# Ocean Tide Loading

Variance reduction in mm$^2$ using various flavors of OTL

7-day arcs, estimating only orbit parameters, 2008-2009  
(reference fixed to LPOD2005 and IERSC04)  
RMS $\sim$10 mm (7-8 mm for best stations)

<table>
<thead>
<tr>
<th></th>
<th>CSR3.0 tides OL (incl. geocenter)</th>
<th>FES2004 tides OL (incl. GCM)</th>
<th>FES2004 tides OL (non incl. GCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS-1</td>
<td>30.1</td>
<td>32.4</td>
<td>21.7</td>
</tr>
<tr>
<td>LAGEOS-2</td>
<td>31.9</td>
<td>34.3</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Not including tidal geocenter is worse than using inferior ocean tide model
Some Comments on Selected Models

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\[
\dot{F} = (I_{xz} / f, I_{yz} / f, 1), \quad I_{xz} = -C_{21} MR^2, \quad I_{xy} = -S_{21} MR^2, \quad f = C - A
\]

Modeling of the variations of $C_{21}$ and $S_{21}$

\[
C_j(t) = \overline{C}_j(t_0) + \bar{Y}_j(t - t_0) + \Delta C_j^{st} + \Delta C_j^{ot} + \Delta C_j^{spt} + \Delta C_j^{opt} + \Delta C_j^L
\]

where
- st: solid Earth tides, IERS 2003
- ot: ocean tides from FES2004 model
- spt: solid Earth pole tide, IERS2003
- opt: ocean pole tide, [Desai et al., 2004]
- $L$: surface mass loading changes

The linear approximation will be valid only over a limited time span.
C21/S21 and the Mean Pole

In the absence of external torques, the Earth’s mean figure axis and the mean rotation axis will be aligned (Wahr, 1987). Thus, the mean values for the C21 and S21 gravity coefficients (normalized) are related to the mean pole and can be expressed as

\[(\overline{C}_{21}, \overline{S}_{21}) = \sqrt{3} \overline{C}_{20}(\overline{x}_p, -\overline{y}_p) + h.o.t.\]

This is purely a reference frame effect; in a reference frame whose z axis is aligned with the mean pole, \(C21=S21=0\).

This is the formulation adopted for C21/S21 and (their rates) in the IERS2003 conventions, where the mean pole drift was defined as purely secular, and C22/S22 were also included in h.o.t..

This was adopted since these harmonics were too small to estimate accurately (order 1e-9 – 1e-10)
C21/S21 from SLR and GRACE

- SLR estimates agree well at epoch 2000.0 with conventions but significant slope difference observed (reference rate is IERS2003).
- Extrapolating GRACE results to 2000.0 probably speculative but results not bad, especially for S21.
- Seasonal signal is small in C21 so agreement is not as apparent as for S21.
- Both series indicate correction to rate.

GRACE 30-day estimates from GRACE; SLR 60-day estimates using LAGEOS-1/2; AOD restored to GRACE.
Comparison of rates for C21/S21 (unit of $10^{-12}$/year).

<table>
<thead>
<tr>
<th>Data</th>
<th>$\dot{C}_{21}$</th>
<th>$\dot{S}_{21}$</th>
<th>Time span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Pole (IERS)</td>
<td>-3.4</td>
<td>16.6</td>
<td>1976-2000</td>
</tr>
<tr>
<td>Mean Pole</td>
<td>-8.1</td>
<td>5.8</td>
<td>1993-2009</td>
</tr>
<tr>
<td>Mean pole</td>
<td>-16.3</td>
<td>2.7</td>
<td>2002-2009</td>
</tr>
<tr>
<td>SLR</td>
<td>-5.2</td>
<td>8.4</td>
<td>1993-2009</td>
</tr>
<tr>
<td>SLR</td>
<td>-9.3</td>
<td>5.8</td>
<td>2002-2009</td>
</tr>
<tr>
<td>GRACE</td>
<td>-17.2</td>
<td>4.0</td>
<td>2002-2009</td>
</tr>
<tr>
<td>GIA</td>
<td>-0.2</td>
<td>1.2</td>
<td>millennia</td>
</tr>
</tbody>
</table>
‘Mean’ Pole (Yp)

Mean Yp JR(solid), ZM(dash), IERS2003(dot), DG(short dash), CB(dash–dot)
Summary

• Who is (or should be) using ocean tide loading with tidal geocenter included?
  – VLBI should not (no sensitivity to geocenter)
  – Satellite techniques should (though for GPS this is not completely clear)

• Current IERS Conventions are inconsistent with observed mean pole beyond ~2000.0
  – 2003 Conventions based on linear fit from 1976-1999
  – Is being updated to use higher order model; affects both solid earth pole tide and ocean pole tide
  – Principal effect is on dynamical models but mean pole affects station displacement model as well
C21/S21 and the Mean Pole (2)

Is there a Love number involved? i.e.,

\[
(\bar{C}_{21}, \bar{S}_{21}) = \frac{k}{k_0} \sqrt{3} \bar{C}_{20}(\bar{x}_p, \bar{y}_p) + \text{h.o.t.}
\]

If the averaging interval is not adequately long, perhaps dynamics become involved and a non-unity \(k/k_0\) applies; i.e., no longer purely ‘geometric’

Additional caveats:
(1) what averaging interval is needed to compute a good ‘mean’ (>>14 months)
(2) rotational axis pertains to the mantle; core may have different rotation axis

Using GRACE and SLR, we can estimate C21/S21 directly for the entire Earth (mantle+core) and compare to values inferred from mean pole determination, and perhaps observe something about the core.
Contribution of the Core

Assuming that there are no contributions to the relative angular momentum vector from the mantle, any residual mean C21 and S21 are proportional to the relative angular momentum vector of the core, caused by a dynamic torque from the mantle and a misalignment of the core figure axis [Wahr, 1987]. The products of inertia of the core can be expressed as follows (N is the mean torque on the core)

$$ (I^c_{xz}, I^c_{yz}) = \left( N^c_y, -N^c_x \right) / \Omega^2 - \sqrt{5/3} M_{ae}^2 (\overline{C}_{21}, \overline{S}_{21}) $$