Comparison of “Old” and “New” Concepts: Celestial Ephemeris Origin (CEO), Terrestrial Ephemeris Origin (TEO), Earth Rotation Angle (ERA)

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The following pages, with their notes, summarise the definitions of the CEO, TEO and ERA, their properties in theory and in practice. They show comparisons with traditional concepts, equinox, origin of terrestrial longitudes (not rigorously defined), Greenwich Sidereal Time.

These elements intervene in the transformation between terrestrial and celestial systems and frames.

This transformation does not require necessarily the use of celestial and terrestrial intermediate reference systems, and corresponding frames, attached to the pole of rotation and to some origins on the equators : the use of three Eulerian angles is sufficient and has been suggested. However, such intermediate systems are convenient for practical and scientific reasons. They introduce quantities which vary slowly or almost linearly with time, and which have a astronomical and geophysical interest. This is, in particular, the case of UT1, historically intended to be proportional to the angle of sidereal rotation of the Earth (Earth Rotation Angle, ERA).

The five parameters of the transformation are noted :

- X, Y, direction cosines of the Celestial Intermediate Pole (CIP) in the Geocentric Celestial Reference System (GCRS) or Frame (GCRF) [usually multiplied by 1296000’’/2π and referred to as coordinates of the CIP] ;
- x, y, direction cosines [or coordinates] of the CIP in the International Terrestrial Reference System (ITRS) or Frame (IRTF) :
- θ Earth Rotation Angle, ERA.

The observations bring corrections to approximate values of these quantities provided by models (X, Y) or previsions (x, y, θ). In the spirit of the IAU resolutions of the XXIVth IAU General Assembly (2000), the goal is to ensure that definitions and processes of the involved quantities maintain an accuracy at the level of the microarcsecond.

The main differences between old and new concepts, in this field, are :

- the use of the CIP in place of the Celestial Ephemeris Pole ;
- the direct use of the coordinates X, Y of the CIP ;
- the adoption of the Celestial Ephemeris Origin (CEO) in place of the equinox on the equator of the CIP, and of the corresponding longitude origin, TEO, on the terrestrial equator of the CIP ;
- the adoption of the ERA in place of the Greenwich Sidereal Time.

The explicit use of the CEO is not the only possible route between the Geocentric Celestial Reference Frame (GCRF) and the International Terrestrial Reference Frame (ITRF), using an intermediate frame attached to the CIP, but it has the advantage of avoiding the complexities introduced by a conventional ecliptic which plays no direct role in the transformation between frames and in the observations of the

Earth rotation. In particular, *GST* depends on the observed position of the CIP and, therefore, the conventional relation *GST-UT1* needs to be corrected when precession-nutation model is changed and when the position of the CIP is measured. In the relation *GST-UT1*, the use of the CEO is implicit in order to ensure that *UT1* is proportional to the sidereal rotation of the Earth, represented by θ.

Main references

Guinot B. 1979, Basic problems in the kinematics of the rotation of the Earth, in *Time and the Earth's rotation,* D.D. McCarthy and J.D. Pilkington eds, Reidel Pub. Co., pages 7-18.

Capitaine N., Guinot B. Souchay J., 1986, A non-rotating origin on the instantaneous equator : definition, properties and use, *Celestial Mechanics*, 39, 283-307.

Capitaine N., Guinot B., McCarthy D.D., 2000, Definition of the Celestial Ephemeris Origin in the International Terrestrial Reference Frame, *Astron. Astrophys.*, 355, 398-405.

Celestial Ephemeris Origin

Definitions and Basic Properties

CEO	Equinox

Definitions

CEO	Equinox
On the CIP equator at t (Terrestrial Time).	On the CIP equator at t.
Defined cinematically : The Celestial Intermediate Reference System linked to the CIP and the CEO has no component of instantaneous rotati along the axis of the CIP in the GCRS, at any time.	Defined geometrically : Intersection of the true equator with a conventional plane passing through the geocentre, called ‘ecliptic’,
Requires a quantity s.	Usual expressions for precession-nutation.
s is the composite arc CEO-N (on the tru equator) minus Σ_0N (on the equator of GCRS), N being the node and Σ_0 the orig of right ascensions in the GCRS.	
s obtained by an integration along the CI (X,Y) trajectory from t_0 = J2000.0 to t. (Note 1)	Requires only the position of the CIP at t.
X , Y and s are the three parameters for the transformation GCRS - Celestial Intermediate Reference System. They include precession and nutation	Usual matrices.

Realisations

CEO	Equinox
s is computed from a model of precession-nutation.	
s is a small quantity (2) (s = 0 at J2000.0, reaches 70 mas in 2100).	Large variation of right ascensions (reaches 1.3° in 2100).
s is practically insensitive (at the microarcsecond level until 2100) - to changes of model of precession-nutation, - to the difference observation-model.	Sensitivity of the equinox to differences (models of the motion of the CIP and to observed pole offsets.

Terrestrial Ephemeris Origin

Definitions and Basic Properties

TEO	Usual longitude origin
Definitions	
On the CIP equator at *t*.	On the CIP equator at *t*.
Defined cinematically :	No precise definition
The Terrestrial Intermediate Reference System linked to the CIP and the TEO has no component of rotation along the axis of the CIP in the ITRS	
Requires an integration along the CIP (coordinates *x* and *y*) trajectory from t_0 to *t* providing a quantity denoted *s'*.	
x, y and *s'* are the three parameters for the transformation ITRS <-> Terrestrial Intermediate Reference System.	
Realisations	
s' is a very small quantity which varies almost linearly with time (<0,4 mas in 2100) (3).	Errors at the microarcsecond level on UT1.
s' is computed from the observed pole coordinates by numerical integration and can be extrapolated several years in advance at the microsecond level.	

COMPLETE SYMMETRY WITH CEO

UT1

Use of CEO and TEO	Use of the equinox
Definitions and Theory	
UT1 based on the Earth Rotation Angle θ.	*UT1* based on the Greenwich Sidereal Time *GST*
θ is the hour angle of CEO reckoned from TEO.	*GST* is the hour angle of the equinox reckoned from ?
The time derivative of θ is strictly the component of the rotation vector of the Earth along the axis of the CIP.	
The relationship between θ and *UT1* is linear (conceptual definition of UT1).	The relationship between *GST* and *UT1* involves polynomial, periodic and Poisson terms.
	The relationship between *GST* and *UT1* is based on the implicit use of the CEO (4).
Application	
The linear relationship between θ and *UT1* is practically insensitive to the model and to the observations of the CIP path (at the level of 1 μas until 2100). It provides a stable operational definition of UT1.	The relationship between *GST* and *UT1* is very sensitive to the model and to the observations of the CIP position.

NOTES

1 Integration for CEO

The fact that the CEO depends on an integration along the CIP path is sometimes seen as a major drawback. One may argue that an interruption of the observations of the motion of the CIP would lead to the loss of the CEO and therefore of UT1. The answers to that objection are as follows.

- The model of precession-nutation we have presently would suffice to bridge a gap of several decades in the observations, without error larger than 1 microarcsecond in UT1 (70 ns).
- Anyway, the CEO is required for evaluating UT1, even when the equinox is used.
- Let us remark also that the values of UT1 are referred to TAI, which is another integrated quantity and which would be lost without possibility of precise recovery if all atomic clocks stop.

2 Form of *s*

The form of $s(t)$ compatible with IAU 2000A precession-nutation model, including all terms exceeding 0.5 µas during the interval (1975-2025) is (Capitaine et al., 2000) :

$s(t) = -X(t)\,Y(t)/2$ + polynomial terms up to t^3
+ 13 periodic terms (largest term : amplitude 2641 µas)
+ 6 Poisson terms in t^2.

The extension of validity to (1900-2100) requires the additional terms

$\Delta s(t) =$ polynomial terms in t^4 and t^5
+ 9 periodic terms
+ 2 Poisson terms in t^3.

3 Maximum value

The maximum value of 0.4 mas in 2100 is computed assuming an amplitude of 0.5’’ of the Chandlerian nutation. The maximum observed amplitude since 1900 is about 0.25’’.

4 Equation of the equinox

The concept of the CEO is required for providing the ‘equation of the equinox’. It was already implicitly used in 1982 in ‘The New Definition of Universal Time’, Aoki et al., Astron. Astrophys., 105, 359-361.