II. Current methodology for TRF combination

The current strategy adopted for Terrestrial Reference Frame combination analysis is twofold: simultaneous combination of station positions and velocities using full variance / covariance matrices; rigorous weighting scheme based on the analysis and estimation of the variance components using Helmert method.

II.1. Physical model for simultaneous combination of station positions and velocities

Assuming that for each individual solution s, and each point i, we have position X_s^i at epoch t_s and velocity $\dot{X}_s^i$, expressed in a given TRF k.

The combination consists on estimating:

- Positions X_{itrf}^i at a given epoch t_0 and velocities $\dot{X}_{itrf}^i$ in ITRS
- Transformation parameters T_k at an epoch t_k and their rates $\dot{T}_k$, from the ITRF to each individual frame k

The general physical model used is given by the following equation (II.1):

$$\begin{cases} X_s^i &= X_{itrf}^i + (t_s - t_0)\dot{X}_{itrf}^i + T_k + D_k X_{itrf}^i + R_k X_{itrf}^i \\ & \quad +(t_s - t_k)\left[\dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i\right] \\ \dot{X}_s^i &= \dot{X}_{itrf}^i + \dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i \end{cases} \tag{II.1}$$

where for each individual frame k, D_k is the scale factor, the translation vector T_k and rotation matrix R_k are respectively defined (following IERS conventions) by :

$$T_k = \begin{pmatrix} T1_k \\ T2_k \\ T3_k \end{pmatrix} \quad \text{and} \quad R_k = \begin{pmatrix} 0 & -R3_k & R2_k \\ R3_k & 0 & -R1_k \\ -R2_k & R1_k & 0 \end{pmatrix}$$

The dotted parameters designate their derivatives with respect to time. $T1, T2, T3$ are the 3 origin components, $R1, R2, R3$ are the three small rotations according to the 3 axes, respectively X, Y, Z.

II.2. Statistical and variance estimation modeling

Combining heterogeneous data has been, since the beginning of ITRF combinations, one of the key element of ITRF analysis. The problem is, by essence, statistic. Up to now, the data were homogenized by the use of an empirical weighting scheme. For the ITRF96, a theoretical estimate of weights has been built and used in a rigorous stochastic framework.

The individual solutions, as well as the eccentricities are considered to be statistically independent populations. In addition, their variance matrix is supposed to be known except from a multiplicative factor (the variance factor). Therefore, for the ℓ^{th} set, we can postulate[1]:

$$\begin{cases} E(Y_\ell) &= \dot{Y}_\ell \\ cov(Y_k, Y_\ell) &= \delta_\ell^k \sigma_\ell^2 \Sigma_\ell \end{cases} \tag{II.2}$$

where Y_ℓ is the vector of observations (coordinates and velocities) in the ℓ^{th} set and δ_ℓ^k is the Kronecker symbol ($1 \leq \ell \leq p$). The variance factor σ_ℓ^2 is unknown and estimated iteratively through the Helmert-variance components estimation technique (Grafarend, 1984; Rao, 1973; Rao and Kleffe, 1988). The estimate $\hat{\mathbf{s}}$ of the vector $\mathbf{s} = (\sigma_1^2, \ldots, \sigma_p^2)^T$ is given by :

$$\begin{cases} \hat{\mathbf{s}} = \begin{pmatrix} \widehat{\sigma_1^2} \\ \vdots \\ \widehat{\sigma_p^2} \end{pmatrix} = H^{-1}.q \\ \\ H_{\ell,k} = \delta_\ell^k [n_\ell - 2tr(\nu N_\ell)] + tr(\nu N_\ell \nu N_k) \\ q_\ell = V_\ell^T \Sigma_\ell^{-1} V_\ell \end{cases} \tag{II.3}$$

where tr is the function trace and ν is the inverse of normal matrix of the whole adjustment. N_ℓ is the normal matrix of the ℓ^{th} solution and for which n_ℓ is the total number of observations; V_ℓ is the position and velocity residual vector of the combination. $\hat{\mathbf{s}}$ is an unbiased estimate of s and the basic idea of constructing such estimator is to minimize, in the sense of the Gauss-Markov theorem, the variance of the resulting least square estimate (ν).

[1] $E(Y)$ is the expectancy of the random variable Y and $\dot{Y}$ is the exact value of Y ; $cov(X, Y)$ is the covariance matrix of X and Y.