

8 Combination and Comparison of Contributed Catalogs (SOL, SLB, DG)

The following section describes the preliminary catalogs submitted by seven different analysis centers using four independent software analysis packages, the construction of a combination catalog from seven contributed catalogs generated at seven different VLBI analysis centers, and comparisons of individual catalogs amongst themselves and the combined catalog. The main purpose of this analysis is to investigate systematic effects in individual solutions and estimate a precision of the combined and the individual realizations of the celestial reference frame.

8.1 Contributed Catalogs

The analysis centers involved in ICRF2 were asked to generate and submit two catalogs, one without the VCS sessions and one with the VCS sessions. The data and models used were to be as similar as possible. The VCS catalogs were to be used to construct a combination catalog at Main Astronomical Observatory. Lists of database sessions, sources to solve as arc parameters, and sources to exclude were distributed by GSFC. The solutions were to use group delays only, use only sources with three or more “good” observations, be a TRF solution using VTRF2008, and apply a no-net-rotation constraint using the 205 ICRF1 defining sources that were not classified as special handling sources. The solutions also were to solve for atmosphere gradients, apply pressure loading, use the VMF1 model, and apply thermal deformation. Seven analysis centers generated catalogs using four independent software analysis packages and submitted them in time for use in constructing a combination catalog. Table 3 lists the particulars of the contributed solutions. It can be seen that no two analysis centers used the same data span, the same sessions, or obtained the same number of estimated sources. One of the catalogs also had an editing problem and used some observations normally considered bad. Also, most analysis centers used different analysis models. Some did not use the thermal deformation model, or the VMF1 model, or pressure loading, or solved for baselines instead of the TRF. The model comparisons section showed that these analysis differences should not produce any significant systematic differences, but may increase the noise level of the differences between solutions. Seven contributed catalogs were used to produce the combined catalog, listed with an “*” in Table 3. Because of larger differences seen in the aus007a solution, the Geoscience Australia group produced two additional solutions, aus008a and aus009a, which are included in the comparisons later in this section. Later, in §10, we will present comparisons of the corresponding TRF and EOP solutions.

8.2 Creation of a Combined Catalog

The seven catalogs used to generate a combination catalog are given in Table 4. The first line is the combination catalog itself, designated maoC08a. There are two columns for the number of sources. The first gives the number of sources in the catalog and the second gives the number of sources included in the combination catalog and used in the comparisons.

In the combination procedure, only sources which were observed three or more times (number of group delays) were used. The procedure was performed recursively, eliminating outlier radio sources (5σ) from individual catalogs. The outliers are sources with small (3 – 15) numbers of observations in one or two sessions with poor network configuration

Table 3: Contributed Catalogs

Solution ID	# Sessions	# Sources	Time Range	Software	Analysis Center
aus007a*	3712	1564	1979.7-2008.7	OCCAM6.2	GA
aus008a	3774	2869	1979.7-2008.7	OCCAM6.2	GA
aus009a	3774	537	1979.7-2008.7	OCCAM6.2	GA
bkg001a*	3823	3039	1984.0-2009.2	CALC 10, SOLVE rev. 2007.10.31	BKG
gsf007a	4516	1219	1979.7-2009.2	CALC 10, SOLVE rev. 2008.12.05	GSFC
gsf007b*	4540	3414	1979.7-2009.2	CALC 10, SOLVE rev. 2008.12.05	GSFC
iaa008a	...	3009	1980.0-2009.2	QUASAR	IAA
iaa008b	...	3009	1980.0-2009.2	QUASAR	IAA
iaa008c*	...	3009	1980.0-2009.2	QUASAR	IAA
mao008a*	4541	3555	1979.7-2009.3	SteelBreeze	MAO
opa008b*	4528	3244	1979.7-2009.2	CALC 10, SOLVE rev. 2008.12.05	OP
opa008c	4434	1188	1979.7-2009.2	CALC 10, SOLVE rev. 2008.12.05	OP
usn010b*	4465	3414	1979.7-2009.2	CALC 10, SOLVE rev. 2007.11.08	USNO

Table 4: General characteristics of the combination catalog and the seven contributed solutions used to construct it.

Solution ID	$N_{sources}$		Software	Analysis Center
	total	in comb.		
maoC08a	3572	3572	Combination	MAO
aus007a	1564	1516	OCCAM6.2	GA
bkg001a	3019	2978	CALC/SOLVE	BKG
gsf007b	3414	3378	CALC/SOLVE	GSFC
iaa008c	2961	2918	QUASAR	IAA
mao008a	3555	3512	SteelBreeze	MAO
opa008b	3244	3214	CALC/SOLVE	OP
usn010b	3414	3380	CALC/SOLVE	USNO

(usually, one-baseline sessions). The combined catalog, maoC08a, consists of the coordinates of 3572 radio sources. The combined solution, maoC08a, was created using the arc-length method. The method of arc-lengths was developed at the Main Astronomical Observatory of the National Academy of Sciences of Ukraine and is described in Kur'yanova & Yatskiv [1993]. The principles of the arc-length method are:

- calculation of the arc lengths (distances on the celestial sphere) of the common ICRF1 defining sources for all individual solutions;
- construction of an intermediate reference frame, with an orientation defined by the positions of two radio sources;
- building of a combined catalog in the intermediate reference frame;
- transition from the combined catalog frame of two sources to a frame given by the positions of the ICRF1 defining radio sources.

The list of ICRF1 defining sources used consisted of 204 objects. From the 212 ICRF1 defining sources we eliminated eight sources: seven are from the special handling sources list (0014+813, 0235+164, 0637–752, 0738+313, 1308+326, 1448+762 and 2145+067) plus the source 1903–802, which is missing in bkg001a solution.

8.3 Comparison of Individual Solutions

A comparison of catalogs was performed in the following way. First, the parameters of a transformation model between two catalogs were esti-

mated with the least-squares method. Then, the model was applied to coordinates of one of the catalogs and wrms residuals for right ascension and declination were calculated. And lastly, from the comparison of three catalogs at a time (combined and the two individual ones), the so-called “external” dispersions have been evaluated.

8.3.1 Systematic Effects

For evaluation of systematic effects a transformation model was applied. The model assumes the following systematic effects: rotation of one catalog relative to another, slopes in Right Ascension and declination, a bias in declination, and harmonic terms in both coordinates (see Bolotin & Lytvyn [2008]). The differences in Right Ascension, $\Delta\alpha$, and declination, $\Delta\delta$, are presented as:

$$\Delta\alpha = A_1 \tan \delta \cos \alpha + A_2 \tan \delta \sin \alpha - A_3 + D_\alpha(\delta - \delta_0) + C_\alpha \sin(\alpha + \varphi_\alpha) \quad (2)$$

$$\Delta\delta = -A_1 \sin \alpha + A_2 \cos \alpha + D_\delta(\delta - \delta_0) + B_\delta + C_\delta \sin(\alpha + \varphi_\delta), \quad (3)$$

where A_1 , A_2 and A_3 are the rotation angles about the three axes; D_α and D_δ are the slopes in right ascension and declination as functions of the declination; B_δ is a bias in declination; C_α , φ_α and C_δ , φ_δ are amplitudes and phases of harmonic oscillations in right ascension and declination.

Table 5: Number of common sources in the catalogs (all and defining).

ID	aus008a	aus009a	bkg001a	gsf007b	iaa008c	mao008a	opa008b	usn010b
maoC08a	2847 203	536 177	2977 204	3375 204	2918 204	3505 204	3214 204	3377 204
aus008a		537 177	2736 203	2836 203	2583 203	2829 203	2804 203	2839 203
aus009a			536 177	536 177	536 171	536 177	536 177	536 177
bkg001a				2945 204	2747 204	2933 204	2883 204	2945 204
gsf007b					2897 204	3340 204	3202 204	3367 204
iaa008c						2899 204	2848 204	2898 204
mao008a							3193 204	3345 204
opa008b								3209 204

To calculate the parameters of the model the coordinates of the common (for both catalogs) ICRF1 defining sources were used. Then, after the model was applied, the wrms was evaluated for the entire set of common radio sources. The numbers of common defining sources and all sources for each pair of catalogs are presented in Table 5.

Table 6: Weighted post-fit residuals ($\Delta\alpha \cos \delta$, $\Delta\delta$), μas .

ID	aus008a	aus009a	bkg001a	gsf007b	iaa008c	mao008a	opa008b	usn010b
maoC08a	103 127	57 59	39 37	27 30	45 42	43 54	27 39	27 41
aus008a		26 19	129 128	104 109	108 115	98 102	106 108	115 110
aus009a			66 68	58 58	60 69	53 56	58 58	64 62
bkg001a				40 39	47 46	59 61	42 42	42 69
gsf007b					49 64	41 46	15 15	24 29
iaa008c						59 52	46 40	49 49
mao008a							41 46	46 55
opa008b								24 28

The results of least square estimation of model parameters are presented in Table 7 and Table 8. Table 7 shows comparison of the combined catalog, maoC08a, with the individual solutions. Mutual comparisons between individual solutions are presented in Table 8. In the tables the first lines for each pair of catalogs present the estimated values, and the second lines present the standard deviations. Parameters A_1 , A_2 , A_3 ,

Table 7: Comparison of catalogs: maoC08a vs. individual solutions. The first row for each pair presents the estimated parameters of the transformation model. The second rows present the corresponding standard deviations.

A_1	A_2	A_3	D_α	D_δ	B_δ	C_α	φ_α	C_δ	φ_δ
maoC08a – aus008a									
290.5	111.4	–164.5	–37.8	37.8	–23.3	13.0	301.7	32.3	18.5
20.1	16.6	11.6	21.3	13.1	10.6	15.2	65.1	22.1	37.4
maoC08a – aus009a									
29.0	8.1	–11.6	–26.1	–8.0	18.4	12.8	261.5	35.7	353.8
10.9	9.8	7.5	12.6	7.8	6.9	9.3	40.0	12.5	18.6
maoC08a – bkg001a									
–34.2	13.9	–14.4	–5.2	13.0	–30.0	9.2	146.3	13.2	128.3
6.8	6.0	4.1	7.2	4.8	4.3	5.0	34.9	7.7	30.6
maoC08a – gsf007b									
–1.2	–3.4	0.3	–0.9	11.2	–2.2	8.9	185.8	1.8	79.5
5.2	4.6	3.3	5.6	3.5	3.1	4.2	27.1	5.3	181.4
maoC08a – iaa008c									
–7.1	11.9	5.0	21.3	–16.7	2.5	7.5	225.8	9.4	305.8
7.8	6.9	5.1	8.6	5.4	4.8	6.8	46.3	8.8	49.2
maoC08a – mao008a									
8.8	–25.3	–6.6	1.1	–24.5	24.5	19.2	148.7	7.2	53.3
8.8	7.9	6.0	9.7	5.9	5.3	7.1	23.9	9.1	81.6
maoC08a – opa008b									
9.6	–11.8	2.3	–0.8	18.6	–12.1	5.4	191.5	6.3	73.3
6.3	5.6	4.1	6.8	4.2	3.7	5.2	54.6	6.5	65.4
maoC08a – usn010b									
–6.3	26.4	7.4	2.7	3.6	–1.6	33.7	350.5	18.2	259.5
6.5	5.7	4.2	7.0	4.5	4.0	5.2	9.2	6.7	23.3

B_δ , C_α and C_δ are in units of μas ; units for D_α and D_δ are $\mu\text{as}/\text{rad}$; and phases φ_α and φ_δ are in degrees.

In Table 6 weighted post-fit residuals for each comparison pair are shown. The residuals have been evaluated for each pair of catalogs after removing the estimated systematic effects.

As one can see from the tables, there are significant systematic effects in catalog aus008a. The angles of rotation are about 150–300 μas between aus008a and other individual solutions, while for other individual catalogs (including aus009a) the mutual rotation is about 50 μas or less. Also, standard deviations of estimated parameters for catalog aus008a are greater than the corresponding deviations of parameters for other solutions by about 2–3 times.

On the other hand, catalog aus009a shows relatively good agreement with the other individual catalogs. Catalogs aus008a and aus009a differ only in the minimum number of observations per source (> 3 for aus008a and > 100 for aus009a, which eliminated many VCS sources). This could indicate the influence of *a priori* information on results in solutions obtained by Geoscience Australia caused either by design of the least squares collocation method or its implementation. In any case, if catalog aus008a is omitted, then the remaining mutual systematic effects between seven individual catalog solutions obtained with four independent software packages do not exceed the 50 μas level.

Also we note considerably large (up to 40 μas) angles of rotation between the bkg001a catalog and other individual solutions. The reason of this change in orientation is the absence of one ICRF1 defining source, 1903–802, in the BKG solution. All the other analysis centers included observations of this source and its *a priori* coordinates were used in the

Table 8: Comparison of catalogs: comparisons between individual solutions. The first rows of each comparison present the estimated parameters of the transformation model. The second rows present the corresponding standard deviations.

A_1	A_2	A_3	D_α	D_δ	B_δ	C_α	φ_α	C_δ	φ_δ
aus008a – aus009a									
–266.4	–109.7	146.7	11.0	–18.3	22.5	12.3	138.8	4.0	234.8
4.1	3.5	2.5	4.5	2.8	2.3	3.1	14.9	4.1	67.8
aus008a – bkg001a									
–332.9	–106.2	155.3	39.1	–21.0	–9.9	27.2	118.7	43.4	167.2
21.9	18.4	12.7	23.4	14.6	11.8	16.6	34.1	25.5	28.9
aus008a – gsf007b									
–289.4	–114.7	162.3	33.9	–22.5	19.8	16.0	154.5	28.4	190.6
18.1	15.1	10.5	19.3	11.9	9.7	13.3	49.8	20.4	37.9
aus008a – iaa008c									
–287.9	–97.9	165.4	60.0	–63.5	32.1	18.2	146.7	24.7	236.5
19.0	15.9	11.1	20.3	12.6	10.3	14.1	45.9	19.2	50.7
aus008a – mao008a									
–277.4	–138.2	158.3	41.5	–71.8	59.6	25.9	134.2	23.2	153.0
16.9	14.1	10.0	18.1	11.1	9.1	12.8	28.5	19.6	41.9
aus008a – opa008b									
–277.8	–120.3	162.9	33.7	–13.8	7.9	14.4	154.3	25.0	190.3
18.2	15.1	10.6	19.3	11.9	9.7	13.3	55.3	20.5	43.1
aus008a – usn010b									
–292.9	–85.4	167.9	36.6	–21.1	11.3	26.1	1.0	46.3	226.7
19.1	15.9	11.1	20.3	12.6	10.2	13.8	32.3	19.6	26.7
aus009a – bkg001a									
–59.4	10.3	–0.4	25.8	20.5	–48.4	18.0	120.6	41.1	166.7
12.4	11.3	8.6	14.6	9.3	8.2	9.7	35.3	14.5	18.5
aus009a – gsf007b									
–31.1	–13.4	10.2	23.1	16.3	–17.8	14.6	109.1	38.1	167.7
10.7	9.7	7.4	12.6	8.0	7.1	8.5	37.1	12.5	17.2
aus009a – iaa008c									
–34.7	4.6	18.6	50.4	–10.1	–15.7	7.6	124.3	28.9	192.3
12.0	10.9	8.4	14.1	8.9	8.0	9.4	82.2	13.3	27.0
aus009a – mao008a									
–23.4	–39.0	2.3	23.7	–17.1	6.6	26.6	111.8	38.7	156.3
10.0	9.2	7.1	11.9	7.5	6.7	8.1	19.6	12.0	15.7
aus009a – opa008b									
–19.6	–24.4	12.7	23.8	21.9	–26.2	12.9	86.1	38.8	157.6
10.7	9.8	7.5	12.6	8.0	7.1	9.1	40.2	12.7	16.7
aus009a – usn010b									
–39.4	18.9	18.4	27.9	6.9	–15.7	36.8	17.8	46.6	198.1
11.6	10.5	8.0	13.6	8.7	7.7	10.2	14.5	12.6	16.4
bkg001a – gsf007b									
29.9	–19.3	14.9	4.0	–3.5	30.0	2.6	257.7	8.4	308.9
6.6	6.1	4.3	7.6	5.3	4.7	5.5	115.2	7.9	49.2
bkg001a – iaa008c									
25.1	–4.2	19.7	24.5	–31.2	34.4	10.8	287.3	19.5	306.8
7.8	7.1	5.2	9.0	6.1	5.4	6.3	35.5	9.2	24.8
bkg001a – mao008a									
36.0	–44.0	5.7	2.0	–37.7	54.8	12.1	120.8	4.6	358.1
10.1	9.3	6.9	11.6	7.8	6.9	7.9	42.9	11.6	140.8

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(Table 8: continued)

A_1	A_2	A_3	D_α	D_δ	B_δ	C_α	φ_α	C_δ	φ_δ
bkg001a – opa008b									
41.4	-29.0	17.8	5.4	2.5	21.2	5.2	323.4	6.9	351.8
6.9	6.4	4.6	7.9	5.5	4.8	5.4	65.5	8.1	63.8
bkg001a – usn010b									
22.9	11.0	22.0	7.2	-11.7	30.6	41.7	351.6	26.7	269.9
7.5	6.8	4.9	8.5	6.0	5.3	6.0	8.4	8.4	18.5
gsf007b – iaa008c									
-5.2	15.2	5.6	22.6	-26.6	3.5	7.7	297.1	11.1	302.6
7.1	6.4	4.9	8.2	5.5	4.9	5.6	46.7	8.3	39.3
gsf007b – mao008a									
7.9	-23.1	-7.1	1.7	-36.2	27.0	13.7	122.5	4.7	71.8
7.3	6.7	5.1	8.4	5.6	5.0	5.8	28.2	8.0	103.7
gsf007b – opa008b									
10.9	-9.5	2.6	0.6	6.0	-8.6	4.5	359.0	5.6	74.7
2.5	2.3	1.7	2.9	1.9	1.7	2.1	26.4	2.7	29.9
gsf007b – usn010b									
-7.1	30.5	7.6	3.9	-7.8	0.9	41.3	355.1	20.9	255.1
4.5	4.1	3.0	5.2	3.6	3.1	3.7	5.2	4.9	14.4
iaa008c – mao008a									
12.4	-38.7	-14.1	-23.8	-7.1	21.7	21.8	119.3	16.3	113.9
9.3	8.5	6.6	10.8	7.1	6.3	7.5	22.4	10.9	35.9
iaa008c – opa008b									
16.6	-25.1	-3.2	-22.1	33.2	-12.7	6.7	79.5	15.3	106.3
7.2	6.6	5.0	8.4	5.5	4.9	6.3	51.3	8.3	30.0
iaa008c – usn010b									
-2.6	15.9	1.9	-18.9	18.0	-2.0	37.4	5.4	16.5	224.5
8.4	7.6	5.8	9.7	6.5	5.8	7.2	10.7	8.9	34.3
mao008a – opa008b									
2.2	13.4	9.2	-2.1	43.8	-37.0	16.4	316.9	1.1	124.7
7.2	6.7	5.1	8.4	5.5	4.9	5.8	23.7	8.6	399.4
mao008a – usn010b									
-14.7	53.6	15.6	3.6	29.7	-27.0	50.6	342.0	25.3	255.0
8.5	7.8	6.0	9.9	6.6	5.9	7.1	8.6	9.4	22.7
opa008b – usn010b									
-17.5	39.9	5.1	3.5	-13.6	9.3	37.0	354.1	26.2	255.9
4.4	4.1	3.0	5.1	3.5	3.1	3.7	5.8	4.9	11.4

no-net-rotation constraints to fix the orientation of the obtained celestial reference frame.

Significant differences in the harmonic oscillation parameters are obtained for the usn010b catalog. Comparing with gsf007b, they are $41 \pm 4 \mu\text{as}$ and $21 \pm 5 \mu\text{as}$ for Right Ascension and declination respectively. Such deformations could be caused either by the absence of diurnal and semi-diurnal tidal EOP variations or by using an obsolete model of nutation (see Bolotin [2007]).

8.3.2 External Uncertainties

The so-called “external” uncertainties can be evaluated in the following way. For a pair of catalogs we can write (with some assumptions):

$$\overline{d_{12}^2} = \sigma_1^2 - 2\rho_{12}\sigma_1\sigma_2 + \sigma_2^2 \quad (4)$$

where $\overline{d_{12}^2}$ is the weighted mean of the squared differences between a pair of catalogs; σ_1 and σ_2 are the “external” uncertainties of the catalogs; and ρ_{12} is the corresponding correlation coefficient. By writing such equations for three catalogs, it is possible to construct a system of equations and to solve it with respect to σ_1 , σ_2 and σ_3 . The results of such calculations of external uncertainties are presented in Table 9. In these comparisons the combined solution has been used as third catalog. The calculations were done for all common radio sources in the three catalogs.

“External” uncertainties for almost all catalogs except bkg001a, aus008a, and aus009a are at the level of $50 \mu\text{as}$. For bkg001a they are about twice as great, and for aus009a catalog they are about 1.5 times greater. So, in addition to the systematic effects, these catalogs are also noisier.

Table 9: Comparison of catalogs: external uncertainties

Coordinate	index		σ_1 μas	σ_2 μas	σ_3 μas
	1	2			
α	aus008a	aus009a	58	61	6
δ	aus008a	aus009a	73	76	3
α	aus008a	bkg001a	188	89	14
δ	aus008a	bkg001a	220	73	7
α	aus008a	gsf007b	189	22	10
δ	aus008a	gsf007b	223	29	6
α	aus008a	iaa008c	192	64	14
δ	aus008a	iaa008c	219	70	6
α	aus008a	mao008a	199	57	17
δ	aus008a	mao008a	227	62	10
α	aus008a	opa008b	190	20	10
δ	aus008a	opa008b	224	30	6
α	aus008a	usn010b	190	23	11
δ	aus008a	usn010b	223	40	8
α	aus009a	bkg001a	58	24	9
δ	aus009a	bkg001a	77	27	5
α	aus009a	gsf007b	57	15	7
δ	aus009a	gsf007b	75	18	3
α	aus009a	iaa008c	57	33	11
δ	aus009a	iaa008c	76	36	4
α	aus009a	mao008a	56	38	11
δ	aus009a	mao008a	73	42	7
α	aus009a	opa008b	57	15	6
δ	aus009a	opa008b	75	16	3

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(Table 9: continued)

Coordinate	index		σ_1	σ_2	σ_3
	1	2	μas	μas	μas
α	aus009a	usn010b	57	17	7
δ	aus009a	usn010b	75	30	6
α	bkg001a	gsf007b	88	23	10
δ	bkg001a	gsf007b	115	30	7
α	bkg001a	iaa008c	85	65	14
δ	bkg001a	iaa008c	110	74	8
α	bkg001a	mao008a	94	56	13
δ	bkg001a	mao008a	119	64	9
α	bkg001a	opa008b	92	20	10
δ	bkg001a	opa008b	120	33	7
α	bkg001a	usn010b	89	23	11
δ	bkg001a	usn010b	117	42	8
α	gsf007b	iaa008c	25	64	10
δ	gsf007b	iaa008c	30	73	7
α	gsf007b	mao008a	26	55	12
δ	gsf007b	mao008a	33	62	10
α	gsf007b	opa008b	23	21	10
δ	gsf007b	opa008b	28	32	8
α	gsf007b	usn010b	24	25	11
δ	gsf007b	usn010b	29	41	9
α	iaa008c	mao008a	59	47	15
δ	iaa008c	mao008a	68	53	10
α	iaa008c	opa008b	64	23	10
δ	iaa008c	opa008b	71	32	8
α	iaa008c	usn010b	64	26	10
δ	iaa008c	usn010b	74	43	8
α	mao008a	opa008b	52	23	12
δ	mao008a	opa008b	59	32	10
α	mao008a	usn010b	56	27	12
δ	mao008a	usn010b	64	45	10
α	opa008b	usn010b	21	24	10
δ	opa008b	usn010b	33	40	9

8.4 Conclusions

Comparison of individual contributed catalog solutions have showed that the individual catalogs are very close to each other. The systematic effects in general are at the level of $50 \mu\text{as}$. The weighted post-fit residuals, evaluated after removing systematic effects for all common sources of pairs of catalogs are at the same level. That indicates good agreement between the different solutions. Considering that the individual catalogs were obtained with four independent software packages, and used slightly different data sets and analysis models, one could conclude that systematic effects and additional random errors in the newly generated celestial reference frame ICRF2 will not exceed $50 - 100 \mu\text{as}$.