tide generating potential has a mean (time average) value which is nonzero. This permanent (time independent) potential produces a permanent deformation which is reflected in the static figure of the Earth, and a corresponding time independent contribution to the geopotential which can be considered as part of the adopted value of $\bar{C}_{20}$, as in the JGM-3 model. Therefore, for $(nm) = (20)$, the zero frequency part should be excluded from the expression (1). Hereafter the symbol $\Delta\bar{C}_{20}$ is reserved for the temporally varying part of the tidal contribution to $\bar{C}_{20}$; the expression (1) for $(mn) = (20)$ will be redesignated as $\bar{C}_{20}^*$.

$$\Delta\bar{C}_{20}^* = \frac{k_{20}}{5}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^3 \bar{P}_{20}(\sin\Phi_j).$$

Its zero frequency part is

$$\langle\Delta\bar{C}_{20}\rangle = A_0 H_0 k_{20} = (4.4228 \times 10^{-8})(-0.31460)k_{20}. \tag{6}$$

To represent the tide induced changes in the (20) geopotential, one should then use only the time variable part

$$\Delta\bar{C}_{20} = \Delta\bar{C}_{20}^* - \langle\Delta\bar{C}_{20}\rangle. \tag{7}$$

In evaluating it, the same value should be used for k_{20} in both $\Delta\bar{C}_{20}^*$ and $\langle\Delta\bar{C}_{20}\rangle$. If the elastic Earth value $k_{20} = 0.29525$ is used, $\langle\Delta\bar{C}_{20}\rangle = -4.108 \times 10^{-9}$, while with the value $k_{20} = 0.30190$ of the anelastic case, $\langle\Delta\bar{C}_{20}\rangle = -4.201 \times 10^{-9}$.

The restitution of the indirect effect of the permanent tide is done to be consistent with the XVIII IAG General Assembly Resolution 16; but to obtain the effect of the permanent tide on the geopotential, one can use the same formula as equation (6) using the fluid limit Love number which is $k = 0.94$.

Effect of the Ocean Tides

The dynamical effects of ocean tides are most easily incorporated by periodic variations in the normalized Stokes' coefficients. These variations can be written as

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = F_{nm}\sum_{s(n,m)}\sum_{+}^{-}(C_{snm}^{\pm} \mp iS_{snm}^{\pm})e^{\pm i\theta_f}, \tag{8}$$

where

$$F_{nm} = \frac{4\pi G\rho_w}{g}\sqrt{\frac{(n+m)!}{(n-m)!(2n+1)(2-\delta_{om})}}\left(\frac{1+k'_n}{2n+1}\right),$$

g and G are given in Chapter 4, ρ_w = density of seawater = 1025 kg m^{-3},

k'_n = load deformation coefficients ($k'_2 = -0.3075, k'_3 = -0.195, k'_4 = -0.132, k'_5 = -0.1032, k'_6 = -0.0892$),

$C^{\pm}_{snm}, S^{\pm}_{snm}$ = ocean tide coefficients (m) for the tide constituent s

θ_s = argument of the tide constituent s as defined in the solid tide model (Chapter 7).

The summation over + and - denotes the respective addition of the retrograde waves using the top sign and the prograde waves using the bottom sign. The $C^{\pm}_{snm}$ and $S^{\pm}_{snm}$ are the coefficients of a spherical harmonic decomposition of the ocean tide height for the ocean tide due to the constituent s of the tide generating potential.

For each constituent s in the diurnal and semi-diurnal tidal bands, these coefficients were obtained from the CSR 3.0 ocean tide height model (Eanes *et al.*, 1996), which was estimated from the TOPEX/ Poseidon satellite altimeter data. For each constituent s in the long period band, the self-consistent equilibrium tide model of Ray and Cartwright (1994) was used. The list of constituents for which the coefficients were determined was obtained from the Cartwright and Tayler (1971) expansion of the tide raising potential.

These ocean tide height harmonics are related to the Schwiderski convention (Schwiderski, 1983) according to

$$C^{\pm}_{snm} - iS^{\pm}_{snm} = -i\hat{C}^{\pm}_{snm} e^{i(\epsilon^{\pm}_{snm} + \chi_s)}, \tag{9}$$

where

$\hat{C}^{\pm}_{snm}$ = ocean tide amplitude for constituent s using the Schwiderski notation,

$\epsilon^{\pm}_{snm}$ = ocean tide phase for constituent s,

and χ_s is obtained from Table 6.3, with H_s being the Cartwright and Tayler (1971) amplitude at frequency s.

Table 6.3. Values of χ_s for long-period, diurnal and semidiurnal tides.

Tidal Band	$H_s > 0$	$H_s < 0$
Long Period	π	0
Diurnal	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
Semidiurnal	0	π

For clarity, the terms in equation 1 are repeated in both conventions:

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [(C^{+}_{snm} + C^{-}_{snm})\cos\theta_s + (S^{+}_{snm} + S^{-}_{snm})\sin\theta_s] \tag{10a}$$

or

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^{+}_{snm}\sin(\theta_s + \epsilon^{+}_{snm} + \chi_s) + \hat{C}^{-}_{snm}\sin(\theta_s + \epsilon^{-}_{snm} + \chi_s)], \tag{10b}$$

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [(S^+_{snm} + S^-_{snm})\cos\theta_s - (C^+_{snm} - C^-_{snm})\sin\theta_s] \qquad (10c)$$

or

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^+_{snm}\cos(\theta_s + \epsilon^+_{snm} + \chi_s) - \hat{C}^-_{snm}\cos(\theta_s + \epsilon^-_{snm} + \chi_s)]. \qquad (10d)$$

The orbit element perturbations due to ocean tides can be loosely grouped into two classes. The resonant perturbations arise from coefficients for which the order (m) is equal to the first Doodson's argument multiplier n_1 of the tidal constituent s (See Note), and have periodicities from a few days to a few years. The non-resonant perturbations arise when the order m is not equal to index n_1. The most important of these are due to ocean tide coefficients for which $m = n_1 + 1$ and have periods of about 1 day.

Certain selected constituents (*e.g.* S_a and S_2) are strongly affected by atmospheric mass distribution (Chapman and Lindzen, 1970). The resonant harmonics (for $m = n_1$) for some of these constituents were determined by their combined effects on the orbits of several satellites. These multi-satellite values then replaced the corresponding values from the CSR3.0 altimetric ocean tide height model.

Based on the predictions of the linear perturbation theory outlined in Casotto (1989), the relevant tidal constituents and spherical harmonics were selected for several geodetic and altimetric satellites. For geodetic satellites, both resonant and non-resonant perturbations were analyzed,whereas for altimetric satellites, only the non-resonant perturbations were analyzed. For the latter, the adjustment of empirical parameters during orbit determination removes the errors in modeling resonant accelerations. The resulting selection of ocean tidal harmonics was then merged into a single recommended ocean tide force model. With this selection the error of omission on TOPEX is approximately 5 *mm* along-track, and for Lageos it is 2 *mm* along-track. The recommended ocean tide harmonic selection is available via anonymous ftp from ftp.csr.utexas.edu.

For high altitude geodetic satellites like Lageos, in order to reduce the required computing time, it is recommended that out of the complete selection, only the constituents whose Cartwright and Tayler amplitudes H_s is greater than 0.5 *mm* be used, with their spherical harmonic expansion terminated at maximum degree and order 8. The omission errors from this reduced selection on Lageos is estimated at approximately 1 cm in the transverse direction for short arcs.

NOTE: The Doodson variable multipliers ($\bar{n}$) are coded into the argument number (A) after Doodson (1921) as:

$$A = n_1(n_2 + 5)(n_3 + 5).(n_4 + 5)(n_5 + 5)(n_6 + 5).$$

Conversion of tidal amplitudes defined according to different conventions

The definition used for the amplitudes of tidal terms in the recent high-accuracy tables differ from each other and from Cartwright and Tayler (1971). Hartmann and Wenzel (1995) tabulate amplitudes in units of the potential (m^2s^{-2}), while the amplitudes of Roosbeek (1996), which follow the Doodson (1921) convention, are dimensionless. To convert them to the equivalent tide heights H_f of the Cartwright-Tayler convention, multiply by the appropriate factors from Table 6.4. The following values are used for the constants appearing in the conversion factors: Doodson constant D_1 = 2.63358352855 $m^2\ s^{-2}$; $g_e \equiv g$ at the equatorial radius = 9.79828685 (from $GM = 3.986004415 \times 10^{14}$ $m^3\ s^{-2}$, $R_e = 6378136.55$ m).

Table 6.4 Factors for conversion to Cartwright-Tayler amplitudes from those defined according to Doodson's and Hartmann and Wenzel's conventions

From Doodson	From Harmann & Wenzel
$f_{20} = -\frac{\sqrt{4\pi}}{\sqrt{5}}\frac{D_1}{g_e} = -0.426105$	$f'_{20} = \frac{2\sqrt{\pi}}{g_e} = 0.361788$
$f_{21} = -\frac{2\sqrt{24\pi}}{3\sqrt{5}}\frac{D_1}{g_e} = -0.695827$	$f'_{21} = -\frac{\sqrt{8\pi}}{g_e} = -0.511646$
$f_{22} = \frac{\sqrt{96\pi}}{3\sqrt{5}}\frac{D_1}{g_e} = 0.695827$	$f'_{22} = \frac{\sqrt{8\pi}}{g_e} = 0.511646$
$f_{30} = -\frac{\sqrt{20\pi}}{\sqrt{7}}\frac{D_1}{g_e} = -0.805263$	$f'_{30} = \frac{2\sqrt{\pi}}{g_e} = 0.361788$
$f_{31} = -\frac{\sqrt{720\pi}}{8\sqrt{7}}\frac{D_1}{g_e} = -0.603947$	$f'_{31} = -\frac{\sqrt{8\pi}}{g_e} = -0.511646$
$f_{32} = \frac{\sqrt{1440\pi}}{10\sqrt{7}}\frac{D_1}{g_e} = 0.683288$	$f'_{32} = \frac{\sqrt{8\pi}}{g_e} = 0.511646$
$f_{33} = -\frac{\sqrt{2880\pi}}{15\sqrt{7}}\frac{D_1}{g_e} = -0.644210$	$f'_{33} = -\frac{\sqrt{8\pi}}{g_e} = -0.511646$

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