Local tie adjustment: from principle to implementation

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Introduction

- Physical def. of Ref. point
- Local survey
- Geometrical constraints
- SINEX

- What to measure?
- How to measure it?
- How to parametrize the problem?
- How to compute?
- What is the final product?
Reference point definition

- Completely linked with the way it is modeled in SG softwares
- Goal= to realize from terrestrial measurement, the physical definition
- Trying to reproduce in the sequence of terrestrial measurements what is done in SG Analysis softwares
Types of instruments

- "Big" instruments (VLBI & SLR) which have degrees of freedom and cannot be removed to observe a physically \textit{linked} ground marker
- "Small" instruments which can be removed and which are potentially physically \textit{linked} to a ground marker

- Recovering the information reproducing the physical definition
- Direct measurement of the ground marker
Example

Medicina 2001 and 2002 surveys

VLBI + GPS
Basic observations

- Direct measurement of VLBI antenna targets
- Direct measurements of GPS antenna

CLASSICAL GEODESY OBSERVATIONS
Scheme

\[ \Sigma^{-1}_\xi = \Sigma^{-1}_\xi \]

- Normal equation coming from classical geodesy observations with respect to target coordinates (\( \xi \)).

- Additional constraints which link previous coordinates (\( \xi \)) to Ref. Points coordinates (\( \pi \)).
Optimisation problem

\[
\min_{\xi, \pi} (\xi - \tilde{\xi})^T \Sigma^{-1} (\xi - \tilde{\xi})
\]

u.c. \( F(\xi, \pi) = 0 \)

\[
F(\xi, \pi) = 0 \iff \begin{align*}
g(\xi, \pi) &= 0 & (\lambda) \\
q(\pi) &= 0 & (\mu) \\
h(\xi, \pi) &= 0 & (\eta)
\end{align*}
\]
Constraints

- While the VLBI antenna is moving around one axis, a target belongs to a circle.
- The centers are aligned and are connected to the antenna Ref. Point.
- Link between GPS antenna observations and GPS Ref. Point.

π = Planes par. Circle centers VLBI Ref. Point GPS Ref. Point
System

\[
\begin{pmatrix}
\Sigma^{-1} & 0 & G_1^T & 0 & H_1^T \\
0 & 0 & G_2^T & Q^T & H_2^T \\
G_1 & G_2 & 0 & 0 & 0 \\
0 & Q & 0 & 0 & 0 \\
H_1 & H_2 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\xi \\
\pi \\
\lambda \\
\mu \\
\eta
\end{pmatrix}
=
\begin{pmatrix}
\Sigma^{-1} \tilde{\xi} \\
0 \\
K \\
L \\
J
\end{pmatrix}
\]
Solution

Inversion of

\[
\begin{pmatrix}
\Sigma^{-1} & 0 & G_1^T & 0 & H_1^T \\
0 & 0 & G_2^T & Q^T & H_2^T \\
G_1 & G_2 & 0 & 0 & 0 \\
0 & Q & 0 & 0 & 0 \\
H_1 & H_2 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\xi \\
\pi \\
\lambda \\
\mu \\
\eta
\end{pmatrix}
= \begin{pmatrix}
\Sigma^{-1} \tilde{\xi} \\
0 \\
K \\
L \\
J
\end{pmatrix}
\]

Gives estimates \( \hat{\xi}, \hat{\pi} \)

And residuals

\[
\hat{V} = \tilde{\xi} - \hat{\xi}
\]
Covariance of $\pi$

(must be multiplied by a variance factor $\sigma^2$)

$$\sigma^2 = \frac{\hat{V}^T \Sigma^{-1} \hat{V}}{n_{obs} + n_{cons} - n_{unkn}}$$
Example: Medicina 2001 and 2002 surveys

<table>
<thead>
<tr>
<th>Local tie</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stations</td>
<td>110</td>
<td>105</td>
</tr>
<tr>
<td>azimuth angles</td>
<td>297</td>
<td>308</td>
</tr>
<tr>
<td>zenith angles</td>
<td>289</td>
<td>339</td>
</tr>
<tr>
<td>distances</td>
<td>272</td>
<td>327</td>
</tr>
<tr>
<td>unknowns</td>
<td>324</td>
<td>312</td>
</tr>
</tbody>
</table>

\[ \sigma^2 \approx 3.1 \]
## Uncertainties

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Evlbi</th>
<th>Nvlbi</th>
<th>Uvlbi</th>
<th>Egps</th>
<th>Ngps</th>
<th>Ugps</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ (mm) 2001</td>
<td>0.30</td>
<td>0.40</td>
<td>0.82</td>
<td>1.02</td>
<td>1.07</td>
<td>0.85</td>
</tr>
<tr>
<td>σ (mm) 2002</td>
<td>0.22</td>
<td>0.29</td>
<td>0.56</td>
<td>0.82</td>
<td>0.56</td>
<td>0.53</td>
</tr>
</tbody>
</table>

\[
\rho_{2002} = \begin{pmatrix}
1 & 0.0595 & -0.0004 & 0.0035 & -0.0131 & -0.0005 \\
1 & -0.0054 & -0.0149 & 0.1773 & -0.0002 \\
1 & 0.0003 & -0.0009 & 0.3985 \\
1 & -0.0587 & -0.0169 \\
1 & -0.0122 \\
1
\end{pmatrix}
\]
Transformation into the ITRF

- The underlying reference frame is arbitrary
- So, an arbitrary transformation can be applied
- Anyway, a set a local tie should be assumed to be expressed in a fuzzy reference frame
- In our case, application of a mean rotation and a mean translation without introducing associated uncertainty
%SNX 1.00

+FILE/COMMENT
* The DOMES are correct
* Variance factor equal to 1.00
-FILE/COMMENT

+SITE/ID
*CODE PT __DOMES__ T __STATION DESCRIPTION__ APPROX_LON__ APPROX_LAT__ APP_H__
VLBR  A 12711S001
GPSR  A 12711M003

-SITE/ID

+SOLUTION/ESTIMATE
*INDEX TYPE __ CODE PT SOLN __ REF_EPOCH__ UNIT S __ESTIMATED VALUE__ STD_DEV
1   STAX   VLBR  A    1  0:000:00000 m    2  4.461369978747157E+06  2.67134E-004
2   STAY   VLBR  A    1  0:000:00000 m    2  9.195968255608416E+05  5.44588E-004
3   STAZ   VLBR  A    1  0:000:00000 m    2  4.449559200474912E+06  2.80537E-004
4   STAX   GPSR  A    1  0:000:00000 m    2  4.461400895152843E+06  6.81391E-004
5   STAY   GPSR  A    1  0:000:00000 m    2  9.195934231391583E+05  5.43265E-004
6   STAZ   GPSR  A    1  0:000:00000 m    2  4.449504680825088E+06  7.14008E-004

-SOLUTION/ESTIMATE

+SOLUTION/MATRIX_ESTIMATE L COVA
*PARA1 PARA2 __ PARA2+0__________ PARA2+1__________ PARA2+2__________
1   1  7.136055600058919E-08
2   1  2.715051926328614E-09  7.801475999667776E-07
3   1 -1.95794692585922E-08  6.082195561979286E-08  9.883862016862772E-08
4   1  1.258598565862185E-08  2.544143821923472E-09 -1.306511842288357E-08
5   1  8.642941241946729E-07
6   1 -1.501342154124068E-08  2.450421396973857E-08  2.347635228442708E-08

-SOLUTION/MATRIX_ESTIMATE L COVA
%ENDSNX
Conclusion

- Definition of the Reference Point
- Design of local survey
- Implementation
- documentation