

## 9 Tropospheric Model

### 9.1 Optical Techniques

The formulation of Marini and Murray (1973) is commonly used in laser ranging. The formula has been tested by comparison with ray tracing radiosonde profiles.

The correction to a one-way range is

$$\Delta R = \frac{f(\lambda)}{f(\phi, H)} \cdot \frac{A + B}{\sin E + \frac{B/(A + B)}{\sin E + 0.01}}, \quad (1)$$

where

$$A = 0.002357P_0 + 0.000141e_0, \quad (2)$$

$$B = (1.084 \times 10^{-8})P_0T_0K + (4.734 \times 10^{-8})\frac{P_0^2}{T_0} \frac{2}{(3 - 1/K)}, \quad (3)$$

$$K = 1.163 - 0.00968 \cos 2\phi - 0.00104T_0 + 0.00001435P_0, \quad (4)$$

where

- $\Delta R$  = range correction (meters),
- $E$  = true elevation of satellite,
- $P_0$  = atmospheric pressure at the laser site  
(in  $10^{-1}$  kPa, equivalent to millibars),
- $T_0$  = atmospheric temperature at the laser site  
(degrees Kelvin),
- $e_0$  = water vapor pressure at the laser site  
( $10^{-1}$  kPa, equivalent to millibars),
- $f(\lambda)$  = laser frequency parameter  
( $\lambda$  = wavelength in micrometers),
- $f(\phi, H)$  = laser site function, and
- $\phi$  = geodetic latitude.

Additional definitions of these parameters are available. The water vapor pressure,  $e_0$ , should be calculated from a relative humidity measurement,  $R_h(\%)$  by

$$e_0 = \frac{R_h}{100} \times e_s f_w,$$

where the saturation vapor pressure,  $e_s$ , is computed using the following formula (Giacomo, 1982; Davis, 1992):

$$e_s = 0.01 \exp(1.2378847 \times 10^{-5}T_0^2 - 1.9121316 \times 10^{-2}T_0 + 33.93711047 - 6.3431645 \times 10^3T_0^{-1})$$

The enhancement factor,  $f_w$ , is computed by (Giacomo, 1982):

$$f_w = 1.00062 + 3.14 \times 10^{-6}P_0 + 5.6 \times 10^{-7}(T_0 - 273.15)^2.$$

The laser frequency parameter,  $f(\lambda)$ , is

$$f(\lambda) = 0.9650 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4}.$$

$f(\lambda) = 1$  for a ruby laser, [*i.e.*  $f(0.6943) = 1$ ], while  $f(\lambda_G) = 1.02579$  and  $f(\lambda_{IR}) = 0.97966$  for green and infrared YAG lasers.

The laser site function is

$$f(\phi, H) = 1 - 0.0026 \cos 2\phi - 0.00031H,$$

where  $\phi$  is the geodetic latitude of the site and  $H$  is the height above the geoid (km).

Traditionally the correction of the atmospheric delay at optical wavelengths has been performed using the formulation of Marini and Murray (1973), a model developed for the  $0.6943 \mu\text{m}$  wavelength. The model includes the zenith delay determination and the mapping function, to project the zenith delay to a given elevation angle, in a non-explicit form. In the last few years, the computation of the refractive index at optical wavelengths has received special attention and, as a consequence, the International Association of Geodesy (IUGG, 1999) recommended a new procedure to compute the group refractivity, following Ciddor (1996) and Ciddor and Hill (1999). Based on this formulation, Mendes *et al.* (2002) have derived new mapping functions for optical wavelengths, using a large database of ray tracing radiosonde profiles. These mapping functions are tailored for the  $0.532 \mu\text{m}$  wavelength and are valid for elevation angles greater than 3 degrees, if we neglect the contribution of horizontal refractivity gradients. The new mapping functions represent a significant improvement over other mapping functions available and have the advantage of being easily combined with different zenith delay models. The analysis of two years of SLR data from LAGEOS and LAGEOS 2 indicate a clear improvement both in the estimated station heights and adjusted tropospheric zenith delay biases (Mendes *et al.*, 2002).

For the computation of the zenith delay, the available models seem to have identical precision, but variable biases. Preliminary results indicate that the Saastamoinen (1973) zenith delay model, updated with the dispersion factor given in Ciddor (1996) gives satisfactory results, but further studies are needed to validate it over the entire spectrum of wavelengths encountered in satellite laser ranging today (355 to 1064 nm).

## 9.2 Radio Techniques

The non-dispersive delay imparted by the atmosphere on a radio signal up to 30 GHz in frequency, is divided into “hydrostatic” and “wet” components. The hydrostatic delay is caused by the refractivity of the dry gases in the troposphere and by the nondipole component of water vapor refractivity. The dipole component of the water vapor refractivity is responsible for the wet delay. The hydrostatic delay component typically accounts for about 90% of the total delay at any given site but is highly predictable based on surface pressure. For the most accurate *a priori* hydrostatic delay, desirable when the accuracy of the estimate of the zenith wet delay is important, the formula of Saastamoinen (1972) as given by Davis *et al.* (1985) should be used.

$$D_{hz} = \frac{[(0.0022768 \pm 0.0000005)]P_0}{f_s(\phi, H)}$$

where

$$\begin{aligned} D_{hz} &= \text{zenith hydrostatic delay in meters,} \\ P_0 &= \text{total atmospheric pressure in millibars at the antenna} \\ &\quad \text{reference point (e.g. intersection of the axes of rotation} \\ &\quad \text{for a radio antenna),} \\ f_s(\phi, H) &= (1 - 0.00266 \cos 2\phi - 0.00028H), \text{ where } \phi \text{ is the geodetic} \\ &\quad \text{latitude of the site and } H \text{ is the height above the} \\ &\quad \text{geoid (km).} \end{aligned}$$

In precise applications where millimeter accuracy is desired, the delay must be estimated with the other geodetic quantities of interest. The estimation is facilitated by a simple parameterization of the tropospheric delay, where the line of sight delay,  $D_L$ , is expressed as a function of four parameters as follows:

$$D_L = m_h(e)D_{hz} + m_w(e)D_{wz} + m_g(e)[G_N \cos(a) + G_E \sin(a)].$$

The four parameters in this expression are the zenith hydrostatic delay,  $D_{hz}$ , the zenith wet delay,  $D_{wz}$ , and a horizontal delay gradient with components  $G_N$  and  $G_E$ .  $m_h$ ,  $m_w$  and  $m_g$  are the hydrostatic, wet, and gradient mapping functions, respectively, and  $e$  is the elevation angle at which the signal is received.  $a$  is the azimuth angle in which the signal is received, measured east of north. The estimation of tropospheric gradients was shown by Chen and Herring (1997) and MacMillan (1995) to be beneficial in VLBI, and by Bar-sever *et al.* (1998) to be beneficial in GPS. Davis *et al.* (1993) and MacMillan (1995) recommend using either  $m_g(e) = m_h(e) \cot(e)$  or  $m_g(e) = m_w(e) \cot(e)$ . Chen and Herring (1997) propose using  $m_g(e) = 1/(\sin e \tan e + 0.0032)$ . The various forms agree to within 10% for elevation angles higher than  $10^\circ$ , but the differences reach 50% for  $5^\circ$  elevation due to the singularity of the  $\cot(e)$  function. The estimate of gradients is only worthwhile when using data lower than  $15^\circ$  in elevation. In the case of GPS analyses, such low-elevation data should be deweighted because of multipath effects.

Comparisons of many mapping functions with the ray tracing of a global distribution of radiosonde data have been made by Mendes and Langley (1998b). For observations below  $10^\circ$  elevation, which may be included in geodetic programs in order to increase the precision of the vertical component of the site position, the mapping functions of Lanyi (1984) as modified by Sovers and Jacobs (1996), Ifadis (1986), Herring (1992, designated MTT) and Niell (1996, designated NMF) are the most accurate. Only the last three were developed for observations below an elevation of  $6^\circ$ , with MTT and NMF being valid to  $3^\circ$  and Ifadis to  $2^\circ$ . Each of these mapping functions consists of a component for the water vapor and a component for either the total atmosphere (Lanyi) or the hydrostatic contribution to the total delay (Ifadis, MTT, and NMF). In all cases the wet mapping should be used as the function partial derivative for estimating the residual atmosphere zenith delay.

The most commonly used hydrostatic and wet mapping functions in precise geodetic applications are those derived by Lanyi (1984) as modified by Sovers and Jacobs (1996), Herring (1992), and Niell (1996). The first two allow for input of meteorological data although Lanyi's function requires information on the vertical temperature profile for best results, whereas Herring's requires only surface data. Niell's mapping function is based on global climatology of the delay and requires only input of time and location. Only the wet zenith delay is typically estimated, and an *a priori* value is used for the hydrostatic zenith delay.

The parameters of the atmosphere that are readily accessible at the time of the observation are the surface temperature, pressure, and relative humidity. The mapping functions of Lanyi, Ifadis, and Herring were developed to make use of this information. Lanyi additionally requires parameterization in terms of the height of a surface isothermal layer, the lapse rate from the top of this layer to the tropopause, and the height of the tropopause. If only the surface meteorology is used without also modeling these parameters (as described, for example, in Sovers and Jacobs (1996)), the agreement with radiosonde-derived delays is significantly worse than any of the other mapping functions. Mendes and Langley (1998a) found that the use of nominal values to parametrize the Lanyi mapping function degrades its performance significantly. They concluded that the best results are obtained using either an interpolation scheme developed by Sovers and Jacobs (1996) or having the temperature-profile parameters predicted from surface mean temperature using models (Mendes and Langley, 1998a).

The mapping functions of Niell differ from the other three in being independent of surface meteorology. The hydrostatic mapping function relies instead on the greater contribution by the conditions in the atmosphere above approximately 1 km, which are strongly season dependent, while the wet mapping function depends only on latitude.

Based on comparison with total delays calculated by ray tracing temperature and relative humidity profiles from a globally distributed set of radiosonde data, Mendes and Langley (1998b) conclude that Ifadis, Lanyi (which must be used with temperature profile modeling), and NMF provide the best accuracy down to  $10^\circ$ , while Ifadis and NMF are the most accurate at  $6^\circ$ . Niell (1996) compared the hydrostatic and wet mapping functions directly with ray tracing of radiosonde profiles and found that Ifadis, MTT, and NMF are comparable in accuracy at  $5^\circ$  elevation. (Lanyi was tested without temperature profile modeling.)

A recent assessment study using more than 32,000 traces corresponding to a one-year data set of radiosonde profiles from 50 stations distributed worldwide (Mendes and Langley, 1998b) concluded that none of these mapping functions has a clear supremacy over the others, for all elevation angles and at all latitudes. Nevertheless, the Ifadis mapping function yields the best overall performance, both in bias and rms scatter, especially for lower elevation angles. In the absence of reliable meteorological data, NMF is preferred.

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