Reliability of Atmospheric Torque for Geodesy

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1 Angular momentum balance approach and torque approach

The effect of the atmosphere on the Earth rotation can be computed from the equation of angular momentum conservation of an isolated system. For the system composed of the Earth and atmosphere, we have:

\[
\frac{dH_{\text{Earth}}}{dt} + \frac{dH_{\text{atmosphere}}}{dt} = 0
\]

The angular momentum of the atmosphere is computed from the surface atmospheric pressure and the three-dimension horizontal wind fields at different altitudes, themselves coming from a general circulation model (GCM) constrained by observations. Alternatively, the effect of the atmosphere can be computed as an external forcing acting on the solid Earth. In that case, we have:

\[
\frac{dH_{\text{Earth}}}{dt} = \Gamma_{\text{atmosphere\to Earth}}
\]

When the Earth-atmosphere system is considered as isolated, the two methods can be linked together by

\[
\frac{dH_{\text{atmosphere}}}{dt} = \Gamma_{\text{Earth\to atmosphere}} = -\Gamma_{\text{atmosphere\to Earth}}
\]

The interaction torque results from the action of three forces: the pressure force acting on the topography, the gravitational interaction between the solid Earth and the atmosphere, and the friction at the fluid-solid interface. The mathematical formula to compute those torques are given in the literature (see for instance de Viron et al., 2001).

The relative sizes of the different effects are very different for the different components of the torque affecting the Earth rotation rate (axial torque) or affecting polar motion/nutation (equatorial components of the torque). The torque associated with the Earth equatorial bulge and consisting of the sum of the pressure and the gravitational effects on that bulge, dominates the equatorial component of the budget at the 95% level in the time domain. It does not affect the axial budget. As shown by Bell (1994), the term associated with the equatorial bulge appears in both sides of the angular momentum budget equation (eq. 3). This so-called ellipsoidal torque being much larger than any of the other effects in the equation, it is obvious that eq. (3) is automatically verified with a very high precision (at least at the 95% level), whatever the quality of the atmospheric model could be.

2 Advantage and disadvantage of the torque approach

The main advantage of the torque approach is that it allows to get a physical insight on the interaction processes. In particular, we can tell what process, where and how the angular momentum has been exchanged. Conversely, the angular momentum approach only gives the information that a given quantity of angular momentum has been exchanged. The main disadvantage of the
torque approach is that the computation is very delicate, as it is highly sensitive to small errors in the data, unlike the angular momentum computation, which is more a global-average computation which smoothes out the small errors in the data. Consequently, the atmospheric angular momentum is usually a more accurate quantity, which allows to compute more accurately with a precision that matches the precision of the geodetic Earth rotation measures.

3 Test on torque reliability

In de Viron and Dehant (2003), the torques are tested in order to infer the precision of the computation by comparing the results obtained from three different reanalysis models and by checking the angular momentum budget of each model independently. In order to make the tests more relevant, we also removed, from the equatorial budget, the common ellipsoidal part, associated with the Earth equatorial bulge. A summary of the model comparison results is given by the chart here below.

<table>
<thead>
<tr>
<th>ellipsoidal mountain friction</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>10–50</td>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>50–100</td>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>100–400</td>
<td>2</td>
<td>2</td>
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<td>2</td>
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<td>&gt;400</td>
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<td>2</td>
<td>2</td>
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</tr>
</tbody>
</table>

The angular momentum budget tests (not displayed here) show that the angular momentum of the atmosphere is conserved in X and Y at any period longer than 10 days. When the bulge contribution is removed from the budget, the quality is about the same for the three components, with a reasonable agreement between AAM time derivative and torque between 10 days and the annual period, and not as good for the periods shorter and longer.

4 Conclusion for the torque approach

The torque approach is a powerful tool which enables us to study Earth-atmosphere interaction, but the precision which can be obtained from GCM reanalysis output is not good enough to use this approach in operational geodesy. Nevertheless, it can be used for getting insights about where and how the angular momentum is transferred, and about its causes in particular regions of the world.

References

