9 Determination of Realistic Errors (DSM)

The formal uncertainties of source position estimates based on observation noise tend to improve by a factor of $1/\sqrt{N}$ where N is the number of observations. For sources that have a very large number of observations, the formal uncertainties are generally too small. To obtain a more realistic measure of the uncertainty, we have considered three effects: 1) modeling errors, 2) analysis noise, and 3) statistical consistency (validity) of the formal uncertainties. The sensitivities of source position estimates to different modeling choices was discussed in §6 and summarized in Table 2. These sensitivities are less than 20 μas. They should not be interpreted necessarily as errors in analysis but rather as the level of variation associated with improvements of the state-of-the-art analysis. Unmodeled or mis-modeled errors should be at this level. Analysis noise refers to the cumulative effects of data editing and modeling errors. This is quantified by comparing catalogs generated by different analysis centers and was discussed in detail in §8. Differences will result from different analysis software as well as different analysis strategies. Each analysis center may edit data differently or choose different sets of experiment sessions to include in a solution. However, the raw observation data available to all analysis centers are identical. This means that the source position estimates from the different centers will be correlated. Therefore, differences between position estimates from different solutions will not reflect the true noise in either solution. In the following, we consider how to inflate the formal source position estimates to obtain realistic uncertainties.

9.1 Decimation Test

To determine a realistic level of source position errors, we ran a decimation test in which all experiments were ordered chronologically and divided into two sets selected by even or odd session. This was done for each well-defined session type, where a session type refers to a series of experiments with the same core network of observing stations. This should help ensure that the two full sets of sessions were equivalent in terms of networks and sources observed. The remaining group of sessions not in an obvious category were similarly divided. The source position estimates from the two solutions are independent and the solution position differences provide estimates of the noise of each solution as well as how much the formal uncertainties should be scaled up. In a similar way, Ryan et al. [1993] investigated geodetic solutions to determine the uncertainty of site velocity estimates. Analysis of the differences between site velocities estimated in two terrestrial reference frame solutions that used independent session lists yielded the result that the site velocity component formal errors should be multiplied by a factor of 1.3–1.8.

The differences in source position estimates from the two decimation solutions were scaled by their formal errors and then the standard deviation of the scaled differences was computed. The histograms of the scaled differences are shown in Figure 18. The resulting scaling factors (standard deviations) were 1.6 and 1.5 for declination and Right Ascension, respectively.

The wrms difference between source position estimates, s_i, from the two solutions after removing biases is

$$\sigma^2 = \left\langle (s_1 - s_2)^2 \right\rangle = \sigma_1^2 + \sigma_2^2 \qquad (5)$$

where σ_i^2 are the solution noise variances and the estimates from the two solutions are assumed to be uncorrelated,

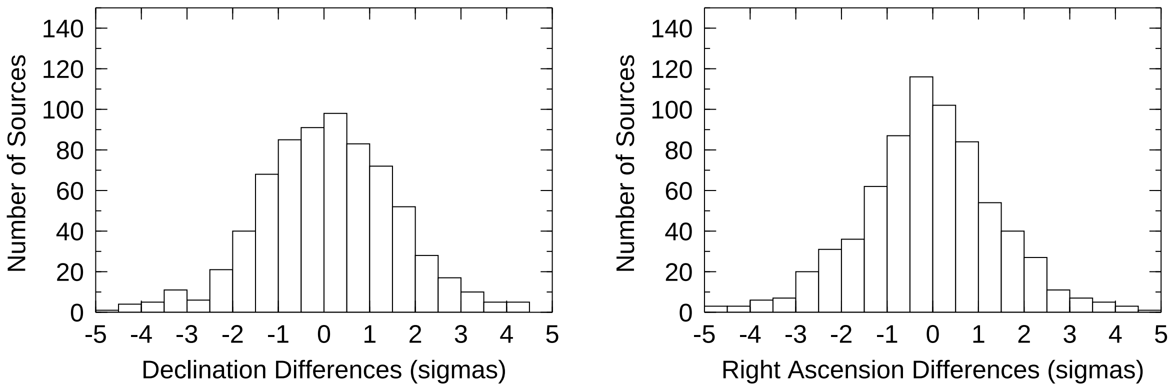

Figure 18: Histograms of declination and Right Ascension differences (scaled by sigmas) between estimates from the two decimation solutions.

Table 10: Solution Difference Statistics

Solution difference	Right Ascension wrms (μas)	Right Ascension scale factor	Declination wrms (μas)	Declination scale factor	Number of Sources
Decimation	67	1.6	52	1.54	730
gsf08b -usn10b	39	0.91	32	1.17	1136
gsf08b - iaa008c	55	1.14	38	1.06	1051
gsf08b - mao008a	66	1.37	48	1.31	1031

wrms differences were scaled by $1/\sqrt{2}$.

$$\langle s_i s_j \rangle = \sigma_i^2 \delta_{ij} \tag{6}$$

If we assume the two solutions have the same noise then we can get an estimate of the noise of each solution

$$\sigma_i \sim \sigma/\sqrt{2} \tag{7}$$

For comparison, we have computed the wrms differences (scaled by a factor of $1/\sqrt{2}$ between the GSFC solution (gsf008b) and several of the other analysis center solutions (usn010b, iaa008c, and mao008a). VCS sources from these solutions were not included in the comparisons. The average wrms differences (scaled by $1/\sqrt{2}$) for the different analysis center solutions are compared with the differences from the decimation test in Table 1.

9.2 Declination Band Noise

In Figure 19, the noise, σ_i, is shown as a function of declination band. One can see that the right ascension wrms differences for the bands north of $-45°$ declination are about 50 μas. For declination, σ_i are about 50 μas north of 30° declination, but are 60–80 μas between $-45°$ and $-30°$ declination. If the scaling factor is computed for different declination bands, one finds that it has a declination dependence, which is shown in Figure 20. The factor tends to increase with declination because higher declination sources have been observed more frequently.

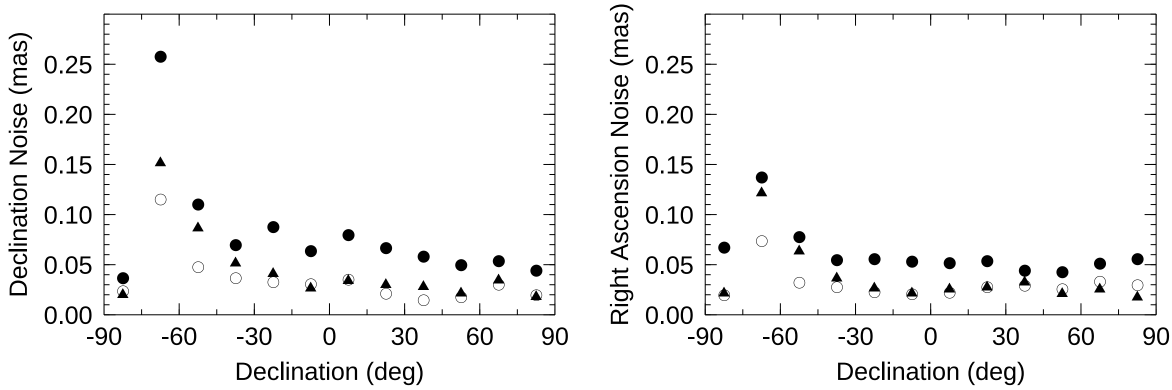

Figure 19: Declination and Right Ascension noise for each 15 degree declination band in each solution derived from differences between positions in the two decimation solutions (solid circles). The average noise for the solution differences gsf08b - usn10b (open circles) and for gsf08b - iaa008c (solid triangles) are shown for comparison.

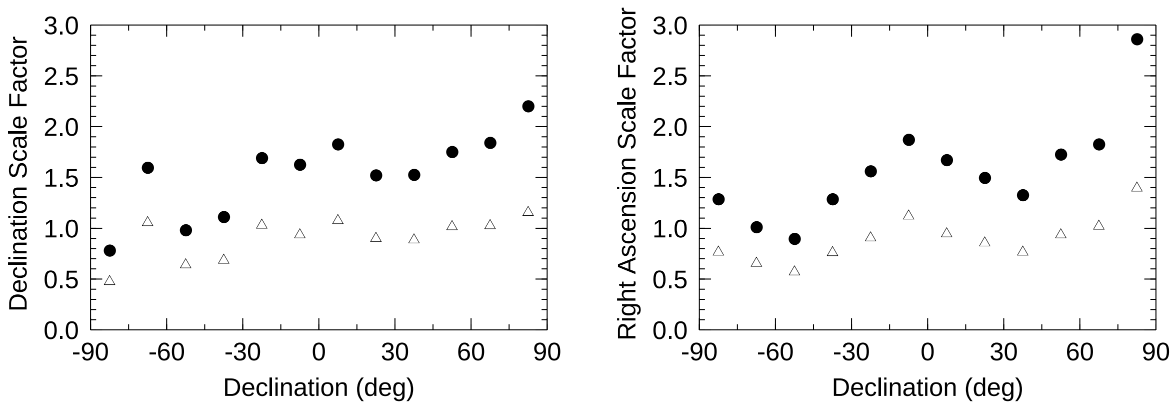

Figure 20: Formal error scaling factor for declination and Right Ascension (solid circles). Also shown is the residual scaling factor after applying a uniform average scaling factor of 1.5 to the formal uncertainties followed by a root-sum-square addition of 40 μas (open triangles).

The differences between the GSFC solution and the other analysis center solutions are shown in Figure 19 and follow the same general trend in declination as for the decimation test difference. The magnitudes of the differences are smaller because each of the analysis center solutions used approximately the same set of data so that the estimates from the two solutions are correlated. The analysis center wrms differences give a measure of analysis noise. The GSFC/USNO differences are generally the smallest since both solutions used the SOLVE analysis software. The MAO and IAA differences tend to be larger probably because these solutions used different analysis software – SteelBreeze for MAO and QUASAR for IAA.

9.3 Dependence of Source Noise on Number of Observing Sessions

The average formal precision of position generally is better as declination increases since observing has been dominated by sites in the Northern hemisphere. However, there is a large range of variation of formal precision in all declination bands. One of the motivations for inflating the position uncertainties and establishing a noise floor is to account for error sources that cannot be averaged down by more frequent observing. If all errors were Gaussian then the uncertainty of position estimates should fall off as $1/\sqrt{N}$ where N is the number of observations. Instead of looking at the dependence of the wrms differences between decimation solutions as a function of declination, we next consider the dependence on the number of sessions that a source was observed. The sources were ordered by the average number of experiment sessions in which a source was observed in the two decimation solutions. The differences in position were analyzed for a running window of 50 sources in this ordered sequence of sources. We computed the wrms difference of positions from the two solutions for each 50 source subset of all the sources common to both decimation solutions. Figure 21 shows the dependence of the wrms difference (scaled by $1/\sqrt{2}$) as a function of the minimum number of sessions in each subset. This is compared to the median formal uncertainty in the subset. The wrms differences are larger than the median formal errors and both fall off approximately as $1/\sqrt{N}$. The observed minimum error of 25 μas for declination and 15 μas for right ascension is reached for sources that have been observed in more than 200 sessions. If one applies an overall scaling factor of 1.5 based on all source position differences, one still needs to add additional noise to account for residual scaling errors that are as large as 1.5 for sources observed in less than 75 sessions. An additional 40 μas of noise in a root-sum-square sense reduces the residual scaling error to what is shown in Figure 22 at the expense of conservative uncertainties for the most observed sources.

9.4 Summary

For ICRF1, a scaling factor of 1.5 was first applied to the formal uncertainties followed by a root-sum-square increase of 250 μas. From the current decimation test, we get a similar scaling factor when averaging over all sources, but we can see that the scaling factor increases with declination since the formal uncertainties of positions tend to increase with declination. To account for this, we need to then add additional noise. Based on the noise shown in Figure 21, a value of 40 μas is a reasonable upper limit on the noise floor. The residual scale factor after applying first a scale factor of 1.5 to the original formal uncertainties and then adding 40 μas in a root-sum-square sense shown in Figure 20 is flatter and closer to unity as a function of declination. As a function of the number of sessions in which a source is observed, the residual scale factor shown in Figure 22 is generally less than unity. After applying these corrections to the formal errors, the average residual scaling factors are 0.95 for declination and 0.88 for Right Ascension.

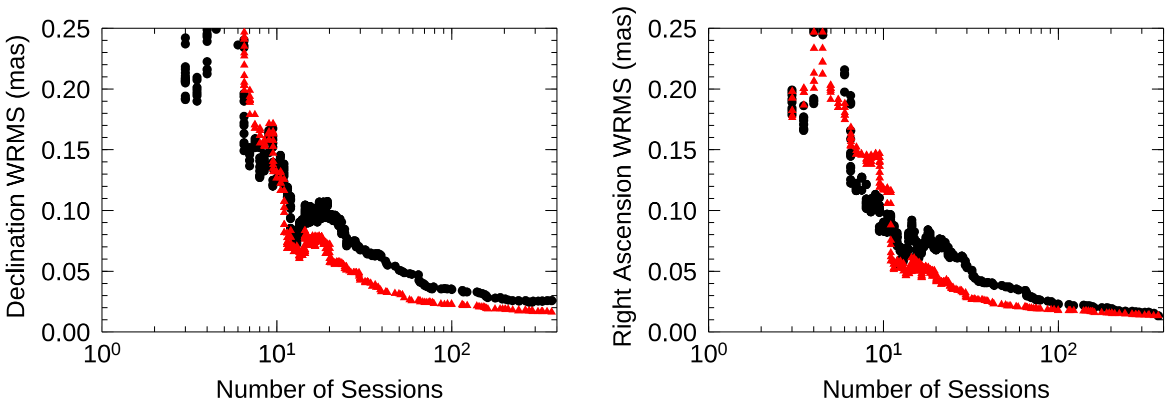

Figure 21: Wrms noise (solid circles) for subsets of 50 sources in each solution as a function of the minimum number of sessions a source was observed. The median formal uncertainty (red triangles) in each subset is shown for comparison. These were derived from differences between positions in the two decimation solutions.

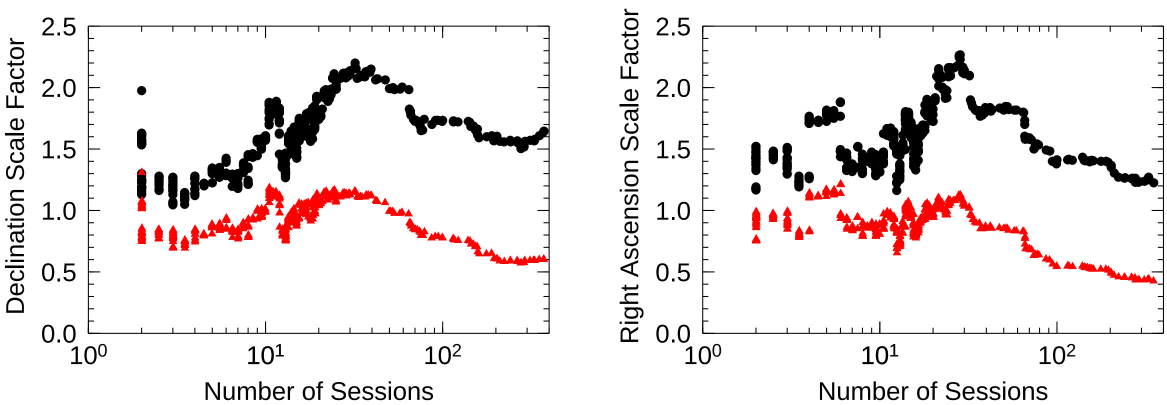

Figure 22: Error scaling factor (solid black circles) for each subset of 50 sources in each solution as a function of the minimum number of sessions a source was observed. The residual scaling factor (red triangles) after application of a scale factor of 1.5 to the formal uncertainties followed by a root-sum-square increase of 40 μas.