CHAPTER 8 TIDAL VARIATIONS IN THE EARTH'S ROTATION

Periodic variations in UT1 due to tidal deformation of the polar moment of inertia were derived by Yoder *et al.* (1981) including the tidal deformation of the Earth with a decoupled core. This model uses effective Love numbers that differ from the bulk value of 0.301 because of the oceans and the fluid core giving rise to different theoretical values of the ratio k/C for the fortnightly and monthly terms. However, Yoder *et al.* recommend the value of 0.94 for k/C for both cases. Oceanic tides also cause variations in UT1 represented by models given by Brosche *et al.* (1991, 1989), Dickman (1993, 1991, 1990, 1989), Gross (1993), Herring and Dong (1994), and Ray *et al.* (1994). The contribution of the oceanic tides is split into a part which is in phase with the solid Earth tides and an out-of-phase part.

Table 8.1 provides corrections for the tidal variations in UT1−UTC with periods between five and 35 days. This is identical to tables in IERS Technical Notes 3 and 13 which defined UT1R−UTC, $\Delta - \Delta$R, and $\omega - \omega$R. Table 8.1 continues to define UT1R−UTC, $\Delta - \Delta$R, and $\omega - \omega$R.

$$\mathrm{UT1} - \mathrm{UT1R} = \sum_{i=1}^{41} A_i \sin \xi_i$$

$$\Delta - \Delta\mathrm{R} = \sum_{i=1}^{41} A_i' \cos \xi_i$$

$$\omega - \omega\mathrm{R} = \sum_{i=1}^{41} A_i'' \cos \xi_i$$

$$\xi_i = \sum_{j=1}^{5} a_{ij} \alpha_j$$

a_{ij} = integer multipliers of the α_j (l, l', F, D or Ω) for the i^{th} tide given in the first five columns of Table 8.1. A_i, A_i', A_i'' are given in columns 7-9 respectively in Table 8.1.

Table 8.2 is identical to Table 8.2 in IERS Technical Note 13 which defined UT1S−UTC, $\Delta - \Delta$S, and $\omega - \omega$S except that the table below corrects one misprint in the out-of-phase term for the 18.6-year tide. Table 8.2 provides corrections for the tidal variations with periods from five days to 18.6 years and incorporates the most significant effects of oceanic tides using the model of Dickman (1993).

$$\mathrm{UT1} - \mathrm{UT1S} = \sum_{i=1}^{62} B_i \sin \xi_i + C_i \cos \xi_i$$

$$\Delta - \Delta\mathrm{S} = \sum_{i=1}^{62} B_i' \cos \xi_i + C_i' \sin \xi_i$$

$$\omega - \omega\mathrm{S} = \sum_{i=1}^{62} B_i'' \cos \xi_i + C_i'' \sin \xi_i$$

ξ_i are defined the same way as for UT1R above except the a_{ij} are given in the first five columns of Table 8.2. B_i, C_i, B_i', C_i', B_i'', and C_i'' are given in columns 7-12 respectively in Table 8.2.

Table 8.3 provides corrections for daily and sub-daily tidal variations in the Earth's rotation from Ray (1995). Values corrected using Table 8.3 are referred to as UT1D, $\Delta - \Delta$D, and $\omega - \omega$D. Those corrected using Table 8.1 and 8.3 will then be called UT1DR, $\Delta - \Delta$DR, and $\omega - \omega$DR, and those corected using Table 8.2 and 8.3 will be called UT1DS, $\Delta - \Delta$DS, and $\omega - \omega$DS.

$$\text{UT1} - \text{UT1D} = \sum_{i=1}^{8} D_i \sin \xi_i + E_i \cos \xi_i$$

$$\Delta - \Delta\text{D} = \sum_{i=1}^{8} D_i' \cos \xi_i + E_i' \sin \xi_i$$

$$\omega - \omega\text{D} = \sum_{i=1}^{8} D_i'' \cos \xi_i + E_i'' \sin \xi_i$$

$$\xi_i = \sum_{j=1}^{6} c_{ij} \gamma_i + \phi_i$$

c_{ij} = integer multipliers of the γ_j (l, l', F, D, Ω, or θ) for the i^{th} tide given in columns 2-7 of Table 8.3. ϕ_i = the phase given in column 8 of Table 8.3.

Table 8.4 provides corrections for daily and subdaily tidal variations in polar motion from Ray (1995). Values corrected by using this table are referred to as xD and yD and are obtained by

$$xD - x = \sum_{i=1}^{8} F_i \sin \xi_i + G_i \cos \xi_i,$$

$$yD - y = \sum_{i=1}^{8} H_i \sin \xi_i + K_i \cos \xi_i,$$

where F_i, G_i, H_i, and K_i are given in the table and

$$\xi_i = \sum_{j=1}^{6} c_{ij} \gamma_i + \phi_i.$$

c_{ij} = integer multipliers of the γ_i (l, l', F, D, Ω, or θ) for the i^{th} tide given in columns 2-7 of Table 8.4. ϕ_i = the phase given in column 8 of Table 8.4.

Table 8.1. Zonal Tide Terms With Periods Up to 35 Days. UT1R, ΔR, and ωR represent the regularized forms of UT1, the duration of the day Δ, and the angular velocity of the Earth, ω. The units are 10^{-4} s for UT, 10^{-5} s for Δ, and 10^{-14} rad/s for ω.

ARGUMENT*					PERIOD	UT1−UT1R	$\Delta - \Delta$R	$\omega - \omega$R
						Coefficient of	Coefficient of	
l	l'	F	D	Ω	Days	Sin (Argument)	Cos (Argument)	
1	0	2	2	2	5.64	-0.02	0.3	-0.2
2	0	2	0	1	6.85	-0.04	0.4	-0.3
2	0	2	0	2	6.86	-0.10	0.9	-0.8
0	0	2	2	1	7.09	-0.05	0.4	-0.4
0	0	2	2	2	7.10	-0.12	1.1	-0.9
1	0	2	0	0	9.11	-0.04	0.3	-0.2
1	0	2	0	1	9.12	-0.41	2.8	-2.4
1	0	2	0	2	9.13	-0.99	6.8	-5.8
3	0	0	0	0	9.18	-0.02	0.1	-0.1
-1	0	2	2	1	9.54	-0.08	0.5	-0.5
-1	0	2	2	2	9.56	-0.20	1.3	-1.1
1	0	0	2	0	9.61	-0.08	0.5	-0.4
2	0	2	-2	2	12.81	0.02	-0.1	0.1
0	1	2	0	2	13.17	0.03	-0.1	0.1
0	0	2	0	0	13.61	-0.30	1.4	-1.2
0	0	2	0	1	13.63	-3.21	14.8	-12.5
0	0	2	0	2	13.66	-7.76	35.7	-30.1
2	0	0	0	-1	13.75	0.02	-0.1	0.1
2	0	0	0	0	13.78	-0.34	1.5	-1.3
2	0	0	0	1	13.81	0.02	-0.1	0.1
0	-1	2	0	2	14.19	-0.02	0.1	-0.1
0	0	0	2	-1	14.73	0.05	-0.2	0.2
0	0	0	2	0	14.77	-0.73	3.1	-2.6
0	0	0	2	1	14.80	-0.05	0.2	-0.2
0	-1	0	2	0	15.39	-0.05	0.2	-0.2
1	0	2	-2	1	23.86	0.05	-0.1	0.1
1	0	2	-2	2	23.94	0.10	-0.3	0.2
1	1	0	0	0	25.62	0.04	-0.1	0.1
-1	0	2	0	0	26.88	0.05	-0.1	0.1
-1	0	2	0	1	26.98	0.18	-0.4	0.3
-1	0	2	0	2	27.09	0.44	-1.0	0.9
1	0	0	0	-1	27.44	0.53	-1.2	1.0
1	0	0	0	0	27.56	-8.26	18.8	-15.9
1	0	0	0	1	27.67	0.54	-1.2	1.0
0	0	0	1	0	29.53	0.05	-0.1	0.1
1	-1	0	0	0	29.80	-0.06	0.1	-0.1
-1	0	0	2	-1	31.66	0.12	-0.2	0.2
-1	0	0	2	0	31.81	-1.82	3.6	-3.0
-1	0	0	2	1	31.96	0.13	-0.3	0.2
1	0	-2	2	-1	32.61	0.02	0.0	0.0
-1	-1	0	2	0	34.85	-0.09	0.2	-0.1

Table 8.2. Zonal Tide Terms. UT1S1, ΔS1, and ωS1 represent the regularized forms of UT1, the duration of the day Δ, and the angular velocity of the Earth, ω. The units are 10^{-4} s for UT, 10^{-5} s for Δ, and 10^{-14} rad/s for ω.

ARGUMENT*					PERIOD	UT1−UT1S		$\Delta - \Delta$S		$\omega - \omega$S	
								Coefficient of			
l	l'	F	D	Ω	Days	Sin	Cos	Cos	Sin	Cos	Sin
1	0	2	2	2	5.64	-0.02		0.3		-0.2	
2	0	2	0	1	6.85	-0.04		0.4		-0.3	
2	0	2	0	2	6.86	-0.10		0.9		-0.8	
0	0	2	2	1	7.09	-0.05		0.4		-0.4	
0	0	2	2	2	7.10	-0.12		1.1		-0.9	
1	0	2	0	0	9.11	-0.04		0.3		-0.2	
1	0	2	0	1	9.12	-0.40	0.01	2.7	0.1	-2.3	-0.1
1	0	2	0	2	9.13	-0.98	0.03	6.7	0.2	-5.7	-0.2
3	0	0	0	0	9.18	-0.02		0.1		-0.1	
-1	0	2	2	1	9.54	-0.08		0.5		-0.5	
-1	0	2	2	2	9.56	-0.20		1.3		-1.1	
1	0	0	2	0	9.61	-0.08		0.5		-0.4	
2	0	2	-2	2	12.81	0.02		-0.1		0.1	
0	1	2	0	2	13.17	0.03		-0.1		0.1	
0	0	2	0	0	13.61	-0.30		1.4		-1.2	
0	0	2	0	1	13.63	-3.20	0.09	14.7	0.4	-12.4	-0.4
0	0	2	0	2	13.66	-7.73	0.21	35.6	1.0	-30.0	-0.8
2	0	0	0	-1	13.75	0.02		-0.1		0.1	
2	0	0	0	0	13.78	-0.34		1.5		-1.3	
2	0	0	0	1	13.81	0.02		-0.1		0.1	
0	-1	2	0	2	14.19	-0.02		0.1		-0.1	
0	0	0	2	-1	14.73	0.05		-0.2		0.2	
0	0	0	2	0	14.77	-0.72	0.02	3.1	0.1	-2.6	-0.1
0	0	0	2	1	14.80	-0.05		0.2		-0.2	
0	-1	0	2	0	15.39	-0.05		0.2		-0.2	
1	0	2	-2	1	23.86	0.05		-0.1		0.1	
1	0	2	-2	2	23.94	0.10		-0.3		0.2	
1	1	0	0	0	25.62	0.04		-0.1		0.1	
-1	0	2	0	0	26.88	0.05		-0.1		0.1	
-1	0	2	0	1	26.98	0.18		-0.4		0.3	
-1	0	2	0	2	27.09	0.44		-1.0		0.9	
1	0	0	0	-1	27.44	0.53		-1.2		1.0	
1	0	0	0	0	27.56	-8.33	0.12	19.0	0.3	-16.0	-0.2
1	0	0	0	1	27.67	0.54		-1.2		1.0	
0	0	0	1	0	29.53	0.05		-0.1		0.1	
1	-1	0	0	0	29.80	-0.06		0.1		-0.1	
-1	0	0	2	-1	31.66	0.12		-0.2		0.2	
-1	0	0	2	0	31.81	-1.84	0.02	3.6	0.0	-3.0	0.0
-1	0	0	2	1	31.96	0.13		-0.3		0.2	
1	0	-2	2	-1	32.61	0.02		0.0		0.0	
-1	-1	0	2	0	34.85	-0.09		0.2		-0.1	
0	2	2	-2	2	91.31	-0.06		0.0		0.0	

0	1	2	-2	1	119.61	0.03		0.0		0.0	
0	1	2	-2	2	121.75	-1.88		1.0		-0.8	
0	0	2	-2	0	173.31	0.25		-0.1		0.1	
0	0	2	-2	1	177.84	1.17		-0.4		0.3	
0	0	2	-2	2	182.62	-48.84	0.11	16.8	0.0	-14.2	0.0
0	2	0	0	0	182.63	-0.19		0.1		-0.1	
2	0	0	-2	-1	199.84	0.05		0.0		0.0	
2	0	0	-2	0	205.89	-0.55		0.2		-0.1	
2	0	0	-2	1	212.32	0.04		0.0		0.0	
0	-1	2	-2	1	346.60	-0.05		0.0		0.0	
0	1	0	0	-1	346.64	0.09		0.0		0.0	
0	-1	2	-2	2	365.22	0.83		-0.1		0.1	
0	1	0	0	0	365.26	-15.55	0.02	2.6	0.0	-2.2	0.0
0	1	0	0	1	386.00	-0.14		0.0		0.0	
1	0	0	-1	0	411.78	0.03		0.0		0.0	
2	0	-2	0	0	1095.17	-0.14		0.0		0.0	
-2	0	2	0	1	1305.47	0.42		0.0		0.0	
-1	1	0	1	0	3232.85	0.04		0.0		0.0	
0	0	0	0	2	3399.18	7.90		0.1		-0.1	
0	0	0	0	1	6790.36	-1637.68	-0.10	-10.4	0.0	8.8	0.0

Table 8.3. Diurnal and subdiurnal zonal tide terms. The units are 10^{-4} s for UT, 10^{-5} s for Δ, and 10^{-14} rad/s for ω.

Tide	l	l'	F	D	Ω	θ	phase	Period	UT1−UT1D		$\Delta - \Delta$D		$\omega - \omega$D	
											Coefficients of			
							(deg.)	(hours)	Sin	Cos	Cos	Sin	Cos	Sin
Q_1	-1	0	-2	0	-2	1	-90	26.868	0.02	0.05	-1.4	2.8	1.2	-2.4
O_1	0	0	-2	0	-2	1	-90	25.819	0.12	0.16	-7.1	9.4	6.0	-7.9
P_1	0	0	-2	2	-2	1	-90	24.066	0.03	0.05	-1.8	3.2	1.5	-2.7
K_1	0	0	0	0	0	1	90	23.935	0.09	0.18	-5.4	11.2	4.6	-9.4
N_2	-1	0	-2	0	-2	2	0	12.658	-0.04	-0.02	4.5	-1.8	-3.8	1.6
M_2	0	0	-2	0	-2	2	0	12.421	-0.16	-0.07	19.6	-8.7	-16.6	7.4
S_2	0	0	-2	2	-2	2	0	12.000	-0.08	0.00	9.5	-0.5	-8.1	0.4
K_2	0	0	0	0	0	2	0	11.967	-0.02	0.00	2.5	-0.5	-2.1	0.4

θ = 180° + 360°.993326(MJD-51544.5) Earth's rotation angle
See Chapter 5 for definitions of l, l', F, D, Ω.

Table 8.4. Diurnal and subdiurnal tide terms for polar motion. The units are msec. of arc for Δx and Δy.

Tide	l	l'	F	D	Ω	θ	phase	Period	Δx		Δy	
									Coefficients of			
							(deg.)	(hours)	Sin	Cos	Sin	Cos
Q_1	-1	0	-2	0	-2	1	-90	26.868	-0.026	0.006	-0.006	0.026
O_1	0	0	-2	0	-2	1	-90	25.819	-0.133	0.049	-0.049	0.133
P_1	0	0	-2	2	-2	1	-90	24.066	-0.050	0.025	-0.025	0.050
K_1	0	0	0	0	0	1	90	23.935	-0.152	0.078	-0.078	0.152
N_2	-1	0	-2	0	-2	2	0	12.658	-0.057	-0.013	0.011	-0.033
M_2	0	0	-2	0	-2	2	0	12.421	-0.330	-0.028	0.037	-0.196
S_2	0	0	-2	2	-2	2	0	12.000	-0.145	0.064	0.059	-0.087
K_2	0	0	0	0	0	2	0	11.967	-0.036	0.017	0.018	-0.022

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