New Precession Formula

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In order to find a substitute of IAU 1976 precession formula (Lieske *et al.* 1977) to be used with latest nutation theories such as SF2001 (Shirai and Fukushima 2001), we modified J.G. Williams' formulation of the precession and the nutation by using the 3-1-3-1 rotation (Williams 1994) so as to express them in an arbitrary inertial frame of reference.

It gives the precession-nutation matrix as the product of four rotation matrices as

$$\mathcal{NP} = \mathcal{R}_1(-\epsilon)\mathcal{R}_3(-\psi)\mathcal{R}_1(\varphi)\mathcal{R}_3(\gamma), \tag{1}$$

and the precession one similarly as

$$\mathcal{P} = \mathcal{R}_1\left(-\bar{\epsilon}\right)\mathcal{R}_3\left(-\bar{\psi}\right)\mathcal{R}_1(\varphi)\mathcal{R}_3(\gamma). \tag{2}$$

Here φ and γ are the angles to specify the location of the ecliptic pole of date in the given inertial frame, ψ and $\bar{\psi}$ are the true and mean ecliptic angles of precession, respectively, and ϵ and $\bar{\epsilon}$ are the true and mean obliquities of the ecliptic, respectively. In other words, we (1) first specify the ecliptic pole of date in the given inertial reference frame and shift from the inertial frame to an ecliptic reference frame of date, (2) then specify the mean or true equatorial pole of date, $\bar{\mathrm{P}}$ or P, in the ecliptic reference frame and shift from the ecliptic frame to the mean or true equatorial reference frame of date. Although the expression of nutation matrix is unchanged, we recommend the usage of the above form of $\mathcal{NP}$ instead of preparing $\mathcal{P}$ and $\mathcal{N}$ separately because of faster evaluation.

As Williams stressed, this unified treatment is a merit of the original Williams' formulation, which is inherited to the modified one. Note that the usual nutations are used in this formulation. This is another merit. Further the number of rotational operations needed to express the precession-nutation matrix reduces from six of the IAU formulation to four of the new one. This is yet another merit of the new formulation.

The formulation is robust in the sense it avoids a singularity caused by finite pole offsets near the epoch. Facing the singularity is inevitable in the current IAU formulation. This is clear from the differential relation $\delta\zeta_{\mathrm{A}} \approx -\delta\epsilon_{\mathrm{A}}/\sin\theta_{\mathrm{A}}$, which shows the possibility of divergence in the correction to ζ_{A} due to a small divisor, $\sin\theta_{\mathrm{A}}$, in the vicinity of the epoch.

By using SF2001, we converted the true pole offsets referred to the ICRF, observed by VLBI for 1979-2000, and compiled by USNO, to the offests in the above three angles of precession, $\bar{\psi}$, φ, and γ, while we fixed $\bar{\epsilon}$ as the combination of the linear part provided in SF2001 and the quadratic and higher terms derived by Williams (1994) as

$$\bar{\epsilon}(t) = 84381.4428 - 46.8388\,t - 0.0002t^2 + 0.0020t^3, \tag{3}$$

where we set the cut-off level so that the resulting precision be 0.1 mas for 1900-2100. From the converted offsets, we determined the best-fit polynomial expressions of the differences of the three precession angles in the ICRF from the polynomial expressions of Williams' counterparts, $\eta_{\rm A}, \epsilon'_{\rm A}, \xi_{\rm A}$, by a weighted least square method as

$$\bar{\psi} - \eta_{\rm A} = -(0.0431 \pm 0.0006) + (0.0174 \pm 0.0100)\ t, \tag{4}$$

$$\varphi - \epsilon'_{\rm A} = +(0.0389 \pm 0.0003) - (0.0044 \pm 0.0040)\ t, \tag{5}$$

$$\gamma - \xi_{\rm A} = -(0.0000 \pm 0.0000_1) - (0.0052 \pm 0.0000_2)\ t. \tag{6}$$

Then, by adding the Williams' original polynomial expressions, we obtained the final forms as

$$\bar{\psi}(t) = -0.0431 + 5038.4739\ t + 1.5584\ t^2 - 0.0002\ t^3, \tag{7}$$

$$\varphi(t) = 84381.4479 - 46.8140\ t + 0.0511\ t^2 + 0.0005\ t^3, \tag{8}$$

$$\gamma(t) = 10.5525\ t + 0.4932\ t^2 - 0.0003\ t^3, \tag{9}$$

where we again set the cut-off level so as to keep the precision as 0.1 mas for 1900-2100. Note that the constant term of γ is practically equal to zero. This means that the ecliptic pole of the epoch precisely lies on the yz-plane of the ICRF.

In conclusion, the above set of four formulas constitute a new set of fundamental expressions of the precessional quantities. The combination of the new precession formula and the periodic part of SF2001 serves a good approximation of the precession-nutation matrix in the ICRF.

Note that the procedure to determine the polynomial forms of the new precession angles described here is applicable to any combination of the observation and the nutation theory as long as the latter gives the corrections to the IAU precession formula in polynomial form of the corrections in nutation. Thus the results presented here will be easily updated.

References

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