Practical Consequences of Resolutions
B1.3, B1.4, B1.5 and B1.9
Concerning “Relativity in Astrometry, Celestial Mechanics and Metrology”

Michael H. Soffel

Lohrmann observatory, TU Dresden
e-mail: soffel@rcs.urz.tu-dresden.de

1 Resolutions

Many arguments can be given why the old IAU 1991 Resolutions concerning relativity are not sufficient for present and future applications, both because of conceptual reasons and considerations of accuracy. The new IAU framework is given by 4 Resolutions that were adopted at the GA 2000.

1.1 Resolution B1.3

Resolution B1.3 defines the barycentric celestial reference system BCRS and the geocentric celestial reference system GCRS. The BCRS might be associated with a celestial sphere of remote astronomical objects (quasars etc.) and represents the ICRS. It should be used as basic reference system for astrometry and solar system ephemerides. On the other hand the GCRS is only a local system moving with the geocenter. Dynamical processes occuring in the Earth’s system and its immediate vicinity such as the dynamics of the Earth and its constituents should be referred to the GCRS. The GCRS replaces classical concepts used for the definition of classical Earth orientation parameters (such as ‘ecliptic and equinox at a certain epoch etc.). Note, that the coordinates of the BCRS and of the GCRS are related by a complex 4-dimensional space-time transformation.

1.2 Resolution B1.4

Resolution B1.4 defines post-Newtonian potential coefficients for the Earth. They generalize the classical potential coefficients $C_{lm}$ and $S_{lm}$ in the expansion of the Newtonian potential that is now replaced by $W_E(T, X)$. They are chosen to keep deviations from the Newtonian framework at a minimum. Note, that in the old IAU framework not even the mass of the Earth was defined with post-Newtonian accuracy. The gravito-magnetic potential $W^g_E$ is expressed in terms of the spin vector $S_E$ of the Earth. The recommended expansions of $W^g_E$ lead to the post-Newtonian accelerations of Earth’s satellites including the usual Schwarzschild and Lense-Thirring terms.

1.3 Resolution B1.5

Resolution B1.5 applies the formalism recommended in Resolutions B1.3 and B1.4 to the problem of time transformations and realizations of coordinate times in the solar system.
1.4 Resolution B1.9

Resolution B1.9 contains a re-definition of Terrestrial Time $TT$. It is recommended that $TT$ be a time scale differing from $TCG$ by a constant rate $d(TT)/d(TCG) = 1 - L_G$, where $L_G = 6.969290134 \times 10^{-10}$ is a defining constant. This removes the reference to the geoid in the $TT$ definition.

2 Practical aspects

Fields of applications concern: 1. clock rates, definition and realization of time scale in the solar system, 2. astrometry and problems of reference systems and 3. celestial mechanics. A very detailed explanatory supplement concerning these new resolutions will be submitted to the Astronomical Journal in the near future.

2.1 Clock rates, definition and realization of time scales

Various aspects of $TCG$, $TT$, $TAI$, $TCB$ and proper time have been treated in detail in the Resolutions.

2.2 Astrometry and problems of reference systems

2.2.1 The celestial sphere

Usually one considers the solar system to be isolated, i.e., one neglects other stars, galaxies etc. In other words one usually forgets about the cosmological problem. In that case the BCRS is a global coordinate system that contains all the 'far away regions'. Consider some light-ray in the BCRS and follow it back in time. That region where all light-rays originate might be called the celestial sphere. Since the BCRS is a global coordinate system it contains the celestial sphere. In practice the BCRS represents a theoretical backbone of the ICRS.

The situation, however, is very different for the GCRS. The origin of the GCRS is accelerated with $g \sim GM_S/(AU)^2$ and one can show that any reasonably defined accelerated coordinates become meaningless for $d \gg c^2/g$. Accelerated coordinates are only local ones and the remote celestial spheres cannot be described in the GCRS.

For the problem of Earth’s rotation and the introduction of Earth’s orientation parameters the GCRS is the primary celestial system to be used. Only in a second step by means of space-time coordinate transformations the relations with the ICRS/BCRS can be obtained.

2.2.2 Observables - coordinate quantities

In relativity one has to distinguish carefully observables from coordinate dependent objects. In astrometry one fundamental observable is the angle between two incident light-rays as seen by some observer. This observable can be derived in any coordinate system as a coordinate independent object. Usually, however, in the field of astrometry one talks about a large number of quantities, such as parallax, proper motion or radial velocity that are pure coordinate objects. For that reason it is essential to fix the coordinates (theoretically by the corresponding metric tensors) by convention.

Other coordinate objects have to be introduced for the description of Earth’s rotation. The GCRS coordinates $X$ clearly can be used for the introduction of diverse quantities, like polar angles $r, \theta, \phi$ and a corresponding directional sphere, coordinate vectors $X = (X, Y, Z)$, the equator of the GCRS etc. The
EOP should be defined with respect to the GCRS and NOT with respect to the ICRS/BCRS.

2.2.3 Astrometry at the $\mu$as level

The resolutions allow for the calculation of practically every astrometric effect at the $\mu$as level. Details can be found in the literature.

2.3 Celestial mechanics

2.3.1 Ecliptic, equator, equinox etc.

For celestial mechanics the travel time of some electromagnetic signal between various points in space as measured by some observer is a fundamental observable. This observable can be derived directly from the metric tensor and the equations for photons and the observer. Obviously in celestial mechanics one talks about diverse quantities such as orbits of satellites, planets, spacecrafts etc., that are mere coordinate objects. The same applies for concepts like the ecliptic or some celestial equator. An ecliptic might be defined as $t = TCB = \text{const.}$ coordinate plane by means of $x_E(t_0)$ and $v_E(t_0)$ in the BCRS. I.e., one is forced to consider the ecliptic only as a BCRS coordinate object related with the Earth’s ephemeris. A transformation of this object into the GCRS yields nothing reasonable. On the other hand some celestial equator might be defined in the GCRS only. The classical equinox should be considered as an obsolete quantity and should not be used in the future.

2.3.2 Equations of motion

The new resolutions lead to post-Newtonian equations of motion for solar system bodies. Keeping only the masses in the BCRS equations of motion it can be shown that they reduce to the well known EIH equations of motion that have been used for the JPL DE ephemerides. In satellite theory the post-Newtonian Schwarzschild acceleration caused by the mass of the Earth and the Lense-Thirring acceleration due to the Earth’s total angular momentum that dominate the relativistic effects in the orbital motion can be derived from the resolutions. Expressions for post-Newtonian tidal forces that can also be obtained from the resolutions can be found in the literature.

Finally I would like to state that much more work is needed in order to formulate theories of Earth’s rotation compatible with the resolutions. Even the best theories for the rotational motion of a rigid Earth definitively are NOT compatible though all the tools are available for that goal.

3 Consequences for the IERS

Presently used models for observing techniques (VLBI etc.) are practically in agreement with the IAU2000 resolutions. Presently used theories of Earth’s rotation, however, are not. To be consistent the two celestial systems, BCRS and GCRS, should carefully be distinguished. This involves a clear distinction between TCG and TCB.