

# A Simple Method for Conversion Between the Celestial Ephemeris Origin, the ICRF, and the Equinox of Date

James L. Hilton  
*Astronomical Applications Dept.*  
*U.S. Naval Observatory*  
*3450 Massachusetts Ave. NW*  
*Washington, DC 20392 USA*

Eq. 21 of Capitaine et al. (2000) simplifies the equation for the CEO parameter  $s$  based on the assumption that the position of the CEO pole is the same as the Celestial Intermediate Pole at J2000.0. For this particular case, comparison of Fig. 2 of Lieske et al. (1977) and Fig. 1 of Capitaine et al. (2000) shows that:

$$d = \theta_A + \Delta\psi \quad \text{and} \quad E = \zeta_A + \Delta\varepsilon$$

where  $\Delta\psi$  and  $\Delta\varepsilon$  are the nutations in longitude and latitude, respectively. Thus, the precession parameters  $\theta_A$  and  $\zeta_A$  are orthogonal and in the same directions as  $\Delta\psi$  and  $\Delta\varepsilon$ , respectively.

The third precession parameter,  $z_A$ , contains all of the information on the motion of the equinox both with respect to the ecliptic of date and none of the information on the motion of the pole itself.

Inspection of Fig. 2 of Lieske et al. (1967) also shows that the position of the node of the equator of date on the ecliptic of epoch is simply  $z_A + \chi_A$ .  $z_A$  is also easy to derive since, as shown in Lieske (1967) the parameters  $\zeta_A$  and  $z_A$  are derived from  $(z_A + \zeta_A)$  and  $(z_A - \zeta_A)$ .

The IAU 2000 precession-nutation theory provides the precession and nutation with respect to the Lieske et al. (1977) precession using the ecliptic angles  $\Delta\psi$  and  $\Delta\varepsilon$ . The conversion to  $X$ ,  $Y$ , and  $Z$  is given as equation (4.16) of Capitaine et al. (1986):

$$\begin{aligned} X &= \sin(\omega_A + \Delta\varepsilon) \sin(\psi_A + \Delta\psi) \\ Y &= -\sin \varepsilon_0 \cos(\omega_A + \Delta\varepsilon) + \cos \varepsilon_0 \sin(\omega_A + \Delta\varepsilon) \cos(\psi_A + \Delta\psi) \\ Z &= \cos \varepsilon_0 \cos(\omega_A + \Delta\varepsilon) + \sin \varepsilon_0 \sin(\omega_A + \Delta\varepsilon) \cos(\psi_A + \Delta\psi) \end{aligned}$$

where  $\varepsilon_0$  is the obliquity at epoch,  $\omega_A$  is the angle between the ecliptic pole of epoch and the rotational pole of date, and  $\psi_A$  is the luni-solar precession angle (see Fig. 1). The coefficients for  $\psi_A$  and  $\omega_A$  through third order are (Lieske, 1967; Andoyer, 1911):

$$\begin{aligned} \psi_1 &= P + \chi_1 \cos \varepsilon_0 \\ \psi_2 &= \frac{c_1 \psi_1}{\tan 2\varepsilon_0} + \frac{1}{2} (P_1 \cos \varepsilon_0 - c_1 p_g \tan \varepsilon_0) \\ \psi_3 &= \frac{1}{3} \left\{ \left[ c_1 P_1 + \frac{1}{2} \left( c_2 - \frac{1}{2} s_1 P \right) \frac{(P + p_g)}{\cos \varepsilon_0} + \chi_1 \right] \frac{\cos 2\varepsilon_0}{\sin \varepsilon_0} \right. \\ &\quad \left. - \psi_1 \left( \frac{\omega_2}{\tan \varepsilon_0} + 2c_1^2 + \frac{1}{2} \chi_1^2 \right) - \frac{1}{2} p_g \left[ 3c_1^2 + \frac{(2c_2 - s_1 P)}{\tan \varepsilon_0} \right] \right\} \end{aligned}$$

$$\begin{aligned}\omega_1 &= 0^* \\ \omega_2 &= \frac{1}{2}s_1\psi_1 \\ \omega_3 &= \frac{\sin \varepsilon_0}{3}(2\psi_2\chi_1 + \psi_1\chi_2)\end{aligned}$$

where  $P$  is the rate of general precession in longitude at J2000.0,  $P_1$  is the rate of change of Newcomb's precession constant [due mainly to changes in the Earth's eccentricity (de Sitter & Brouwer, 1938)],  $p_g$  is the geodetic precession (Lieske et al., 1977),  $\varepsilon_0$  is the obliquity of the ecliptic at J2000.0,  $c_1$  and  $c_2$  are the first and second order coefficients for  $\sin i \cos \Omega$  (where  $i$  is the inclination of the mean ecliptic of date on the mean ecliptic of epoch and  $\Omega$  is the node of the mean ecliptic of date on the mean ecliptic of epoch),  $s_1$  and  $s_2$  are the first and second order coefficients of  $\sin i \sin \Omega$ , and  $\chi_1$  and  $\chi_2$  are the first and second order coefficients for the planetary precession,  $\chi_A$  given by (Lieske et al., 1977; Andoyer, 1911):

$$\begin{aligned}\chi_1 &= \frac{s_1}{\sin \varepsilon_0} \\ \chi_2 &= s_2 + \frac{c_1 P}{\sin \varepsilon_0}\end{aligned}$$

Thus conversion of the IAU 2000 precession nutation from its  $\Delta\psi$ - $\Delta\varepsilon$  form to  $X$ - $Y$ - $Z$  form requires knowledge of the motion of the mean ecliptic of date. Thus when  $P_0$  is the pole of the inertial reference system, the angles  $d$  and  $E$  used in the CEO paradigm are simply:

$$d = \theta_A + \Delta\psi \quad \text{and} \quad E = \zeta_A + \Delta\varepsilon$$

where  $\theta_A$  and  $\zeta_A$  are two of the Lieske et al. (1977) precession angles. The third precession angle,  $z_A$ , contains all of the information on the motion of the equinox on the mean ecliptic of date. The parameter  $z_A$  is a natural byproduct of the easiest way to determine either  $\zeta_A$  or  $E$ .

The equations to determine the position of the pole and the CEO requires knowledge of the motion of the mean ecliptic of date. The planetary precession is a byproduct of the information needed.

## References

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\*  $\omega_1$  is not strictly 0 because, as shown by Williams (1994), the Moon has a slight proper inclination with respect to the ecliptic. This inclination causes a small linear rate of change in the Earth's obliquity with respect to inertial space.