Practical Consequences of Resolution B1.6 "IAU2000 Precession-Nutation Model," Resolution B1.7 "Definition of Celestial Intermediate Pole," and Resolution B1.8 "Definition and Use of Celestial and Terrestrial Ephemeris Origin"

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1 Practical consequences of Resolution B1.6 "IAU2000 Precession-Nutation Model"

Resolution B1.6 (see Annex) recommends a new precession/nutation model.

Beginning on 1 January 2003, the IAU 1976 Precession Model and the IAU 1980 Theory of Nutation are replaced by the precession-nutation model IAU 2000 A (MHB 2000) based on the transfer functions of Mathews *et al.* (2002) for those who need a model at the 0.2 mas level. Alternatively one can use a shorter version IAU 2000 B (McCarthy and Luzum, 2002) if precision at the 1 mas level is required. In both cases the associated celestial pole biases must be used.

The IAU2000 A model contains 678 luni-solar terms and 687 planetary terms and provides the direction of the celestial pole in the Geocentric Celestial Reference System (GCRS) with an accuracy of 0.2 mas. The series includes the geodesic nutation contributions to the annual, semi-annual, and 18.6-year terms. (Fukushima 1991). The Free Core Nutation (FCN), being a free motion that cannot be predicted rigorously, is not a part of the IAU2000A model.

The abridged model, designated IAU2000B includes 80 luni-solar terms plus a planetary bias to account for the effect of the planetary terms in the time period under consideration. It provides the celestial pole motion with an accuracy that does not result in a difference greater than 1 mas with respect to the IAU2000A model during the period 1995-2050.

Currently the International Earth Rotation Service (IERS) provides “celestial pole offsets” for routine use in transformations between the terrestrial and celestial reference systems. These are corrections to the angles ψ (longitude) and ε (obliquity) that one currently computes using the IAU 1976 Precession (Lieske *et al.* 1977) and the IAU 1980 Nutation Theory (Seidelmann, 1982). For most users these corrections will no longer be necessary if they use the IAU2000A/B models. The IERS will continue to publish celestial pole offsets with respect to IAU2000A, but these are expected to be less than 1 mas.

The IAU 2000 series of nutation must be used with improved numerical values for the precession rate of the equator in longitude and obliquity:

$\Delta\psi_A = (-0.2997 \pm 0.0008)$ "/c, and $\Delta\omega_A = (-0.0252 \pm 0.0003)$"/c,

as well as with the following biases in the direction of the Celestial Intermediate Pole (CIP) at J2000.0 from the direction of the pole of the Geocentric Celestial Reference System (GCRS)

$\xi_0 = (-0.016617 \pm 0.000010)$", and $\eta_0 = (-0.006819 \pm 0.000010)$".

The IAU 2000 nutation model is given by series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\varepsilon$, referred to the mean equator and equinox of date, with t measured in Julian centuries from epoch J2000.0:

$$\Delta\psi = \sum_{i=1}^{N} (A_i + A_i' t)\sin(\text{ARGUMENT}) + (A_i'' + A_i''' t)\cos(\text{ARGUMENT}),$$

$$\Delta\varepsilon = \sum_{i=1}^{N} (B_i + B_i' t)\cos(\text{ARGUMENT}) + (B_i'' + B_i''' t)\sin(\text{ARGUMENT}).$$

The IAU2000A subroutine, provided by T. Herring, is available electronically on the IERS Convention Center website at
ftp://maia.usno.navy.mil/conv2000/chapter5/IAU2000A.f. It produces the total nutation in longitude and obliquity based on the adopted MHB2000 model with the exception of the FCN. The software can also be used to model the expected FCN based on the most recent astronomical observations. The IAU2000B subroutine is available at ftp://maia.usno.navy.mil/conv2000/chapter5/IAU2000B.f.

There are two ways to implement the IAU 2000A /B models. If one uses the revised procedure to transform between reference systems that makes use of the Celestial Intermediate Pole (CIP) and Celestial Ephemeris Origin (CEO) (see below), the expressions for the positions of the CIP and CEO in the Celestial Reference System will already contain the proper expressions for the new precession-nutation model. If one elects to continue using the classical expressions based on the IAU 1976 Precession Model and IAU 1980 Theory of Nutation, one should proceed as in the past and then apply the corrections to the IAU 1980 Theory of Nutation $\Delta\psi$ and $\Delta\varepsilon$ provided by the appropriate IAU2000A/B software. This process is outlined in McCarthy (1996). For either mode, the user should consider the additional application of observed corrections to the IAU2000A model provided by the IERS.

2 Practical consequences of Resolution B1.7 "Definition of Celestial Intermediate Pole"

Resolution B1.7 (see Annex) recommends that the Celestial Intermediate Pole (CIP) be used in place of the Celestial Ephemeris Pole (CEP) on 1 January 2003 and specifies how to implement its definition through its direction at J2000.0 in the GCRS as well as the realization of its motion both in the GCRS and ITRS. Its definition is an extension of that of the CEP in the high-frequency domain and coincides with that of the CEP in the low-frequency domain (Capitaine, 2000). The reason for the adoption of the CIP is to clarify the current ambiguity about the difference between nutation and polar motion at high frequencies (motions with periods less than two days).

The realized celestial pole is to be the CIP. This requires an offset at epoch in the conventional model for precession-nutation as well as diurnal and higher frequency variations in the Earth's orientation. According to this resolution, the direction of the CIP at J2000.0 is offset from the direction of the GCRS in a manner consistent with the IAU2000A/B precession-nutation model. The motion of the CIP in the GCRS is to be realized by the IAU 2000A/B model for precession and forced nutation for periods greater that two days plus additional time-dependent corrections provided by the IERS through appropriate observations. The motion of the CIP in the ITRS is to be provided by the IERS through appropriate observations and models including high frequency variations.

The realization of the CIP thus requires that the IERS monitor the observed differences (reported as "celestial pole offsets") with respect to the conventional celestial position of the CIP in the GCRS based on the IAU 2000A precession-nutation model together with its observed offset at epoch. It also requires that the motion of the CIP in the TRS be provided by the IERS by observations taking into account a predictable part specified by a model including the terrestrial motion of the pole corresponding to the forced nutations with periods less than two days (in the GCRS) as well as the tidal variations in polar motion.

Recent models for rigid Earth nutation (Bretagnon *et al.* 1997, Souchay *et al.* 1999, Roosbeek 1999) include prograde diurnal and prograde semi-diurnal terms with respect to the GCRS with amplitudes up to ~15μas in $\Delta\psi \sin\varepsilon_0$ and $\Delta\varepsilon$. In order to realize the CIP as recommended by Resolution B1.7, nutations with periods less than two days are to be considered using a model for the corresponding motion of the pole in the ITRS. The prograde diurnal nutations correspond to prograde and retrograde long periodic (13 d to 3300 d) variations in polar motion and the prograde semi-diurnal nutations correspond to prograde diurnal variations in polar motion. The amplitudes for the non-rigid Earth can be derived from the rigid Earth values by applying the transfer function of Mathews *et al.* (2002) extended for the high frequency nutations. Preliminary numbers have been provided by Folgueira *et al.* (2001). An *ad hoc* Working Group has been working to provide a definitive table for operational use.

3 Practical consequences of Resolution B1.8 “Definition and use of Celestial and Terrestrial Ephemeris Origin”

Resolution B1.8 recommends the use of the “non-rotating origin”' (Guinot, 1979) both in the GCRS and the ITRS and these origins are designated as the Celestial Ephemeris Origin (CEO) and the Terrestrial Ephemeris Origin (TEO). The “Earth Rotation Angle” is defined as the angle measured along the equator of the CIP between the CEO and the TEO. It recommends that UT1 be linearly proportional to the Earth Rotation Angle and that the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS, and the Earth Rotation Angle. It is recommended that the IERS takes steps to implement this by 1 January 2003 and that the IERS continue to provide users with data and algorithms for the conventional transformation.

Two equivalent procedures were given in the IERS Conventions (1996) (McCarthy, 1996) for the transformation from the TRS to the CRS using the procedure

$$[\mathrm{CRS}] = \mathrm{PN}(t)\mathrm{R}(t)\mathrm{W}(t)[\mathrm{TRS}],$$

where PN(t), R(t) and W(t) are the transformation matrices describing the position of the celestial pole in the celestial system, the rotation of the Earth around the axis of the pole, and polar motion respectively. The classical procedure makes use of the equinox for realizing the intermediate reference frame of date t. It uses apparent Greenwich Sidereal Time (GST) in the transformation matrix R(t) and the classical precession and nutation parameters in the transformation matrix PN(t).

The second procedure makes use of the "non-rotating origin" to realize the intermediate reference frame of date t. It uses the "Earth Rotation Angle" originally referred to as "stellar angle" in the matrix R(t) and the two coordinates of the celestial pole in the CRS (Capitaine, 1990) in the transformation matrix PN(t).

Resolutions B1.6 and B1.7 can be implemented in either of these procedures if the requirements described above are followed for the space-time coordinates in the geocentric celestial system, for the precession and nutation model on which are based the precession and nutation quantities used in the transformation matrix PN(t) and for the polar motion used in the matrix W(t). However, only the second procedure can be in agreement with Resolution B1.8, which requires the use of the "non-rotating origin" both in the CRS and the TRS as well as the position of the CIP in the

GCRS and in the ITRS. The IERS must also provide users with data and algorithms for the conventional transformation. This implies that the expression of GST has to be consistent with the new procedure.

4 Implementation of the Resolutions

A direction vector given in a terrestrial reference system (TRS) can be expressed in a celestial reference system (CRS) by the following

$$[CRS] = PN(t)R(t)W(t)[TRS],$$

where PN(t), R(t) and W(t) are the transformation matrices describing the position of the celestial pole in the celestial system, the rotation of the Earth around the axis of the pole, and polar motion respectively. The parameter t is defined by

$$t = (\text{ TT} - 2000 \text{ January 1d 12h TT) in days}/36525.$$

This complies with IAU Resolution C7 (1994) recommending that the epoch J2000.0 be defined at the geocentre and at the date 2000 January 1.5 TT = Julian Date 2451545.0 TT.

4.1 Polar Motion

Referring to (Capitaine, 1990), if we designate R_1, R_2 and R_3 as rotations about the axes 1, 2 and 3 of the coordinate frame, then,

$$W(t)=R_3(-s')\, R_1(y_p)\, R_2(x_p),$$

x_p and y_p being the polar coordinates of the Celestial Intermediate Pole (CIP) in the TRS and s' the accumulated displacement of the TEO on the true equator due to polar motion. The use of the quantity s', which is neglected in the classical form, is necessary to provide an exact realization of the "instantaneous prime meridian." The pole coordinates to be used for the parameters x_p and y_p, if not estimated by analysis of the observations, are those published by the IERS. Models for the diurnal and semi-diurnal tidal variations in pole coordinates are described in the IERS Conventions and are available on the website of the IERS Conventions. The quantity s' is only slightly dependent on polar motion. Some components of s' have to be evaluated, in principle, from the measurements and can be extrapolated using the IERS data. Its main component can be written as

$$s' = -\,0.0015(a_c^2/1.2 + a_a^2)\, t,$$

a_c and a_a being the average amplitudes (in arc seconds) of the Chandlerian and annual wobbles, respectively in the period considered (Capitaine *et al.*, 1986). The value of s' is less than 0.4 mas after a century. Using the current mean amplitudes for the Chandlerian and annual wobbles gives s'= - 47 µas t, t being in Julian centuries from J2000. (see Lambert & Bizouard, this Volume).

4.2 Earth Rotation

Using the same rotational designations as for polar motion, R(t) is given by

$$R(t) = R_3(-\theta),$$

θ being the Earth Rotation Angle between the CEO and the TEO at date t on the equator of the CIP, which provides a rigorous definition of sidereal rotation of the Earth. It is obtained by using its relationship with UT1 as given by Capitaine *et al.*, (2000),

$$\theta(T_u) = 2\pi\, (0.7790572732640 + 1.00273781191135448\, T_u),$$

where T_u = (Julian UT1 date - 2451545.0),

and UT1 = UTC + (UT1-UTC), or equivalently

$$\theta(T_u) = 2\pi\ (\text{UT1 Julian day number elapsed since } 2451545.0 + 0.7790572732640 + 0.00273781191135448\, T_u).$$

The quantity UT1-UTC to be used (if not estimated in the analysis of the observations) is the value published by the IERS. This definition of UT1 is insensitive at the microarcsecond level to the precession-nutation model and to the observed celestial pole offsets. Therefore in the processing of observational data, the quantity s (see below) must be considered as independent of observations.

4.3 Precession-Nutation

Again, using the same rotational designations,

$$PN(t) = R_3(-E)\, R_2(-d)\, R_3(E)\, R_3(s),$$

E and d being such that the coordinates of the CIP in the CRS are

$$X = \sin d \cos E, \qquad Y = \sin d \sin E, \qquad \text{and} \qquad Z = \cos d.$$

s is the accumulated rotation, between the epoch and the date t, of the CEO on the true equator due to the celestial motion of the CIP. Its expression, accurate to 1 microarcsecond after one century, is given as a function of the coordinates X and Y by

$$s = -\frac{1}{2}\left[X(t)Y(t) - X(t_0)Y(t_0)\right] + \int_{t_0}^{t} \dot{X}(t)Y(t)\,dt\,,$$

Equivalently, PN(t) can be given by

$$PN(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \cdot R_3(s),$$

where $a = ½ + ⅛(X^2 + Y^2)$.

Developments of X and Y based on the IAU 2000 A or IAU 2000 B model for precession-nutation and on their corresponding pole offset at J2000.0 will be available on the website of the IERS Conventions. These developments have the form (Capitaine *et al.* 2002)

$$\begin{aligned} X = {} & -0.''016617 + 2004.''191743\, t - 0.''4272190\, t^2 \\ & - 0.''1986205\, t^3 - 0.''0000460\, t^4 + 0.''0000060\, t^5 \\ & + \sum_i \left[(a_{s,0})_i \sin(ARGUMENT) + (a_{c,0})_i \cos(ARGUMENT)\right] \\ & + \sum_i \left[(a_{s,1})_i\, t \sin(ARGUMENT) + (a_{c,1})_i\, t \cos(ARGUMENT)\right] \\ & + \sum_i \left[(a_{s,2})_i\, t^2 \sin(ARGUMENT) + (a_{c,2})_i\, t^2 \cos(ARGUMENT)\right] + \ldots \end{aligned}$$

$$\begin{aligned} Y = {} & -0.''006951 - 0.''025376\, t - 22.''407251\,0\, t^2 \\ & + 0.''0018423\, t^3 + 0.''0011131\, t^4 + 0.''0000099\, t^5 \\ & + \sum_i \left[(b_{c,0})_i \cos(ARGUMENT) + (b_{s,0})_i \sin(ARGUMENT)\right] \\ & + \sum_i \left[(b_{c,1})_i\, t \cos(ARGUMENT) + (b_{s,1})_i\, t \sin(ARGUMENT)\right] \\ & + \sum_i \left[(b_{c,2})_i\, t^2 \cos(ARGUMENT) + (b_{s,2})_i\, t^2 \sin(ARGUMENT)\right] + \ldots \end{aligned}$$

where t = (TT –2000 January 1d 12h TT) in days/36525 and ARGUMENT represents the fundamental arguments of the nutation theory. Each of the lunisolar terms in the nutation series is characterized by a set of five integers N_j that determine the ARGUMENT for the term as a linear combination of the five fundamental arguments F_j, namely the Delaunay variables l, l', F, D, Ω.

$$\mathrm{ARGUMENT} = \sum_{j=1}^{5} N_j F_j \equiv \vec{N} \cdot \vec{F} ,$$

where $\vec{N}$ is the five-vector composed of the values $N_1 \ldots, N_5$ that characterize the term, and $\vec{F}$ is the five-vector $F_1 \ldots, F_5$, which are functions of time.

Planetary nutation terms differ from the above only in that

$$\mathrm{ARGUMENT} = \sum_{j=1}^{14} N'_j F'_j ,$$

F_6 to F_{13} are the mean longitudes of the planets (Mercury to Neptune) including the Earth (l_{Me}, l_{Ve}, l_E, l_{Ma}, l_{Ju}, l_{Sa}, l_{Ur}, l_{Ne}), and the general precession in longitude, p_a.

The expressions for the fundamental arguments are

$F_1 \equiv$ l = Mean Anomaly of the Moon
$= 134.°96340251 + 1717915923.''2178\,t + 31.''8792\,t^2 + 0.''051635\,t^3 - 0.''00024470\,t^4$,

$F_2 \equiv$ l' = Mean Anomaly of the Sun
$= 357.°52910918 + 129596581.''0481\,t - 0.''5532\,t^2 + 0.''000136\,t^3 - 0.''00001149\,t^4$,

$F_3 \equiv$ F = L - Ω (L is the Mean Longitude of the Moon.)
$= 93.°27209062 + 1739527262.''8478\,t - 12.''512\,t^2 - 0.''01037\,t^3 + 0.''00000417\,t^4$,

$F_4 \equiv$ D = Mean Elongation of the Moon from the Sun
$= 297.°85019547 + 1602961601.''2090\,t - 6.''706\,t^2 + 0.''006593\,t^3 - 0.''0003169\,t^4$,

$F_5 \equiv$ Ω = Mean Longitude of the Ascending Node of the Moon
$= 125.°04455501 - 6962890.''5431\,t + 7.''4722\,t^2 + 0.''07702\,t^3 - 0.''0005939\,t^4$,

where t is measured in Julian Centuries of 36525 days of 86400 seconds of TDB since J2000.0 (Simon *et al.*, 1994).

The arguments for the planetary nutations are those of Souchay *et al.* (1999), based on the mean longitudes of the planets of VSOP82. The developments are given below in radians and t in Julian centuries of TDB.

l_{Me} = 4.402 608 842 + 2608.7903 141 574 t,
l_{Ve} = 3.176 146 697 + 1021.3285 546 211 t,
l_E = 1.753 470 314 + 628.3075 849 991 t,
l_{Ma} = 6.203 480 913 + 334.0612 426 700 t,
l_{Ju} = 0.599 546 497 + 52.9690 962 641 t,
l_{Sa} = 0.874 016 757 + 21.3299 104 960 t,
l_{Ur} = 5.481 293 871 + 7.4781 598 567 t,
l_{Ne} = 5.311886287 + 3.8133035638 t,
p_a = 0.024381750 t +0.00000538691 t^2.

TT can be used in these expressions as the difference TDB -TT (of which the largest term is 1.7 x 10 $^{-5}$ t sin l') has a quite negligible effect (lower that 10 $^{-2}$ µas with a period of one year) in the amplitudes of nutation.

The numerical development of s compatible with the IAU 2000 A precession-nutation model and the corresponding celestial offset at J2000.0, retaining terms larger than 0.5 µas, is provided in Table 1 for terms larger than 0.5 µas over 25 years and Table 2 for additional terms extending the development over one century. The quantity s is such that the CEO moves less than 70 mas from the GCRS prime meridian during one century.

Table 1. Development of all terms of s(t) exceeding 0.5 µas during the interval 1975-2025 compatible with the 2000A precession-nutation model. Units are µas. (Capitaine *et al.* 2002)

$$s(t) = -XY/2 + 2184 + 3812t - 121t^2 + 72574t^3 + \sum_k C_k \sin\alpha_k + 2t\sin\Omega$$
$$+ 4t\cos 2\Omega + 744t^2 \sin\Omega + 57t^2 \sin(2F\text{-}2D + 2\Omega) + 10t^2 \sin(2F + 2\Omega)$$
$$- 9t^2 \sin 2\Omega$$

Argument α_k	Amplitude C_k
Ω	-2641
2Ω	-63
2F-2D+3Ω	-12
2F-2D+Ω	-11
2F-2D+2Ω	+5
2F+3Ω	-2
2F+Ω	-2
3Ω	+2
l'+Ω	+1
l'-Ω	+1
l+Ω	+1
l-Ω	+1

Table 2. Development of all terms of s(t) exceeding 0.5 µas during the intervals 1900-1975 and 2025-2100 compatible with the 2000A precession-nutation model. Units are µas. (Capitaine *et al.* 2002)

$$\Delta s(t) = +28t^4 + 15t^5 - 22t^3 \cos\Omega\text{-}1t^3 \cos(2F\text{-}2D + 2\Omega) + \sum_i D_i t^2 \sin\alpha_i$$

Argument α_i	Amplitude D_i
l'	-6
l	-3
l'+2F-2D+2Ω	+2
2F+Ω	+2
L+2F+2Ω	+1

VLBI observations show that there are deficiencies in the IAU2000 A of the order of 0.2 mas (Mathews *et al.* 2002). The IERS will continue to publish observed estimates of the corrections to the IAU 2000 precession-nutation model. The observed differences with respect to the conventional pole position are monitored and reported by the IERS as celestial pole offsets. These are provided as corrections

$\delta X \approx \delta\psi \sin\varepsilon_0$ and $\delta Y \approx \delta\varepsilon$ to the X and Y coordinates.

Using these offsets the corrected celestial position of the CIP is given by

$$X = X(\mathrm{IAU2000}) + \delta X, \qquad \text{and} \qquad Y = Y(\mathrm{IAU2000}) + \delta Y.$$

This is practically equivalent to replacing the transformation matrix PN with the rotation

$$\mathrm{PN} = \begin{pmatrix} 1 & 0 & \delta X \\ 0 & 0 & \delta Y \\ -\delta X & -\delta Y & 1 \end{pmatrix} \mathrm{PN}_{\mathrm{IAU}}$$

where $\mathrm{PN}_{\mathrm{IAU}}$ represents the PN(t) matrix based on the IAU 2000 precession-nutation model.

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