The "geocenter motion" is the translation motion of the Earth's centre of mass as viewed from a crust fixed frame. In order to estimate the "geocenter motion" we have analysed SLR and GPS data with different approaches to define the best analysis procedure for the estimation of this parameter.

The name used to identify the different data analysis procedures are absolutely conventional for this document; we defined "geometric" and "gravitational" the procedures used in SLR data analysis, while the methods used in GPS data analysis are defined as "geometric" and "direct".

With the "gravitational" and the "direct" methods we directly estimate the time series of the "geocenter motion", while following the "geometric" method these parameters are obtained by a Helmert transformation of the coordinates of the stations estimated in the analysis to the ITRF94.

SLR DATA ANALYSIS

The models adopted in the SLR data analysis generally follow the IERS conventions (McCarthy D., 1996) with the exception of the Earth gravity model since the EGM96 is used. We have analysed the same data set with two different approaches: the "geometric" and the "gravitational".

"Geometric" method

The results reported have been obtained from the analysis of Lageos I and Lageos II SLR data available from January 1993 to December 1996. The software used is the NASA/GSFC GeodynII

In the EGM96 gravity model the coefficients of first degree are equal to zero which means that the centre of mass of the Earth coincides with its centre of volume; this allows to obtain the geocenter motion as a series of translation parameters (Tx, Ty, Tz) between the origin of the Terrestrial Reference Frame (TRF) realised by the SLR stations in each data reduction batch and the origin of ITRF94.

The station coordinates are estimated combining Lageos I and Lageos II data in independent 14-day batches. In addition to station coordinates, we estimate daily EOP, satellites ephemerides, station biases and along-track accelerations with a free-network approach (Heflin M.B. et al, 1992) weakly constraining the apriori station coordinates with a 10 meters sigma. The station velocities are held fixed at the ITRF94 values.

Following a free-network approach, for each solution we obtain a loosely defined internal references frame; a tight defined internal references frame could be obtained through the applications of internal constraints (rotational, translation and scale elements) to the covariance matrix obtained from the reduction procedure. The adopted mathematical procedures are described in Heflin M.B. et al., [1992], Gregorius T. [1996].

In order to obtain a homogeneous series of translation parameters any solution is then transformed into the ITRF94 frame by a Helmert transformation. The minimal constraints application and the Helmert transformation have been performed using GIPSY-OASIS II tools (Webb F.H. et al, 1995)
The mean offsets with respect to ITRF94 for the three series are:

- **Tx series**: \( \text{wmean} = -3.477 \text{ mm}, \text{wrms} = 5.506 \text{ mm} \)
- **Ty series**: \( \text{wmean} = -7.864 \text{ mm}, \text{wrms} = 9.463 \text{ mm} \)
- **Tz series**: \( \text{wmean} = 11.161 \text{ mm}, \text{wrms} = 17.811 \text{ mm} \)

The translation parameter series \((Tx,Ty,Tz)\) obtained by these transformations are shown in figure 1.

The free-network approach in data minimises the possibility to introduce frame errors; but, in order to have good results, it needs a well distributed global network and high quality data for each station. The outliers present in the estimated \(T_x\), \(T_y\) and \(T_z\) series are due to bad configurations of the analysed network.

**"Gravitational" method**

In this SLR solution, the data set covers the same period of the previous solution (January 1993 - December 1996), the software used is the NASA/GSFC Geodyn II/Solve. The analysis procedure can be divided into two phases: arc solution and global solution. In the arc solution we separately analyse the data of Lageos I and Lageos II satellites in 14-day independent batches; the normal equations are built up for all the parameters to be estimated but, at this stage, they are solved only for the parameters related to the orbit (state vectors and empirical along-track accelerations) and to the laser tracking network (station range bias). In a global solution all the matrices are combined with Solve in a global matrix; the Terrestrial References Frame is defined by fixing the ITRF94 coordinates of Greenbelt (7105) and Herstmonceux (7840). The global matrix is inverted by Solve to estimate the site position at a references epoch (1993.0), the daily EOP parameters and the \(C_{10}, C_{11}, S_{11}\) geopotential coefficients every 14 days.

These coefficients are proportional to global translation of the terrestrial references system and therefore they describe the motion of the geocenter.

The \(T_x\), \(T_y\) and \(T_z\) series are obtained from \(C_{10}, C_{11}, S_{11}\) applying the following equations (Heiskanen W.A. et al, 1966):

\[
T_x = \alpha \cdot \sqrt[3]{C_{10}} \\
T_y = \alpha \cdot \sqrt[3]{S_{11}} \\
T_z = \alpha \cdot \sqrt[3]{C_{11}}
\]

\(\alpha = 6378.1364 \text{ Km}\) is the mean terrestrial radius.

These values represent the coordinates of the center of mass in the TRF used in Solve. In order to have time series comparable to those obtained with the "geometric" method we calculate the Helmert transformation parameters between this TRF and ITRF94. The values of this translation are added to those obtained by (1).

The \(T_x, T_y\) and \(T_z\) offsets, in terms of \text{wmean} and \text{wrms} with respect to ITRF94, are:

- **Tx series**: \( \text{wmean} = -6.253 \text{ mm}, \text{wrms} = 8.438 \text{ mm} \)
- **Ty series**: \( \text{wmean} = -3.243 \text{ mm}, \text{wrms} = 6.093 \text{ mm} \)
- **Tz series**: \( \text{wmean} = 15.306 \text{ mm}, \text{wrms} = 21.704 \text{ mm} \)

The series are shown in figure 1.

**Comments**

The two time series obtained from SLR data analysis show a good agreement in terms of signal content but in the series obtained with the "gravitational" method the statistical error in the XY plane is 3 times lower than the estimated error in the series obtained with the "geometric" method and in the Z direction the error improvement is of the order of 40%.
GPS DATA ANALYSIS

We use two different analysis approaches also in the GPS data analysis using the GIPSY-OASIS II software (Webb.F.H., et al, 1995). In both solutions, here defined as “geometric” and “direct”, we have generally adopted the models recommended in the IERS conventions (McCarthy D., 1996). For the solid Earth tide we have used the Williams model. The analysed data set (daily Rinex observations compressed in 5 minutes NP) covers the period from 14 July 1996 to 11 January 1997 (six month of data) coming from a network of stations selected considering a global and homogeneous distribution. The entire data set has been analysed in daily batches.

In the two solutions we solve for the following parameters: satellites and station clocks (one clock has been kept fixed as reference), tropospheric zenith path delay, phase ambiguity, satellites state vectors, solar radiation pressure, pole coordinates and rates, UT1-UTC rate.

"Geometric" method

The “geometric” method estimates the station coordinates using a free-network approach with apriori sigma to Station coordinates from 10 meters to 1 km (Vanicek P. et al 1986). Each daily independent station coordinate estimation is then transformed into a defined references frame following the same procedure described for SLR: minimal inner constraints using the full estimated covariance matrix and Helmert transformation. The offsets obtained in the three GPS series with respect to ITRF94 are:

- Tx series: wmean = 0.100 cm, wrms = 4.741 cm
- Ty series: wmean = 1.200 cm, wrms = 7.281 cm
- Tz series: wmean = -7.771 cm, wrms = 14.770 cm

"Direct” method

The “direct” method directly estimates, in addition to the parameters previous described, the offset between the centre of mass of the Earth ad the ITRF94. In this procedure we used 17 stations globally distributed whose coordinates have been kept fixed at the ITRF94 values and propagated at each epoch of the analysed data. The wmean and wrms obtained for the series of this solution are:

- Tx series: wmean = -2.331 cm, wrms = 4.860 cm
- Ty series: wmean = -0.910 cm, wrms = 4.951 cm
- Tz series: wmean = -6.871 cm, wrms = 8.920 cm

Comments

The two series of parameters (figure 2) obtained from GPS are quite comparable; they show a good agreement in the Tx and Ty series even if the Ty series obtained with the ‘geometric’ method is a little bit noisy. In the Tz series it is evident that the results obtained with the ‘direct’ method are better.
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Fig. 1 The two time series obtained from SLR data analysis show a good agreement in terms of signal content. With the "gravitational" method the statistical error in the XY plane is 3 times lower than the estimated error in the series obtained with the "geometric" method and in the Z direction the error improvement is of the order of 40%.
Fig. 2 The two series of parameters obtained from GPS are quite comparable; they show a good agreement in Tx and Ty series even if the Ty series obtained with the 'geometric' method is a little bit noisy. In the Tz series it is evident that the results obtained with the 'direct' method are better.