INTERPRETATION OF HIGH FREQUENCY POLAR MOTION AND LENGTH OF DAY VARIATIONS

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Abstract: The continuous GPS observations of 1995 completed by the International GPS Service are analysed in terms of polar motion (PM) and length of day (LOD) in view of diurnal and subdiurnal periods first of all. The complex study of these two phenomena shows that

- the most prominent diurnal and subdiurnal components of the record can be found at the tidal frequencies. In case of PM the zonal (long periodic) tides were not detected due to the fairly high noise level. The tesseral waves are of Earth tidal origin (their amplitudes are around 1 mas). The sectorial tidal peaks are generated by oceanic tides and are the order of 0.1–1 mas. The most significant LOD spectral amplitude is at the frequency of the fortnightly Mf wave (~10^{-3} s) but significant LOD variations occur in the semi-diurnal band: for: M2 780.1 mas was found.
- regular not explained yet high frequency anomalies were detected in the frequency band 3-11 cycles/day. It can not be excluded that such effects are present at tidal frequency band too. This circumstance not allows an unambiguous interpretation of the Lunisolar spectral peaks.
- The LOD time series was carefully compared with the seismological data catalogue of the US Geological Survey. It was not possible to found any short periodic (diurnal-subdiurnal) signal which can be connected to earthquake activity.

Keywords: Earth rotation, polar motion, tides, Fourier and wavelet analysis.

1. INTRODUCTION

The study of the short periodic variations of the irregularities of the Earth rotation started in 1935, when Pavel and Uhink (1935) discovered the annual term in this phenomenon. The study of variations in the subannual frequency band of the Earth rotation and its interpretation became possible with the appearance of very long baseline interferometry (VLBI) and methods of space geodesy like satellite and lunar laser ranging (SLR and LLR) (Eubanks et al., 1988; Dickey et al., 1994; Gross, 1993; Hide et al., 1991, Kolaczeck, 1995; Popinski et al., 1994). Precise LOD measurements have showed variations in angular frequency down to days and even sub-daily (Chao et al., 1995a,c ; Barnes et al., 1983; Kotaczek, 1995). In the paper of Chao and Gross (1995) the short periodic changes in the Earth’s rotational energy, which induce earthquakes, are given.

In the study of short-periodic variations of LOD and polar motion (PM) the GPS hourly values of these phenomena produced by the International GPS Service (IGS) are of first importance. The IGS starts its activity in June 1992. At present the formal errors of its EOP data are less than 10 microseconds of arc (μas) and in the case of LOD data they are less than 10 microseconds (μs) (Springer et al., 1999). The IGS data are based on the world-wide network of globally distributed GPS tracking stations (Neilan, 1998; Weber, 1999).

The aim of the present work is the complex study of the PM together with LOD in the tidal frequency band with the use of IGS data set for 1995.

2. NUMERICAL METHODS USED TO STUDY THE HIGH FREQUENCY PART OF PM AND LOD SPECTRA

The data processing procedure used in this study can be subdivided into four steps:
   a) Determination and removal of the annual part.
   b) Data filtering to obtain the high frequency components.
   c) Power spectra calculation for PM and LOD based on Fourier analysis and Wavelet technique.

a) Determination and removal of the annual part

Before examining the high frequency processes the trend has to be removed from the original data set. It was found that the accurate removal of the annual term improves significantly the analysis results at high (diurnal, subdiurnal) frequencies. For this purpose polynomials of different degrees are recommended by various authors. An uncertainty is resulting here from the choice of the degree of the polynomial. It was proved by our experience that using a polynomial of degree 10 selected by least-squares adjustment gives satisfactory results. To prove this also a generalized spline interpolation method was developed for long-term trend removal. The conventional interpolation procedures show undesirable oscillations, especially at the beginning and the end of the records. This boundary problem is not present in the method described below. The time series is divided into three subintervals of approximately equal length. For each subintervals polynomials of degree 3 were determined by least-squares adjustment. To ensure the continuity of the function of the interpolation, the interpolant and its derivatives have to satisfy appropriate equations. The procedure is iterative: in the course of the consecutive steps the coordinates of the pole receive a correction in inverse ratio to their distance from the trend function. With the use of this method the gross errors (outliers) can be detected and removed.

b) Data filtering to obtain the diurnal and subdiurnal components

The next step in the data processing is the filtering of the given frequency range of the pole-coordinates and LOD data. There are many solutions described in the literature for filtering polar motion and length of day data (e.g. Gambis 1992; Gilbert et al. 1998; Kosek 1995). In the present study two different methods were used: the Butterworth high-pass filter and the trigonometric interpolation. In the second case the pole coordinates and the LOD data sets constitute a stochastic process with a zero mean value and the interpolant is a cosine function with unknown amplitude and phase, but with a positive integer period. The amplitude and phase are estimated from the observed GPS data with the least-squares method. This procedure allows to remove components of a given period. For the purpose of present study periods longer than 30 days were removed from the records.

c) Determination of the power spectra of PM and LOD

The Fourier transformation is a common procedure in digital data processing. The Fourier transformation method has some significant limitations such as the requirement of uninterrupted and evenly input data. Another limitation is connected with the stationarity. For a stationary process the frequency decomposition generally produces sufficient information. Processes in the nature, however, are usually not stationary: the characteristics of the analysed signal may show time dependency. This phenomenon is not expressed in Fourier spectra. (Arfa-Kaboodvand, Groten, 1999).
Wavelet analysis with Morlet and Mexican hat mother functions

In case of phenomenon for which the spectral composition changes in time, it is necessary to use time-dependent frequency analysis. The PM and LOD temporal variations are typically such a process. In this case, our goal was to apply to the pole coordinates and to the length of day data the wavelet transformation acting in the time-frequency domain. The main advantage of the wavelet transformation is that it identifies time and frequency concurrently. The signal to be analysed (in case of present work the PM and LOD) is evaluated by a mother function in both time and frequency domain. In the frame of present study different basis functions have been used for the wavelet analysis of the Earth’s pole position and of the length of day (Morlet, Arfa-Kaboodvand and Groten, 1998). Further investigations could be done to answer the question: what is the highest resolution of GPS data to obtain useful information in frequency domain?

3. GEODYNAMICAL PROCESSES INFLUENCING THE POLAR MOTION AND LENGTH OF DAY IN TIDAL FREQUENCY DOMAIN

a) Earth tides

With the use of two horizontal components of the Earth tidal acceleration the latitude \( \phi \) dependence of the tides can be expressed as follows:

\[
\begin{align*}
\text{X (longitudinal) component} & \quad 1.5A \sin 2\phi \\
\text{Y (meridional) component} & \quad 0 \\
\text{Long-periodic tide} & \\
\text{Diurnal tide} & \quad A \cos 2\phi \\
\text{Semi-diurnal tide} & \quad 0.5A \sin 2\phi \\
\text{(A is the amplitude)} & \quad A \cos \phi
\end{align*}
\]

It follows from above expressions, that the horizontal acceleration components of the Earth tide do not generate PM neither in the case of zonal, long-periodic waves (the two principal long-periodic tidal constituents are the monthly \( M_m \) and the fortnightly \( M_f \)) nor in the case of the semi-diurnal, sectorial waves (the biggest constituents are the principal lunar \( M_2 \), the principal solar \( S_2 \) and the principal lunar ellipsoidal \( N_2 \)). The tesseral, diurnal waves (the main waves are the main lunar \( O_1 \) and the lunar-solar declinational \( K_1 \)) are able to produce polar motion. Their amplitudes are \(-7.3\ \sin 2\delta \) mas in case of X(N-S) and \(17.3\ \sin 2\delta \) mas in case of Y(E-W) components (\( \delta \) is the declination of the tide generating body) (Melchior, 1981). From physical point of view, it is clear that the long-periodic and semi-diurnal earth tides – due to their symmetry relative to the poles – are not able to generate any PM.

The long-periodic tide causes axis-symmetric tidal bulge of the solid Earth that produce directly changes in the polar moment of inertia of the Earth. According to the third law of Euler on the angular moment conservation the product of the Earth’s polar moment of inertia (\( C \)) and the angular velocity (\( \omega \)) is constant. Consequently

\[
\frac{\Delta C}{C} = -\frac{\Delta \omega}{\omega} = \frac{\Delta (LOD)}{LOD}
\]

The variation of LOD appears in case of Earth tides for the zonal, long-periodic waves alone. The amplitude of the monthly \( M_m \) and the fortnightly \( M_f \) constituents is of the order of 2-3 ms. The one year long GPS data set used in this study seems to be not optimal for the study of the long-periodic zonal tidal effects on LOD (the period of \( M_m \) wave is \(~27.6 \) days that of \( M_f \) is \(~13.6 \) days).
b) Oceanic tides

The amplitude of PM, generated by the oceanic tides, can be obtained with the use of tidal load theory. Let us consider a spherical layer on the surface of the Earth characterised by a centri angle \( \alpha = \frac{\Delta \phi \Delta \lambda}{\pi} \) (where \( \lambda \) is the longitude). In this case at the \( \Psi \) angular distance from the centre of the loaded area the generated potential \( V_L \) is

\[
V_L = \frac{4\pi G R_E \rho}{g(R_E)} \sum_{n=0}^{\infty} J_n(\Psi) \frac{1}{2n+1}
\]

Here \( G \) is the gravitational constant, \( R_E \) is the mean Earth’s radius, \( \rho \) is the density of the seawater, \( g \) is the gravity, while

\[
H_n(\Psi) = \frac{1}{2} H \sum_{n=0}^{\infty} [P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)] \cdot P_n(\cos \Psi)
\]

(3)

Where \( H \) is the amplitude of the tide on the loaded spherical layer characterised with centri angle \( \alpha \) and \( P_n \) is the \( n \)-th Legendre polynomial. Taking into account the features of the Legendre polynomials (Hobson, 1955) the potential of the load can be written as

\[
V_L = 2\pi G \rho R_E H \sum_{n=0}^{\infty} \frac{1}{2n+1} [P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)] P_n(\cos \Psi)
\]

(4)

The radial and the horizontal displacements \( X \) and \( Y \) can be written in the following form for an elastic Earth (Pertzev et al., 1991):

\[
S_r = 2\pi G \rho R_E H \sum_{n=1}^{\infty} \frac{h'_n}{2n+1} [P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)] P_n(\cos \Psi)
\]

\[
S_\phi = \frac{2\pi G \rho R_E H}{g(R_E)} \sin^2 \alpha \sin \phi \sum_{n=1}^{\infty} n(n+1) \cdot \frac{\ell'_n}{n(n+1)} \cdot \frac{dP_n(\cos \alpha)}{d(\cos \alpha)} \cdot \frac{dP_n(\cos \psi)}{d(\cos \psi)}
\]

(5)

The radial and the horizontal displacements \( X \) and \( Y \) can be written in the following form for an elastic Earth (Pertzev et al., 1991):

\[
S_r = 2\pi G \rho R_E H \sum_{n=1}^{\infty} \frac{h'_n}{2n+1} [P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)] P_n(\cos \Psi)
\]

\[
S_\phi = \frac{2\pi G \rho R_E H}{g(R_E)} \sin^2 \alpha \sin \phi \sum_{n=1}^{\infty} n(n+1) \cdot \frac{\ell'_n}{n(n+1)} \cdot \frac{dP_n(\cos \alpha)}{d(\cos \alpha)} \cdot \frac{dP_n(\cos \psi)}{d(\cos \psi)}
\]

\[
S_\lambda = 0
\]

where \( h'_n \) and \( \ell'_n \) are the load numbers.

With the use of above expressions for the displacements the strain components are

\[
e_{\phi \phi} = \frac{S_r}{R_E} + \frac{1}{R_E} \frac{dS_\phi}{d \phi}
\]

\[
e_{\lambda \lambda} = \frac{S_r + \cot g \phi \cdot S_\phi}{R_E}
\]

(6)

The amplitude value \( H \) can be obtained from the co-tidal maps, the load numbers \( h'_n \) and \( \ell'_n \) can be found in many theoretical studies (e.g. Jentzsch, 1997; Grafarend et al., 1997). To estimate the magnitude of the polar motion \( r = \sqrt{x^2 + y^2} \) due to oceanic tide the co-tidal maps of Schwiderski (1989) for \( M_2 \) and \( O_1 \) waves have been divided by 420 spherical segments \( 10^\circ \times 10^\circ \). It was found that the amplitude of PM generated by \( M_2 \) tide is of the order of \( 15 \cdot 10^{-4} \) as while the polar displacement by the oceanic \( O_1 \) wave is 10 times smaller. It follows from considerations of this chapter that the PM in case of diurnal tides can be generated mainly by Earth tides, while in case of semi-diurnal tides by the oceanic ones.
4. INTERPRETATION OF VARIATIONS IN POLAR MOTION AND IN LENGTH OF DAY

The radius of the "annual polar motion circle" varied during 1995 between (0.2-0.4) as. After the removal of the "annual" component the total residual can be characterised by radius 10 to 20 mas.

According to signal to noise studies carried out in frame of present study the PM data provided by the IGS are accurate to about 0.1 mas.

For the better understanding of the physical background of daily and subdaily variations the temporal changes of DM and LOD in frequency band of tidal variations were investigated. Figure 1 shows PM energy spectra for the long periodic, diurnal, semi-diurnal and subtidal frequencies.

Fig. 1. Power spectrum of polar motion data of the semi-diurnal and diurnal frequency bands. Similar energy spectra for LOD are collected together in Fig. 2.
Fig. 2. Power spectrum of LOD data of the semi-diurnal and diurnal frequency bands. 

In Table 1 PM and LOD spectral peak amplitudes are shown in four different frequency bands.

Table 1. Amplitudes of tidal and non-tidal harmonic components for polar motion (PM) (in 10^{-4} as) and length of day (LOD) (in 10^{-4} s).
Frequencies are in °/h

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>PM</th>
<th>LOD</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long periodic terms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5444</td>
<td>not detected</td>
<td>4.55</td>
<td>M_m (27.55d)</td>
</tr>
<tr>
<td>1.0983</td>
<td>not detected</td>
<td>11.40</td>
<td>M_f (13.66d)</td>
</tr>
<tr>
<td><strong>Diurnal components</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.9430</td>
<td>8.05</td>
<td>3.71</td>
<td>O_1</td>
</tr>
<tr>
<td>15.0410</td>
<td>9.10</td>
<td>3.81</td>
<td>K_1</td>
</tr>
<tr>
<td><strong>Semi-diurnal components</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.4397</td>
<td>3.38</td>
<td>1.92</td>
<td>N_2</td>
</tr>
<tr>
<td>28.9841</td>
<td>17.10</td>
<td>7.75</td>
<td>M_2</td>
</tr>
<tr>
<td>30.0000</td>
<td>9.52</td>
<td>2.61</td>
<td>S_2</td>
</tr>
<tr>
<td><strong>Subtidal frequencies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.</td>
<td>4.10</td>
<td>2.80</td>
<td></td>
</tr>
<tr>
<td>60.</td>
<td>2.80</td>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td>75.</td>
<td>3.10</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>90.</td>
<td>2.10</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>105.</td>
<td>3.10</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>120.</td>
<td>2.10</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>135.</td>
<td>1.90</td>
<td>1.85</td>
<td></td>
</tr>
<tr>
<td>150.</td>
<td>1.10</td>
<td>1.80</td>
<td></td>
</tr>
</tbody>
</table>

In the frequency band of long-periodic tides (up to 0.2 cycles/day) LOD data were detected. The PM spectrum at this range is fairly noisy. The most significant LOD peaks are connected with M_m and M_f tidal waves. To obtain realistic spectral peaks for long-periodic band of PM a significant section of it was filtered out. This way a reliable peak for the fortnightly wave M_f was detected.

PM anomalies of diurnal frequency band – as it follows from section 3 – may be of Earth tidal origin. The oceanic tidal influence in principle can generate semi-diurnal LOD components. In semi-diurnal frequency band the biggest anomaly coincides with the frequency of the main lunar wave M_2. It is remarkable that the S_2 constituent, together with S_1, is much bigger as it is acceptable on the basis of the theory of terrestrial and oceanic tides. This phenomenon possibly can be explained with meteorological influences to a and less extent by atmospheric tides (Zharov et al., 1996). Systematic, until now not explained, anomalies were found in the PM and LOD spectra in higher frequencies as the tidal frequency bands, i.e. for 3, 4, 5, 6, 7, 8, 9 and 10 cycles/day (the corresponding angular frequencies are 45, 60, 75, 90, 105, 120, 135, 150 °/h). The amplitudes in this part of Fourier spectra are significantly big.

Some examples of wavelets calculated by base functions Morlet are shown on Fig. 3.
Fig. 3. Energy density values of Earth rotation data (1995) based on Morlet Wavelet technique; maximum prograde (upper diagrams, positive periods) energy density value lies with the time span and the maximum retrograde (lower diagrams, negative periods) energy density value lies with the time span.

It should be mentioned here, that two different wavelet base functions – Morlet and Mexican hat (Arfa-Kaboodvand, Groten, 1998) show similar results. For the PM it can be concluded that – the strongest and most stable signals are the semi-diurnal prograde and the diurnal retrograde one. Anomalies of another frequency bands – like the diurnal prograde and semi-diurnal retrograde – are present only temporarily. This circumstance proves, that the tidal signal in PM undergo significant irregular – possibly meteorological – influences (Arfa-Kaboodvand, et al., 1999). In the diurnal/semi-diurnal frequency band no significant non-tidal signals were detected above the noise level of 0.1 mas.
5. STUDY OF THE POSSIBLE INFLUENCE OF EARTHQUAKE ON LOD

The influence of earthquakes on short periodic PM and LOD is discussed in many scientific publications (Manshina et al., 1967; Runcorn, 1970; Smylie et al., 1971; Chao et al., 1995).

For the purpose of present research the catalogue of seismic events for 1995 published by the US Geological Survey National Earthquake Information Center has been used. In spite of the fact that during the year 1995 many significant events occurred (first of all should be mentioned the Kuril islands event on 2nd December with a magnitude $M_w = 7.8-7.9$) until now there were not LOD anomalies detected in the frame of present research which can be connected to seismic events without doubt.

From Eqs 2 and 3 - if instead of $H$ the normal hydrostatic stress $P_N = \rho g H$ is introduced the following surface level variation of the loaded area with centri angle $\alpha$ can be obtained (Varga, 1988):

$$D_N = \frac{4\pi G R E}{g^2} \sum_{n=0}^{\infty} \frac{h_n^N}{2n+1} P_N (cos \psi)$$

(7)

Similar equation holds for the horizontal stress caused surface level variations:

$$D_T = \frac{4\pi G R E}{g^2} \sum_{n=0}^{\infty} \frac{h_n^T \cdot n \cdot (n+1)}{2n+1} P_M (cos \psi)$$

(8)

In above two equations $h_n^N$ and $h_n^T$ are the normal and horizontal spheroidal stress load numbers, which describes stress accumulation not connected to mass load (Varga, 1988). $P_N$ and $P_M$ have the meaning as holds in Eq. 7 for level variations. Changes in LOD are directly connected to the variations of the polar moment of inertia due to the stress load acting on a spherical segment:

$$\Delta C = \int_0^{r+D} \int_0^\lambda \int_0^{\lambda_2} \rho r^4 \cos^3 \varphi d\varphi d\lambda d\varphi =$$

$$= \frac{(r + d)^5 - r^5}{5} \rho \Delta \lambda \left[ (\sin \varphi_2 - \sin \varphi_1) - \frac{1}{3} (\sin^3 \varphi_2 - \sin^3 \varphi_1) \right]$$

(9)

Since

$$(r + D)^5 - r^5 \approx r^4 \left[ 1 + \frac{D}{r} \right]^5 - 1 \approx 5r^4 D$$

(10)

$\Delta C$ can be written as

$$\Delta C = r^4 \rho D \Delta \lambda \left[ (\sin \varphi_2 - \sin \varphi_1) - \frac{1}{3} (\sin^3 \varphi_2 - \sin^3 \varphi_1) \right]$$

(11)

Actually the area involved in the surface level variation is sufficiently larger than the territory where the stress accumulation takes place. So the integration along the Earth surface in case of loaded by $D_N$ squares $10^\circ \times 10^\circ$, $1^\circ \times 1^\circ$ and $0.1^\circ \times 0.1^\circ$ leads to following results:
<table>
<thead>
<tr>
<th>Area of stress Accumulation</th>
<th>Corresponding LOD variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6 \times 10^6$ ($\approx 10^9 \text{km}^2$)</td>
<td>$10^4-10^5 \text{s}$</td>
</tr>
<tr>
<td>$1^\circ \times 1^\circ$ ($\approx 10^4 \text{km}^2$)</td>
<td>$10^6-10^7 \text{s}$</td>
</tr>
<tr>
<td>$0.1^\circ \times 0.1^\circ$ ($\approx 10^2 \text{km}^2$)</td>
<td>$10^7-10^8 \text{s}$</td>
</tr>
</tbody>
</table>

As it was mentioned in Section 4 the accuracy of the dataset of LOD is accurate to about $10^{-8} \text{s}$. Due to the fact that no short periodic subdiurnal LOD anomaly was detected above this level in case of Kuril event of 0.2.12.1995 it can be concluded that the area involved into stress accumulation – even in case of such a significant earthquake – is not bigger as 100 km$^2$, because the corresponding $\Delta$LOD is less than $10^{-8} \text{s}$.

6. CONCLUSIONS

The usual way of the study of short periodic variations in Earth rotation is connected with LOD investigations. In this paper changes in length of day were studied together with polar motion data. The following conclusions were formulated.

a) The strongest peaks of high frequency part of the LOD and PM data appears at lunisolar frequencies. The amplitudes of the other anomalies of Fourier spectra are smaller.

b) The nature of systematically distributed anomalies at frequencies 3, 4, 5, 6, 7, 8, 9 and 10 cycle/day is not explained satisfactory yet.

c) In the LOD data set for 1995 it was not possible to find any signal which can be connected to earthquakes without doubt.

Acknowledgements

We appreciate support of Internations Büro of BMBF/Bonn and Deutsche Forschungsgemeinschaft as well as the Hungarian Committee for Scientific and Technological Development.

The activity described in this work was also supported by the Hungarian Science Foundation (Project: OTKA T029049).

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