II. Current methodology for TRF combination

The current strategy adopted for Terrestrial Reference Frame combination analysis is twofold: simultaneous combination of station positions and velocities using full variance / covariance matrices; rigorous weighting scheme based on the analysis and estimation of the variance components using Helmert method.

II.1. Physical model for simultaneous combination of station positions and velocities

Assuming that for each individual solution \( s \), and each point \( i \), we have position \( X^i_t \) at epoch \( t \), and velocity \( \dot{X}^i_t \), expressed in a given TRF \( k \).

The combination consists on estimating:

- Positions \( X^i_{itr} \) at a given epoch \( t_0 \) and velocities \( \dot{X}^i_{itr} \) in ITRS
- Transformation parameters \( T_k \) at an epoch \( t_k \) and their rates \( \dot{T}_k \), from the ITRF to each individual frame \( k \)

The general physical model used is given by the following equation (II.1):

\[
\begin{align*}
X^i_t &= X^i_{itr} + (t - t_0)\dot{X}^i_{itr} + T_k + D_k X^i_{itr} + R_k X^i_{itr} \\
&\quad + (t - t_k) \left[ \dot{T}_k + \dot{D}_k X^i_{itr} + \dot{R}_k X^i_{itr} \right] \\
\dot{X}^i_t &= \dot{X}^i_{itr} + \dot{T}_k + \dot{D}_k X^i_{itr} + \dot{R}_k X^i_{itr}
\end{align*}
\]  

(II.1)

where for each individual frame \( k \), \( D_k \) is the scale factor, the translation vector \( T_k \) and rotation matrix \( R_k \) are respectively defined (following IERS conventions) by:

\[
T_k = \begin{pmatrix} T^1_k \\ T^2_k \\ T^3_k \end{pmatrix} \quad \text{and} \quad R_k = \begin{pmatrix} 0 & -R_3_k & R_2_k \\ R_3_k & 0 & -R_1_k \\ -R_2_k & R_1_k & 0 \end{pmatrix}
\]

The dotted parameters designate their derivatives with respect to time. \( T^1, T^2, T^3 \) are the 3 origin components, \( R_1, R_2, R_3 \) are the three small rotations according to the 3 axes, respectively \( X, Y, Z \).
II.2. Statistical and variance estimation modeling

Combining heterogeneous data has been, since the beginning of ITRF combinations, one of the key element of ITRF analysis. The problem is, by essence, statistic. Up to now, the data were homogenized by the use of an empirical weighting scheme. For the ITRF96, a theoretical estimate of weights has been built and used in a rigorous stochastic framework.

The individual solutions, as well as the eccentricities are considered to be statistically independent populations. In addition, their variance matrix is supposed to be known except from a multiplicative factor (the variance factor). Therefore, for the \( \ell \)-th set, we can postulate\(^1\):

\[
\begin{align*}
E(Y_t) &= Y_t \\
\text{cov}(Y_k, Y_t) &= \delta^k_t \sigma^2_t \Sigma_t
\end{align*}
\]

where \( Y_t \) is the vector of observations (coordinates and velocities) in the \( \ell \)-th set and \( \delta^k_t \) is the Kronecker symbol (\( 1 \leq \ell \leq p \)). The variance factor \( \sigma^2_t \) is unknown and estimated iteratively through the Helmert-variance components estimation technique (Grafarend, 1984; Rao, 1973; Rao and Kleffe, 1988). The estimate \( \hat{s} \) of the vector \( s = (\sigma^2_1, \ldots, \sigma^2_p)^T \) is given by:

\[
\begin{align*}
\hat{s} &= \begin{pmatrix} \hat{\sigma}^2_1 \\ \vdots \\ \hat{\sigma}^2_p \end{pmatrix} = H^{-1}q \\
H_{t,k} &= \delta^k_s \left[ n_t - 2tr(\nu N_t) \right] + tr(\nu N_t \nu N_k) \\
q_t &= V_t^T \Sigma_t^{-1} V_t
\end{align*}
\]

where \( tr \) is the function trace and \( \nu \) is the inverse of normal matrix of the whole adjustment. \( N_t \) is the normal matrix of the \( \ell \)-th solution and for which \( n_t \) is the total number of observations; \( V_t \) is the position and velocity residual vector of the combination. \( \hat{s} \) is an unbiased estimate of \( s \) and the basic idea of constructing such estimator is to minimize, in the sense of the Gauss-Markov theorem, the variance of the resulting least square estimate (\( \nu \)).

\(^1E(Y)\) is the expectancy of the random variable \( Y \) and \( \hat{Y} \) is the exact value of \( Y \); \( \text{cov}(X, Y) \) is the covariance matrix of \( X \) and \( Y \).