10 General relativistic models for space-time coordinates and equations of motion

10.1 Time coordinates

IAU resolution A4 (1991) set the framework presently used to define the Barycentric Reference System (BRS) and the Geocentric Reference System (GRS). Its third recommendation defined Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG) as time coordinates of the BRS and GRS, respectively. In the fourth recommendation another time coordinate is defined for the GRS, namely Terrestrial Time (TT). This framework was further refined by the IAU Resolutions B1.3 and B1.4 (2000) to provide consistent definitions for the coordinates and the metric tensor of the reference systems at the full post-Newtonian level (Soffel, 2000). The BRS was renamed Barycentric Celestial Reference System (BCRS) and the GRS was renamed Geocentric Celestial Reference System (GCRS). At the same time IAU Resolution B1.5 (2000) applied this framework to time coordinates and time transformations between reference systems, and IAU Resolution B1.9 (2000) re-defined Terrestrial Time (Petit, 2000). TT differs from TCG by a constant rate, $dTT/dTCG = 1 - L_G$, where $L_G = 6.969290134 \times 10^{-10}$ is a defining constant (see Chapter 1 Table 1.1). The value of L_G has been chosen to provide continuity with the former definition of TT, *i.e.* that the unit of measurement of TT agrees with the SI second on the geoid. The difference between TCG and TT is equal to

$$\mathrm{TCG} - \mathrm{TT} = \left(\frac{L_G}{1 - L_G}\right) \times (\mathrm{JD_{TT}} - T_0) \times 86400 \text{ s}, \quad (10.1)$$

where $\mathrm{JD_{TT}}$ is the TT Julian date and $T_0 = 2443144.5003725$. To within 10^{-18} in rate, it may be approximated as $\mathrm{TCG} - \mathrm{TT} = L_G \times (\mathrm{MJD} - 43144.0) \times 86400$ s where MJD refers to the modified Julian date of International Atomic Time (TAI). TAI is a realization of TT, apart from a constant offset: TT = TAI + 32.184 s.

Before 1991, previous IAU definitions of the time coordinates in the barycentric and geocentric frames required that only periodic differences exist between Barycentric Dynamical Time (TDB) and Terrestrial Dynamical Time (TDT; Kaplan, 1981). As a consequence, the spatial coordinates in the barycentric frame had to be rescaled to keep the speed of light unchanged between the barycentric and the geocentric frames (Misner, 1982; Hellings, 1986). In these systems, a quantity with the dimension of time or length has a TDB-compatible value which differs from its TDT-compatible value by a scale (see also Chapter 1). This is no longer required with the use of the TCG/TCB time scales.

The relation between TCB and TDB is linear, but no precise definition of TDB had been provided by the IAU. In the IERS Conventions (2003) the relation was given in seconds by

$$\mathrm{TCB} - \mathrm{TDB} = L_B \times (\mathrm{MJD} - 43144.0) \times 86400 \text{ s} + P_0, \quad P_0 \approx 6.55 \times 10^{-5} \text{s}, \quad (10.2)$$

with the provision that no definitive value of L_B exists and such an expression should be used with caution.

In order to remove this ambiguity while keeping consistency with the time scale (formerly known as T_{eph}) used in the Jet Propulsion Laboratory (JPL) solar-system ephemerides (see Chapter 3) and with existing TDB implementations such as (Fairhead and Bretagnon, 1990), IAU Resolution B3 (2006) was passed to re-define TDB as the following linear transformation of TCB:

$$\mathrm{TDB} = \mathrm{TCB} - L_B \times (\mathrm{JD_{TCB}} - T_0) \times 86400 \text{ s} + TDB_0, \quad (10.3)$$

where $\mathrm{JD_{TCB}}$ is the TCB Julian date and where $L_B = 1.550519768 \times 10^{-8}$ and $TDB_0 = -6.55 \times 10^{-5}$s are defining constants (see Chapter 1 Table 1.1).

Figure 10.1 shows graphically the relationships between the time scales. See <[1]> for copies of the resolutions of the IAU General Assemblies (1991, 2000, 2006)

[1] http://www.iau.org/administration/resolutions/general_assemblies/

relating to reference systems and time coordinates. IAU Resolution A4 (1991) may also be found in *IERS Technical Note 13*, pp. 137–142, IAU Resolutions B1 and B2 (2000) in *IERS Technical Note 32*, pp. 117–126, and Resolutions of the 26^{th} IAU General Assembly (2006) in Appendix A of this document.

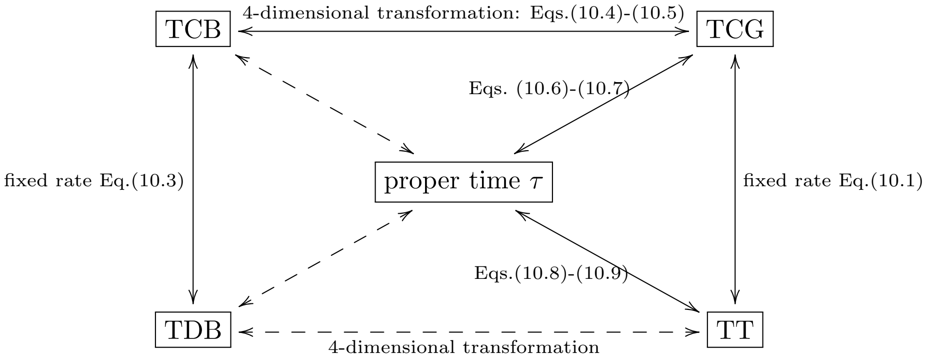

Figure 10.1: Various relativistic time scales and their relations. Each of the coordinate time scales TCB, TCG, TT and TDB can be related to the proper time τ of an observer, provided that the trajectory of the observer in the BCRS and/or GCRS is known. Transformations shown as dashed lines are not explicitly described in this document.

The difference between Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG) for any event (TCB, $\vec{x}$) in the barycentric frame involves a four-dimensional transformation,

$$\mathrm{TCB} - \mathrm{TCG} = c^{-2}\left\{\int_{t_0}^{t}[\frac{v_e^2}{2} + U_{ext}(\vec{x}_e)]dt + \vec{v}_e \cdot (\vec{x} - \vec{x}_e)\right\} + O(c^{-4}), \quad (10.4)$$

where $\vec{x}_e$ and $\vec{v}_e$ denote the barycentric position and velocity of the Earth's center of mass, and U_{ext} is the Newtonian potential of all of the solar system bodies apart from the Earth evaluated at the geocenter. In this formula, t is TCB and t_0 is chosen to be consistent with 1977 January 1, $0^{\mathrm{h}}0^{\mathrm{m}}0^{\mathrm{s}}$ TAI, i.e. the value $T_0 = 2443144.5003725$ given above. Terms not specified in (10.4) are of order 10^{-16} in rate, and IAU Resolution B1.5 (2000) provides formulas to compute the $O(c^{-4})$ terms and Equation (10.4) within given uncertainty limits up to 50000 km from the Earth.

The TCB−TCG formula (10.4) may be expressed as

$$\mathrm{TCB} - \mathrm{TCG} = \frac{L_C \times (TT - T_0) + P(TT) - P(T_0)}{1 - L_B} + c^{-2}\vec{v}_e \cdot (\vec{x} - \vec{x}_e) \quad (10.5)$$

where the values of L_C and L_B may be found in Chapter 1 Table 1.1. Non-linear terms denoted by $P(TT)$ have a maximum amplitude of around 1.6 ms.

Any of the recent solar system ephemerides mentioned in Chapter 3 could be numerically integrated to obtain a realization of Equation (10.4) with ns accuracy, see *e.g.* (Fienga *et al.*, 2009) for INPOP08. For consistency with past versions of this document, we provide in the following different realizations of Equation (10.5):

- The terms $P(TT) - P(T_0)$ may be provided by a numerical time ephemeris such as TE405 (Irwin and Fukushima, 1999), with an accuracy of 0.1 ns from 1600 to 2200. TE405 is available in a Chebyshev form at <[2]> and at the IERS Conventions Center website <[3]>. A similar product for the INPOP08 ephemeris (Fienga *et al.*, 2009) is available at <[4]>.
- The terms $P(TT)$ can be evaluated by the "FB" analytical model (Fairhead and Bretagnon, 1990; Bretagnon 2001). The 2001 version of the FB

[2] ftp://astroftp.phys.uvic.ca/pub/irwin/tephemeris
[3] ftp://tai.bipm.org/iers/conv2010/chapter10/software/
[4] http://www.imcce.fr/inpop/

model is available at the IERS Conventions Center website <[3]> or <[5]>, where the files of interest are fb2001.f, fb2001.dat, fb2001.in, fb2001.out, and README.fb2001.f. The SOFA (Standards of Fundamental Astronomy) service <[6]> also provides a routine iau_DTDB in both Fortran 77 and ANSI C to perform the computation, based on a less accurate version of the FB model.

- A series, HF2002, providing the value of $L_C \times (TT - T_0) + P(TT) - P(T_0)$ as a function of TT over the years 1600–2200 has been fit (Harada and Fukushima, 2002) to TE405. The HF2002 model is available at the IERS Conventions Center website <[3]>, where the files of interest are xhf2002.f, HF2002.DAT and xhf2002.out (However, see below the updated version XHF2002_IERS.F).

Note that TE405 is an integration of Equation (10.4) and does not account for terms in c^{-4}, and neither does HF2002 which was fit to TE405. On the other hand, the L_C value provided in Chapter 1 Table 1.1 includes a 1.15×10^{-16} contribution from terms in c^{-4} and from the effect of asteroids. For best accuracy, the linear term $1.15 \times 10^{-16} \times (TT - T_0)$ should be added to the original TE405 and HF2002 results. For convenience, a version XHF2002_IERS.F is provided at <[3]>, that directly provides the correct result of Equation (10.5) based on HF2002 and can be considered as the conventional TCB-TCG transformation.

Irwin (2003) has shown that TE405 and the 2001 version of the FB model differ by less than 15 ns over the years 1600 to 2200 and by only a few ns over several decades around the present time. HF2002 has been shown (Harada and Fukushima, 2002) to differ from TE405 by less than 3 ns over the years 1600–2200 with an RMS error of 0.5 ns. Note that in this section TT is assumed to be the time argument for computing TCB−TCG, while the actual time argument is that of the underlying solar-system ephemerides, *i.e.* a realization of TDB (see Chapter 3). The resulting error in TCB−TCG is at most approximately 20 ps.

10.2 Transformation between proper time and coordinate time in the vicinity of the Earth

Similarly to the time coordinate transformation, the formalism of the IAU resolutions is used to provide the transformation between the proper time of a clock and coordinate time. Formulas and references are presented here to perform this transformation in the vicinity of the Earth (typically up to geosynchronous orbit or slightly above). Evaluating the contributions of the higher order terms in the metric of the geocentric reference system (see IAU Resolution B1.3 (2000)), it is found that the IAU 1991 framework is sufficient for time and frequency applications in the GCRS in light of present clock accuracies. Nevertheless, in applying the IAU 1991 formalism, some care needs to be taken when evaluating the Earth's potential at the location of the clock, especially when accuracy of order 10^{-18} is required (Klioner, 1992; Wolf and Petit, 1995; Petit and Wolf, 1997; Soffel *et al.*, 2003).

In this framework, the proper time of a clock A located at the GCRS coordinate position $\mathbf{x}_A(t)$, and moving with the coordinate velocity $\mathbf{v}_A = d\mathbf{x}_A/dt$, where t is TCG, is

$$\frac{d\tau_A}{dt} = 1 - 1/c^2 \left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A) + V(X_{\mathrm{A}}) - V(X_{\mathrm{E}}) - x_A^i \partial_i V(X_{\mathrm{E}}) \right]. \quad (10.6)$$

Here, U_{E} denotes the Newtonian potential of the Earth at the position $\mathbf{x}_A$ of the clock in the geocentric frame, and V is the sum of the Newtonian potentials of the other bodies (mainly the Sun and the Moon) computed at a location X in barycentric coordinates, either at the position X_{E} of the Earth's center of mass, or at the clock location X_{A}. Only terms required for frequency transfer with

[5] ftp://maia.usno.navy.mil/conv2010/chapter10/software

[6] http://www.iausofa.org/

uncertainty of order 10^{-18} have been kept. For application to a given experiment, one should also consider the time amplitude of terms in Equation (10.6) that happen to be periodic and compare those terms to the expected time accuracy of the measurements. For example, the contribution of tidal terms (the last three terms in Equation (10.6)) will be limited to below 1×10^{-15} in frequency and a few ps in time amplitude up to the GPS orbit. In such cases, one can keep only the first three terms in relation (10.6) between the proper time τ_A and the coordinate time t:

$$\frac{d\tau_A}{dt} = 1 - 1/c^2 \left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A)\right]. \tag{10.7}$$

When using TT as coordinate time, following its defining relation $dTT/dTCG = 1 - L_G$, equations (10.6) and (10.7) are rewritten with the same accuracy as

$$\frac{d\tau_A}{dTT} = 1+L_G-1/c^2 \left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A) + V(X_{\mathrm{A}}) - V(X_{\mathrm{E}}) - x_A^i \partial_i V(X_{\mathrm{E}})\right] \tag{10.8}$$

and

$$\frac{d\tau_A}{dTT} = 1 + L_G - 1/c^2 \left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A)\right], \tag{10.9}$$

respectively. In general, the relation between the proper time of a clock and coordinate time may be obtained by numerical integration of the adequate differential equation (10.6 to 10.9). In doing so, care should be taken to evaluate the Newtonian potential U_E with the uncertainty required by each use.

For GPS satellites, with nearly circular orbits at an altitude of approximately 20200 km, the combined relativistic frequency shift given by Equation (10.9) is about 4.5×10^{-10} and it consists of a constant offset of about 4.46×10^{-10} and periodical variations with amplitudes up to 10^{-11}. The constant relativistic frequency offset is nearly compensated simply by proportionally offsetting the nominal frequency of all GPS satellite frequency standards by a conventional hardware offset of -4.4647×10^{-10}. However, due to differences of mean orbit altitudes of GPS satellites, the actual relativistic frequency offsets for individual satellites can differ from the above conventional hardware offset by up to 10^{-13}.

When retaining only the first term of the Newtonian potential, assuming a Keplerian orbit and that the constant relativistic offset is exactly compensated, integrating Equation (10.9) yields

$$\mathrm{TT} = \tau_A - \Delta\tau_A^{per}, \quad \Delta\tau_A^{per} = -\frac{2}{c^2}\sqrt{a \cdot GM_{\oplus}} \cdot e \cdot \sin E, \tag{10.10}$$

where a, e and E are the orbit semi-major axis, eccentricity and eccentric anomaly angle. Thus $\Delta\tau_A^{per}$ is the conventional GPS correction (see the GPS Interface Control Document available at <[7]>) for the periodical relativity part, which is equally due to eccentricity induced velocity and potential variations in Equation (10.9). From the above equation, one can readily see that the amplitude of the periodical correction is proportional to the orbit eccentricity, *i.e.* equal to about $2.29\mu\mathrm{s} \times e$. Since the eccentricity e for GPS orbits can reach up to 0.02, consequently the amplitude of $\Delta\tau_A^{per}$ can reach up to 46 ns. An alternative expression for the relativistic periodic correction is

$$\Delta\tau_A^{per} = -\frac{2}{c^2}\mathbf{v}_A \cdot \mathbf{x}_A, \tag{10.11}$$

which is exactly equivalent to the preceding Keplerian orbit formulation, provided that the osculating Keplerian orbit elements are used. This formulation is used *e.g.* by the IGS (International GNSS Service) for its official GPS and GLONASS clock solution products.

[7] http://www.navcen.uscg.gov/pdf/IS-GPS-200D.pdf

By retaining also the oblateness term (J_2) of the potential, one can derive (Ashby, 2003; Kouba, 2004) a simple analytical approximation that contains an apparent relativistic clock rate [8] and a 6-h term due to J_2. Comparing to a complete numerical integration, Kouba (2004) finds that the conventional periodic relativistic correction (10.11) differs by periodic terms with amplitudes of about 0.1 and 0.2 ns, and periods of about 6 hours and 14 days, respectively, and that, for most of the new (Block IIR) GPS satellites, the 6-h term is already detectable by statistical analysis in the recent IGS final clock combinations. The deficiencies of the conventional relativistic correction (10.10, 10.11) will become even more significant for Galileo (see the Galileo Interface Control Document available at <[9]>) as the frequency instability of the Galileo passive Hydrogen maser clocks is at a few parts in 10^{15} for an averaging time of several hours (Droz et al., 2009). As the 6-h J_2 term is of similar magnitude, it should be accounted for when determining and using the broadcast satellite clock model.

For low Earth orbit satellites (see *e.g.* Larson *et al.*, 2007 for GRACE and TOPEX), the term in J_2 is more important than at the GPS altitude so that Equation (10.11) may be significantly in error or even completely misleading. It is necessary to perform a numerical integration of Equation (10.9) using the term in J_2 for the potential.

10.3 Equations of motion for an artificial Earth satellite[10]

The relativistic treatment of the near-Earth satellite orbit determination problem includes corrections to the equations of motion, the time transformations, and the measurement model. The two coordinate systems generally used when including relativity in near-Earth orbit determination solutions are the solar system barycentric frame of reference (BCRS) and the geocentric or Earth-centered frame of reference (GCRS), see Section 5.1.

Ashby and Bertotti (1986) constructed a locally inertial E-frame in the neighborhood of the gravitating Earth and demonstrated that the gravitational effects of the Sun, Moon, and other planets are basically reduced to their tidal forces, with very small relativistic corrections. Thus the main relativistic effects on a near-Earth satellite are those described by the Schwarzschild field of the Earth itself. This result makes the geocentric frame more suitable for describing the motion of a near-Earth satellite (Ries *et al.*, 1989). Later on, two advanced relativistic formalisms have been elaborated to treat the problem of astronomical reference systems in the first post-Newtonian approximation of general relativity. One formalism is due to Brumberg and Kopeikin (Kopeikin, 1988; Brumberg and Kopeikin, 1989; Brumberg, 1991) and another one is due to Damour, Soffel and Xu (Damour, Soffel, Xu, 1991, 1992, 1993, 1994). These allow a full post-Newtonian treatment (Soffel, 2000) and form the basis of IAU Resolutions B1.3 and B1.4 (2000).

In the GCRS, the relativistic correction to the acceleration of an artificial Earth satellite is

$$\begin{aligned}\Delta\ddot{\vec{r}} = \; & \frac{GM_E}{c^2r^3}\left\{\left[2(\beta+\gamma)\frac{GM_E}{r} - \gamma\dot{\vec{r}}\cdot\dot{\vec{r}}\right]\vec{r} + 2(1+\gamma)(\vec{r}\cdot\dot{\vec{r}})\dot{\vec{r}}\right\} + \\ & (1+\gamma)\frac{GM_E}{c^2r^3}\left[\frac{3}{r^2}(\vec{r}\times\dot{\vec{r}})(\vec{r}\cdot\vec{J}) + (\dot{\vec{r}}\times\vec{J})\right] + \\ & \left\{(1+2\gamma)\left[\dot{\vec{R}}\times\left(\frac{-GM_S\vec{R}}{c^2R^3}\right)\right]\times\dot{\vec{r}}\right\},\end{aligned} \qquad (10.12)$$

where

[8] Equation (28) in (Kouba, 2004) has a sign error for the (J_2) rate term. The correct expression may be found in Equation (85) of (Ashby, 2003).

[9] http://ec.europa.eu/enterprise/policies/satnav/galileo/open-service/

[10] The IAU Resolutions B1.3 and B1.4 (2000) and references therein now provide a consistent framework for the definition of the geocentric and barycentric reference systems at the full post-Newtonian level using harmonic coordinates. The equations of motion for spherically-symmetric and uniformly rotating bodies in these systems are the same as those previously derived in a Parameterized Post-Newtonian system.

c = speed of light,

β, γ = PPN (parameterized post-Newtonian) parameters, equal to 1 in General Relativity,

$\vec{r}$ is the position of the satellite with respect to the Earth,

$\vec{R}$ is the position of the Earth with respect to the Sun,

$\vec{J}$ is the Earth's angular momentum per unit mass ($|\vec{J}| \cong 9.8 \times 10^8 \mathrm{m}^2/\mathrm{s}$), and

GM_E and GM_S are the gravitational coefficients of the Earth and Sun, respectively.

For satellites in the vicinity of the Earth (up to geostationary orbit) the terms in Equation (10.12) can be evaluated with respect to the main Newtonian acceleration, as follows. The Schwarzschild terms (first line) are a few parts in 10^{10} (high orbits) to 10^9 (low orbits) smaller; the effects of Lense-Thirring precession (frame-dragging, second line) and the geodesic (de Sitter) precession (third line) are about 10^{11} to 10^{12} smaller. The main effect of the Schwarzschild terms is a secular shift in the argument of perigee while the Lense-Thirring and de Sitter terms cause a precession of the orbital plane at a rate of the order of 0.8 mas/y (geostationary) to 180 mas/y (low orbit) for Lense-Thirring and 19 mas/y (independent of orbit height) for de Sitter. The Lense-Thirring terms are less important than the geodesic terms for orbits higher than Lageos (altitude above 6000 km) and more important for orbits lower than Lageos. The observable effects and their magnitude depend on the particular characteristics of each satellite orbit and on the set-up of the analysis software. *E.g.,* neglecting the Schwarzschild terms while adjusting orbit parameters may cause an apparent reduction of the orbit radius by 4 mm for circular orbits at all heights (Hugentobler, 2008).

The relativistic effects of the Earth's oblateness have been neglected here as they are typically even smaller but, if necessary, they could be included using the full post-Newtonian framework of IAU Resolutions B1.3 and B1.4 (2000). The independent variable of the satellite equations of motion may be, depending on the time transformation being used, either TT or TCG. Although the distinction is not essential to compute this relativistic correction, it is important to account for it properly in the Newtonian part of the acceleration.

10.4 Equations of motion in the barycentric frame (see footnote 10)

The n-body equations of motion for the solar system frame of reference (the isotropic Parameterized Post-Newtonian system with Barycentric Coordinate Time (TCB) as the time coordinate) are required to describe the dynamics of the solar system and artificial probes moving about the solar system (for example, see Moyer, 1971). These are the equations applied to the Moon's motion for lunar laser ranging (Newhall *et al.*, 1987). In addition, relativistic corrections to the laser range measurement, the data timing, and the station coordinates are required (see Chapter 11).

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