Discussion on combination strategies at the normal equation level

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Two discussion issues

1. Polar Motion representation and their link to the reference frame

2. Evaluation of the combination strategy based on a tri-dimensional similarity
IGS05 RF stations

Current IGS05 76 usable RF stations
IGS05 RF stations
Impact of using 37(of 76) IGS RF stations

Drift (µas/yr)
Xpole  Ypole
2.0     2.5
\approx 0.06 \text{ mm/yr}
SLR selected RF stations

What happens to PM estimates if we drop the western (NASA) stations?
What happens to PM if we drop SLR western stations?

Drift ($\mu$as/yr)
Xpole  Ypole
10     12

In mm/yr
0.30 0.36

Origin change
0.30 mm/yr

Scale change
0.12 mm/yr
Number of VLBI sessions 2000.0 - 2009.0

500 sessions

# of VLBI sessions
Number of VLBI sessions 2000.0 - 2009.0

- RF stations
  - more than 100 sessions
  - spanning 2000 - 2009

500 sessions

# of VLBI sessions
Impact on VLBI PM when dropping southern RF stations

Drift (μas/yr)
Xpole  Ypole
3      19

In mm/yr
0.09    0.57

Scale change:
0.24 mm/yr
ITRF2008P PM residuals
ITRF2008P PM residuals
CATREF combination model

\[
\begin{align*}
X_s^i &= X_c^i + (t_s^i - t_0)\dot{X}_c^i \\
&\quad + T_k + D_k X_c^i + R_k X_c^i \\
&\quad + (t_s^i - t_k) \left[ \ddot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \right] \\
\dot{X}_s^i &= \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i
\end{align*}
\]

Station Positions & Velocities

EOPs at epochs \( t_k \)

EOPs time derivatives

\[
\begin{align*}
x_s^p &= x^p + R2_k \\
y_s^p &= y^p + R1_k \\
UT_s &= UT - \frac{1}{f} R3_k
\end{align*}
\]

\[
\begin{align*}
\dot{x}_s^p &= \dot{x}^p + R2_k \\
\dot{y}_s^p &= \dot{y}^p + R1_k \\
LOD_s &= LOD + \frac{\Lambda_0}{f} R3_k
\end{align*}
\]
Discussion

• Q: Should there be any physical link between PM daily rate and the frame orientation time evolution?
• A: The answer is probably no

• Q: Do we need to estimate daily PM rates?
• A: Yes and No (?)
  – Yes: to map PM at common epochs for multi-technique combination
  – No: if all techniques provide PM at the same epochs

• Q: are PM rates estimated by different techniques consistent?
PM rate residuals

1 mas

10 mas

[Graphs showing PM rate residuals for XPOR IVS and XPOR IGS for 1 mas and 10 mas.]
Evaluation of the combination strategy based on a tri-dimensional similarity
Objective:

• To evaluate the combination strategy used to compute ITRF at IGN
• To help answering basic questions such as:
  
  • Can GPS frame origin be accurately tied to the SLR origin?
  • How accurate is the frame datum definition?
  • What is the minimum number of local ties that need to be available?
  • etc…
**Simulation**

Simulate space geodetic daily/weekly TRF with realistic noise at identical time sampling of the real data.

\[ X(t) = V^{\text{Truth}}(t-t_0) + X^{\text{Truth}}(t_0) + \Sigma \text{discont} + \delta X_{\text{spatial}} + \delta X_{\text{flicker noise}} \]

From ITRF2008 prel. solution

From:
- NNR-Nuvel 1A +
- Post-glacial Rebound ICE-5Gv1.2/VM4 (Peltier)

Gaussian noise from real Cov. Matrix (Minimum constraints applied)

GPS only From published model

To simulate local tie data with realistic noise

\[ dX^{1,2} = X^{\text{truth},2} - X^{\text{truth},1} + \delta X \]

Gaussian noise from real uncertainty
What do we want to retrieve?

horizontal

vertical
What do the simulated data look like?

Ex: a GPS time series

![Graphs showing simulated GPS data](image)
Recall ITRF computation strategy

Time series

VLBI
- 1980 - 2009
- \( (X_R, V_R, EOPs) \)

SLR
- 1983 - 2009
- \( (X_L, V_L, EOPs) \)

DORIS
- 1993 - 2009
- \( (X_D, V_D, EOPs) \)

GPS
- 1997 - 2009
- \( (X_P, V_P, EOPs) \)

Sets of positions

Stacking

Local ties

Constrains for Reference Frame definition

COMBINATION

\( (X_{ITRF}, V_{ITRF}, EOPs) \)
Stacking

Mean Repeatability of the time series compared to the real data

DORIS

SLR
Stacking

Mean Repeatability of the time series compared to the real data

VLBI

GPS

Real data

synthetic data
## Combination

### Definition of the combined frame:
**Origin (SLR), Scale (VLBI), Orientation (GPS)**

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<th>Solution</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>D</th>
<th>R1</th>
<th>R2</th>
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Sensibility in the local ties uncertainty clearly visible
Combination

**Definition of the combined frame:**
Origin (SLR), Scale (VLBI), Orientation (GPS)

**Local ties fixed to their true value at 0.1 mm level**
**NO FLICKER NOISE**

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Conclusions:

• No problem in using tri-dimentional similarities for TRF computation
• All technique frame origin tied to SLR origin at the level of 0.2 mm/yr
• Weakness in the scale offset mostly due to local tie uncertainties (0.2 ppb). Scale rate accuracy at the level of 0.01 ppb/yr

Perspectives:

• Add systematic errors (velocity biases, non-modeled small offsets etc…)
• Answer the following questions

  ❑ Which local tie need to be re-measured to add robustness in the multi-technique combination?
  ❑ What is the impact of re-measuring all local ties with the precision of the most recent surveys?