LLR contribution to long-term EOP variations and celestial reference frames

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Lunar Laser Ranging

- 42 years of observations
- Post-newtonian model at cm level
- high long-term stability (orbit, reference frames, Earth orientation)
- relativity tests
Statistics - reflectors and observatories

Time span 1970-2011

- Apollo 15: 78%
- Apollo 14: 10%
- Lunokhod 2: 3%
- Lunokhod 1: <1%
- Apollo 11: 10%

MLRS1: 4%
MLRS2: 16%
Grasse: 54%
McDonald 2.7m: 18%
Matera: <1%

... and a few lunar tracks from
- Orroral
- Wettzell
Number of normal points

- 1970 - 2011: ca. 17,000 normal points
Basic formulas

- Basic equation
  \[ d = \left| r_{EM} - r_{station} + r_{reflector} \right| + c \Delta \tau \approx c \frac{\tau}{2} \]

- Barycentric ecliptical system J2000

- Weighted least-squares adjustment
  \[ \tau + \nu = f(p_0) + \left. \frac{\partial f}{\partial p} \right|_{p_0} \Delta p \]

- Observations \( \tau \)
- Residuals \( \nu \)
- Parameters – initial values \( p_0 \)
- Unknown increments \( \Delta p \)

\[
\begin{align*}
\mathbf{r}_{reflector} & = \mathbf{R}^{moon} \mathbf{r}_{reflector} \\
\mathbf{r}_{station} & = \mathbf{R}^{earth} \mathbf{r}_{ITRF} \\
\mathbf{r}_{station} & = \mathbf{S}(x_p, y_p, UT1) \mathbf{N} \mathbf{P} \mathbf{B} \mathbf{r}_{ITRF} 
\end{align*}
\]

\( \frac{\partial f}{\partial p} \) Part. derivatives
Linear drift in pole coordinates

- Differences between linear drift from IERS C04 and LLR for time span 1970-2008
  - Combined linear drift from LLR: $4.9 \pm 0.3$ mas/a
  - Linear drift from the combination of space geodetic techniques (Gross & Vondrák 1999): $4.123 \pm 0.002$ mas/a for time span 1976.7-1997.1

\[
x_P: 0.3 \pm 0.2 \text{ mas/a}
\]

\[
y_P: 1.2 \pm 0.3 \text{ mas/a}
\]
Earth rotation $\Delta UT$

- Determination of UT1-UTC in global LLR adjustment

- Correction values to introduced UT1-UTC from the IERS C04 series were fitted

- Constraint that at least 10 LLR NP must be available for every considered night
Earth rotation $\Delta UT$

Until 1987 the error bars in the order of ms only
Weighted annual residuals

weighted residuals (observed - computed Earth-Moon distance), annually averaged
After 1988, when the data quality increased, the error bars decrease down to 0.3 µs.
Celestial reference frame

- Alignment of planetary/lunar and ICRS ephemerides < 1 mas
- VLBI and LLR give mean equinox J2000 (angles $\phi$ and $\varepsilon$)
- Mean inertial dynamic ecliptic from LLR based solution
- PN according to equator (ICRS or dynamic), corresponding angles $\phi$ and $\varepsilon$ are determined

- Arc between $\gamma^I_{2000}$ (ICRS) and $\gamma^I_{2000}$ (Eq2000) on the ecliptic 44.5 ± 0.3 mas
  - $\varepsilon^{(ICRS)} = 23^\circ 26' 21".4110 \pm 0.1$ mas; $\phi^{(ICRS)} = -55.4 \pm 0.1$ mas
  - $\varepsilon^{(Eq2000)} = 23^\circ 26' 21".4056 \pm 0.1$ mas; $\phi^{(Eq2000)} = -14.6 \pm 0.1$ mas
- Values adopted by IAU, IERS (2010)

Chapront et al. 2002, Zerhouni et al. 2007
Celestial reference frame

Inertial dynamical mean ecliptic of J2000.0

Mean equator of J2000.0

ICRS equator

ψ = γ′_{2000}(ICRS) γ′_{2000}(Eq2000) = 44.5 mas

φ(Eq2000) = -14.6 mas

φ(ICRS) = -55.4 mas

R_{earth} = B P N S

Chapront et al. 2002, Zerhouni et al. 2007
Nutation

- Differences in nutation angles $\Delta \psi$ and $\Delta \varepsilon$: MHB2000 model minus LLR result

\[ R_{earth} = B \, P \, N \, S \]
Analysis of rotation biases

- Transformation between terrestrial and celestial systems

\[
\mathbf{r}_{ICRF}^{station} = \mathbf{R}^{earth}_{ICRF} \mathbf{r}_{ITRF}^{station} \quad \mathbf{R}^{earth} = \mathbf{BPNs}
\]

- Extension for further rotation

\[
\Omega = D_1(\Theta_x)D_2(\Theta_y)D_3(\Theta_z) \approx \begin{bmatrix}
1 & \Theta_z & -\Theta_y \\
-\Theta_z & 1 & \Theta_x \\
\Theta_y & -\Theta_x & 1
\end{bmatrix}
\]

\[
\Theta_i = \Theta_{i0} + \dot{\Theta}_i \Delta t
\]
Analysis of rotation biases

- Earth orbit fixed

- fit for $\Omega$ rotation (constant term) $\rightarrow$ then fixed

\[ \Theta_x = -6.2 \pm 0.4 \text{ mas} \rightarrow \Delta \varepsilon \]
\[ \Theta_y = 1.1 \pm 0.2 \text{ mas} \rightarrow \Delta \psi \]
\[ \Theta_z = 0.6 \pm 0.1 \text{ mas} \rightarrow \Delta \phi \]

- fit for $\dot{\Theta}_y$ and nutation, correlation up to 30 %

\[ \dot{\Theta}_y = -0.6 \pm 0.1 \text{ mas/y} \]
\[ \dot{\Phi} = -1.6 \pm 0.2 \text{ mas/y} \]
Celestial reference frame

Inertial dynamical mean ecliptic of J2000.0

Mean equator of J2000.0

\[ \gamma_{2000}^I (\text{Eq2000}) \quad \varepsilon(\text{Eq2000}) \quad \alpha(\text{Eq2000}) \]

\[ \phi(\text{Eq2000}) = -14.6 \text{ mas} \]

\[ \psi = \gamma_{2000}^I (\text{ICRS}) \gamma_{2000}^I (\text{Eq2000}) = 44.5 \text{ mas} \]

\[ \varepsilon(\text{ICRS}) \]

\[ \phi(\text{ICRS}) = -55.4 \text{ mas} \]

ICRS equator

Chapront et al. 2002, Zerhouni et al. 2007
LLR tests of general relativity

Strong equivalence principle

\[ \eta = (1 \pm 5) \times 10^{-4} \]

\[ \left[ \frac{M_G}{M_I} \right]_{SEP} - 1 = (-0.5 \pm 2.3) \times 10^{-13} \]

Temporal variation of the gravitational constant

\[ G = G_0 \left( 1 + \frac{\dot{G}}{G} \Delta t + ... \right) \]

\[ \frac{\dot{G}}{G} = (1 \pm 4) \times 10^{-13} \text{ yr}^{-1} \]

Factor 2 improvement due to refined modelling and more LLR data

Hofmann, Müller, Biskupek, Astron. & Astroph., 2010
Conclusions

Open items (recommendations?)
- Added-value for long-term EOP monitoring by LLR?
- Celestial reference frame: better alignment of dynamic and ICRS equator possible/required?
- Change of time scale in planetary/lunar ephemeris ($T_{eph} - TCB$)?

Future steps
- Enhanced studies on ephemeris, earth rotation and relativity
- Software update to include and process novel observations from lunar transponders and orbiters (and VLBI in a more direct way)
Equivalence principle test with LLR

- Earth and Moon have a large amount of gravitational self energy
  → use Earth-Moon system for testing strong equivalence principle (SEP) violation

\[ \Delta r_{EM} = 12.8 \text{ m } \xi \cos D \]

- If \( \eta \neq 0 \)
  - Different accelerations
  - Polarisation of lunar orbit