Combined analysis of GNSS and SLR data using satellite co-locations

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Overview

1. Combination of GNSS and SLR: **General considerations**

2. **Expectation** from a combined analysis of GNSS and SLR@GNSS

3. **Results** from one year of combining GNSS and SLR@GNSS

4. Conclusions and outlook
Combination of GNSS and SLR

Co-location at stations:
→ Application of known local tie values
→ GNSS observations of ground network; SLR observations to Lageos

Problem:
→ Phase center modelling of GNSS antenna
→ Local ties

Co-location at satellites:
1.) GNSS satellite: Transmitting point for GNSS
   Reflecting point for SLR (target)
2.) LEO satellite: Receiving point for GNSS (“kinematic station”)
   Reflecting point for SLR (target)

→ Vector of GNSS and SLR reference points w.r.t. satellite CoM needed
Co-location at GNSS satellites

**SLR to GNSS satellites:**

1. **Validation** of microwave-only GNSS orbits by SLR range residuals
   ⇒ control of a priori models (e.g., radiation pressure)

2. **Combined orbit determination**
   ⇒ only few SLR normal points per day (~ 10-20 per satellite)
   ⇒ problematic on daily basis

Strength of **SLR to geodetic satellites** (Lageos,...):

Reference frame: mainly **scale** and **geocenter**

⇒ Applicable as well for SLR @ GNSS?
## Expectation from combined analysis

<table>
<thead>
<tr>
<th></th>
<th>GNSS @GNSS</th>
<th>SLR @GNSS</th>
<th>SLR @Lageos</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Radiation pressure</strong></td>
<td>Problems in RPR modelling</td>
<td>Problems in RPR modelling</td>
<td>RPR well modelled</td>
</tr>
<tr>
<td>↔ <strong>Geocenter</strong></td>
<td></td>
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<tr>
<td><strong>GNSS satellite antenna phase center</strong></td>
<td>Problems in phase center modelling</td>
<td>independent</td>
<td></td>
</tr>
<tr>
<td>↔ <strong>Scale</strong></td>
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<tr>
<td><strong>Range biases</strong></td>
<td>-</td>
<td>Decorrelated if different elevation angles</td>
<td>For a few sites only</td>
</tr>
<tr>
<td>↔ <strong>Scale</strong></td>
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**SLR @ GNSS:**
- Geocenter is affected as well
- **Scale** is transferred directly to GNSS
- **Independent** control of local ties
Combined analysis of GNSS and SLR

- Studies using one year of data: 2008
- Analysis and combination with the *Bernese GPS Software*

→ **GNSS-only NEQs (daily):**
  - Combined GPS+Glonass analysis performed at CODE

→ **SLR-only NEQs (daily):**
  - SLR data to GPS and Glonass satellites
  - Parameterization identical to GNSS analysis (orbits, ERP, geocenter)
  - In addition: Range biases

→ **Combination:**
  - Daily combination
  - Accumulation to annual solution
  - Use of “space ties” only, no “local ties”!
Network in 2008

Satellites tracked:

- GPS: G05, G06
- GLONASS: R15, R24, R07, R11

Altogether: 25 SLR sites
31,855 normal points (NP)

- 8 sites with > 1000 NP
- 5 sites with > 500 NP
- 6 sites with > 100 NP
- 6 sites with < 100 NP

Northern hemisphere:
16,804 NP

Southern hemisphere:
15,051 NP
Combined analysis of GNSS and SLR

Questions to be answered:

Are satellite co-locations strong enough to fully replace co-locations at ground?  
(space ties instead of local ties)

Is an estimation of **GNSS SAO** together with **SLR range biases** possible or does this cause a rank deficiency for the **scale**?
## Combined analysis of GNSS and SLR

<table>
<thead>
<tr>
<th></th>
<th>GNSS SAO fixed on igs05.atx</th>
<th>GNSS SAO corrections estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M</strong></td>
<td>No SLR range bias</td>
<td>SLR and GNSS deliver information on the scale</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>SLR range bias <em>per satellite</em> (G05, G06, R15, R24, ...)</td>
<td>Scale of combined solution comes only from SLR</td>
</tr>
<tr>
<td><strong>K</strong></td>
<td>SLR range bias <em>per system</em> (GPS, GLONASS)</td>
<td>Scale of combined solution is a weighted mean of the GNSS and SLR scale</td>
</tr>
<tr>
<td><strong>J</strong></td>
<td>SLR range bias <em>per station</em></td>
<td>SLR delivers information on the scale</td>
</tr>
</tbody>
</table>

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**Note:**
- GNSS SAO corrections are estimated on igs05.atx.
- The scale of the combined solution comes only from SLR in case of no SLR range bias.
- The scale of the combined solution is a weighted mean of the GNSS and SLR scale in case of SLR range bias per system (GPS, GLONASS).
- In case of SLR range bias per station, the scale of the combined solution comes only from SLR.
- The scale of the combined solution comes only from SLR in case of no SLR range bias.
Comparison with SLRF2005

SLR station coordinates are well determined by using solely “space ties” (no local ties to GNSS sites; not included in datum definition)

**Prerequisite:** sufficient number of SLR observations (~ 200-300 NP)

Stations sorted according to number of SLR observations (descending order)
Comparison with local ties

Combining GNSS and SLR@GNSS using „space ties“ only
→ Combined yearly GNSS+SLR solution
→ Datum definition: NNR+NNT (GNSS core sites), no Local ties
→ GNSS antenna phase center (SAO) fixed = Scale defined by GNSS and SLR

**Red:**
From annual single-technique solutions

**Blue:**
Combined solution, GNSS SAO fixed
Comparison with local ties

Impact of GNSS antenna phase center (SAO) estimation:
→ No big differences ⇒ Estimation of SAO is possible
→ Slightly better agreement with local ties if scale is NOT defined by GNSS (has to be verified using longer time span of data!)

**Blue:**
Combined solution, GNSS SAO fixed

**Green:**
Combined solution, GNSS SAO estimated
Simultaneous estimation of GNSS antenna phase center offsets and SLR range biases is possible

⇒ Scale of SLR can be transferred into GNSS network
Conclusions and outlook

- Combination using $SLR@GNSS$ works fine
  ⇒ most direct way to combine GNSS and SLR

- Connection via "space ties" only is possible (without local ties)
  ⇒ Independent control of local ties

**BUT:**

*Accurate ties* (in space and on sites) required for combination

*Number of SLR observations* critical (not possible on a daily basis)

- *Scale from SLR* is transferred to GNSS *directly* (not via local ties)
  ⇒ Estimation of GNSS SAO consistent to SLR scale

- Orbit overlap error is slightly improved (mainly GLONASS)

- Studies have to be extended to *longer time span*

- *Inclusion of Lageos*: Geocenter; more stable SLR station coordinates
SLR residuals for satellite G05, Orbit G3

Residual [mm]

Month in 2008
SLR residuals

SLR residuals for satellite G06, Orbit G3

Residual [mm]

Month in 2008

Residuals are plotted against the month in 2008 for satellite G06, Orbit G3.
SLR residuals

SLR residuals for satellite R24, Orbit G3

Residual [mm]

Month in 2008
SLR residuals

SLR residuals for satellite R15, Orbit G3

Residual [mm]

Month in 2008

-120  -90  -60  -30   0   30   60   90  120
1 2 3 4 5 6 7 8 9 10 11 12
SLR residuals: Mean Bias

Satellite-/site-specific bias

Bias [mm]

0  10  20  30  40  50  60  70  80  90  100

-100 -80 -60 -40 -20  0  20  40  60  80  100

7090 7405 7810 7839 7825 7405 7832 7237 7840 8834 1893 7105 7110 7080 7824 7941 7358 7249 7308

G05  G06  R15  R24  R07  R11
SLR residuals: RMS

Satellite-/ site-specific RMS of SLR residuals

RMS [mm]

SLR residuals: RMS
System-specific range biases seem to be sufficient for most sites.
Station-specific range biases seem to be sufficient
Comparison with local ties

Level of agreement when using one year of data only

→ Coordinate differences from GNSS-only and SLR-only solutions (1 year)
→ Datum definition for both solutions using NNR+NNT (core sites)
→ Velocities from ITRF2005

Red: ITRF2008 (prelim.)

Blue: From annual single-technique solutions
Comparison with local ties

Changes when combining GNSS and SLR@GNSS using „space ties“ only
→ Combined yearly GNSS+SLR solution
→ Datum definition: NNR+NNT (GNSS core sites), no Local ties
→ GNSS antenna phase center (SAO) estimated = Scale defined by SLR only!

Red:
From annual single-technique solutions

Blue:
Combined solution, GNSS SAO estimated
# Scale between solutions (GNSS core sites)

$L$ = Satellite-specific SLR range biases  
$K$ = System-specific SLR range biases  
$J$ = Station-specific SLR range biases

<table>
<thead>
<tr>
<th></th>
<th>L1A</th>
<th>K1A</th>
<th>J1A</th>
<th>L1B</th>
<th>K1B</th>
<th>J1B</th>
<th>[ppb]</th>
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<tbody>
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<tr>
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<td>0.9</td>
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<td>0.0</td>
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<tr>
<td>Lageos</td>
<td>0.0 ±0.9</td>
<td>-0.8 ±0.7</td>
<td>-0.4 ±0.7</td>
<td>0.3 ±1.0</td>
<td>-0.6 ±0.6</td>
<td>-0.3 ±0.7</td>
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</table>

A“: GNSS Sat.ant. fixed  
= Scale fixed

B“: GNSS Sat.ant. estimated  
= Scale free ⇒ from SLR