# GPS, DORIS and SLR combined geocenter motion from reference frame time series analysis

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**ABSTRACT** *The improvement in terms of precision of Space Geodesy measurements makes it possible to study global geophysical phenomenas. The Earth's center of mass variations became recently a subject of interest in Space Geodesy. Satellite Laser Ranging analysis centers have computed, for a few years, time series of geocenter positions which are coherent with theoretical models. The DORIS technique, more sensitive than GPS to Earth's gravity, is probably appropriate to compute such time series. The present paper, following a scientific campaign organised by the International Earth Rotation Service during 1997, is focused intercomparison of the dynamic techniques applying various signal analysis methods.*

## 1 INTRODUCTION

The Institut Géographique National team has been involved in the IERS activity for 10 years. The basic philosophy of all the works done in this field at IGN, as well as IERS central bureau, is to intercompare the data from Space Geodesy techniques, in order to detect, and eventually correct, the specific systematic errors that can be observed in this process. This way of thinking is the one chosen in this study and is a key element in the debate relative to the ability of Space Geodesy to monitor geocenter variations. In fact, the aim of this paper is clearly to crosscheck some Space Geodesy test data in order to make it possible to get an objective view of the coherence of Space Geodesy technique in the Geocenter motion determination. Furthermore, a complementary goal of this study is to attempt to combine the information when it is possible. Nevertheless, combining signals resulting of different adjustment is a bet. That's why, if one wants to combine various data in the view of geocenter determination, it is necessary to come back to the reference frame support, namely the set of coordinates, and to combine this type of data. It should be emphasized that this is possible only because there is an identification between the origin of the reference frame and the center of gravity of the Earth. This is generally true, but in a stochastic context, it can be argued that this is only an approximation.

## 2 MEASUREMENT METHODOLOGY

Generally speaking, the Geocenter is identified as the center of masses of the Earth. Its instantaneous position should be accessible to artificial Earth satellites. This assumption leads to the study of the variations of the origin of the frame in which is expressed the orbit of the satellite. When the station coordinates are adjusted in the same estimation process as the satellite orbit, the reference frame of the orbit parametrization should be the same as the underlying reference frame of the set of coordinates. The two approaches to determine reference frame origin variations and described hereafter are based upon time series of station set of coordinates.

Let us suppose that the geocenter is free of motion during the time span of an orbit arc. The gravitation force is radially directed to this "instantaneous" geocenter. If, in addition, we suppose that the gravity field, in steady state, doesn't depend on the geocenter position, the reference system in which the orbit is computed is centred on the instantaneous geocenter. Thus, sets of coordinates carry the instantaneous geocenter information. Let us suppose now, that the orbit of a satellite is analysed through successive orbit arcs. Then each orbit arc can be considered as the realization of the instantaneous reference system. It is then possible to identify the geocenter variations in the way that the tracking stations move together. This leads to directly compute the geocenter motion from the orbit computation.

### *2.1 Comparisons of individual sets to an external reference*

This approach consists of comparing each successive sets of coordinates independently to an external reference. In detail, Helmert transformation parameters are adjusted between individual sets and the reference, and the translation parameters are identified to the instantaneous geocenter position, averaged over the span of the individual set. The adjustment model is presented in Eq. 1. The first equation is the observation equation concerning the $i^{th}$ station at time $t_s$ (i.e. $s^{th}$ set ; difference between the individual set ($X_i^s$) and the reference coordinates ($X_i^0$)), whereas the transformation parameters $\theta^s$ are estimated through a least-square adjustment and by the use of a design matrix $A$ computed with approximate coordinate values[1].

$$\left\{ \begin{array}{ll} X_i^s(t_s) - X_i^0(t_s) \cong A_i.\theta^s(t_s) & (\Sigma_i^s + \Sigma_0) \\ A_i = \begin{pmatrix} 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\ 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\ 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \end{pmatrix} & \\ \theta^s = \begin{pmatrix} T_x^s & T_y^s & T_z^s & k^s & \varepsilon^s & \psi^s & \omega^s \end{pmatrix}^T & \end{array} \right. \qquad (1)$$

The problem of this method is that the estimated transformation parameters ($T_{x,y,z}$ in Eq. 1) are disturbed by the external reference error. The appropriate method is to take into account the variance over the external solution in the estimation process. Nevertheless, the external reference cannot insure the homogeneity of translation parameters in time, especially when the quality and the global distribution of the network used to generate individual sets change with time.

### *2.2 Comparison of individual sets to an internal reference*

In order to avoid the previous problem relative to the disturbance due to the external reference, an internal reference deduced from the whole time series can be used. Basically, it is possible to compute in the same estimation process (Eq. 2) both an internal reference and a set of transformation parameters between individual sets and the combined one.

$$X_i^s(t) \cong \begin{pmatrix} I \mid (t_s - t_0)I \mid A_i \end{pmatrix} . \begin{pmatrix} X_i \\ \dot{X}_i(t_0) \\ \theta^s(t_s) \end{pmatrix} \quad (\Sigma_i) \qquad (2)$$

The method is exactly the method used for ITRF-type combinations. Extending this principle, it is therefore possible to combine informations from various sources, while these sources have the same time sampling by the use of a constraint between transformation parameters. This method has also the advantage to smooth the data, while a temporal connection between sets of data is introduced by the use of the model (positions + velocities), as it is not the case with the different comparisons computed at different times of section 2.1.

## 3 SIGNAL ANALYSIS

### *3.1 Regression analysis*

For each time series that have been obtained through the method explained above, a regression has been carried out in order to determine the amplitude and phase of a semi-annual and an annual term of excitation. A constant term as well as a trend has also been derived in order to express all the time series with respect to the same reference system. For a signal $s(t)$, the analysis is carried

[1]The sign $\cong$ means equality between random variables

out through the following way (annual term indexed $_1$ and semi-annual term indexed $_{1/2}$ ; the phase is computed with respect to the first of January) :

$$s(t) \cong A_0 + A_0' t + A_1 \cos(\omega_1 t + \varphi_1) + A_{1/2} \cos(\omega_{1/2} t + \varphi_{1/2}) \tag{3}$$

### 3.2 *Wavelet analysis*

In order to investigate the whole spectrum of previous signals, a wavelet analysis has been carried out. The interest of wavelets with respect to Fourier analysis is that it does not postulate the time invariability of a particular frequency during the whole data span. The idea is to associate an instantaneous frequency to the signal and let the possibility to analyse the signal as a frequency changing signal.

- A signal $[t \in I \subset \mathbb{R} \longmapsto s(t)]$
- Wavelet transform $\mathcal{T}_s(t,a)$ :

$$\left\{ \begin{array}{lcll} \mathcal{T}_s(t,a) & = & \int_{\mathbb{R}} s(x)\overline{\Psi}_{(t,a)}(x)dx & \\ \Psi_{(t,a)}(x) & = & \frac{1}{a}\Psi\left(\frac{x-t}{a}\right) & \\ \Psi(x) & = & e^{i\omega_0 x}e^{-x^2/2} & \text{(Morlet Wavelet)} \end{array} \right.$$

- graph of $|\mathcal{T}_s(t,a)|$ with $\left| \begin{array}{lll} t & : & \text{X-axis} \quad \text{(time)} \\ a & : & \text{Y-axis} \quad \text{(scale} \leftrightarrow \text{period)} \end{array} \right.$

Table 1: Wavelet transform formulae

Wavelet transform can be seen as a generalization of Fourier transform : it is a basic decomposition into the frequency space, but this space has an other dimension which is time (see Tab. 1).

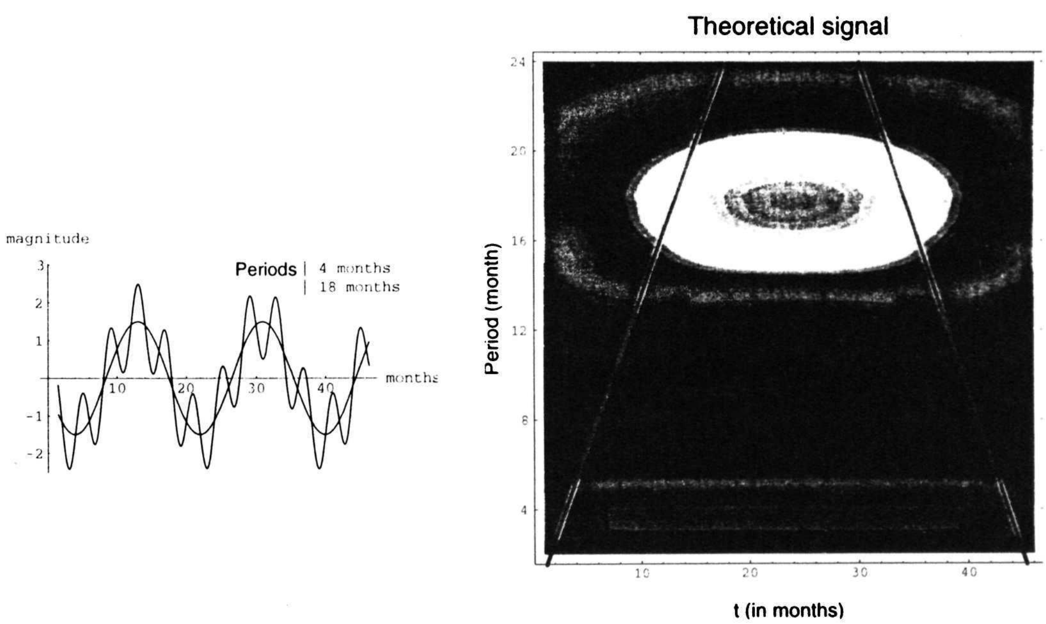


Figure 1: Wavelet analysis of a signal without frequency change

Fig. 1 shows the graph of the wavelet transform of a pure invariant frequency signal, and Fig. 2 shows the impact, on the wavelet transform, of a change in the frequency, during the whole span, in the modulation signal. The gap is perfectly identified by the wavelet transform, while, of course, Fourier analysis couldn't identify it as it pre-supposes the invariability of the signal's frequency during the whole span. An other property of wavelets, showed in Fig. 1 and that has to be kept in mind, is the limitation that the bounded interval of the signal induces on the detection of a specific period : an information window is defined, in which an existing signal can be detected ; outside this window (red lines), there is not enough data to find anything.

Finally, one can say that wavelet transform is a technique that is closer to the physical behaviour of stochastic processes and then, is a major tool for analysing such signals.

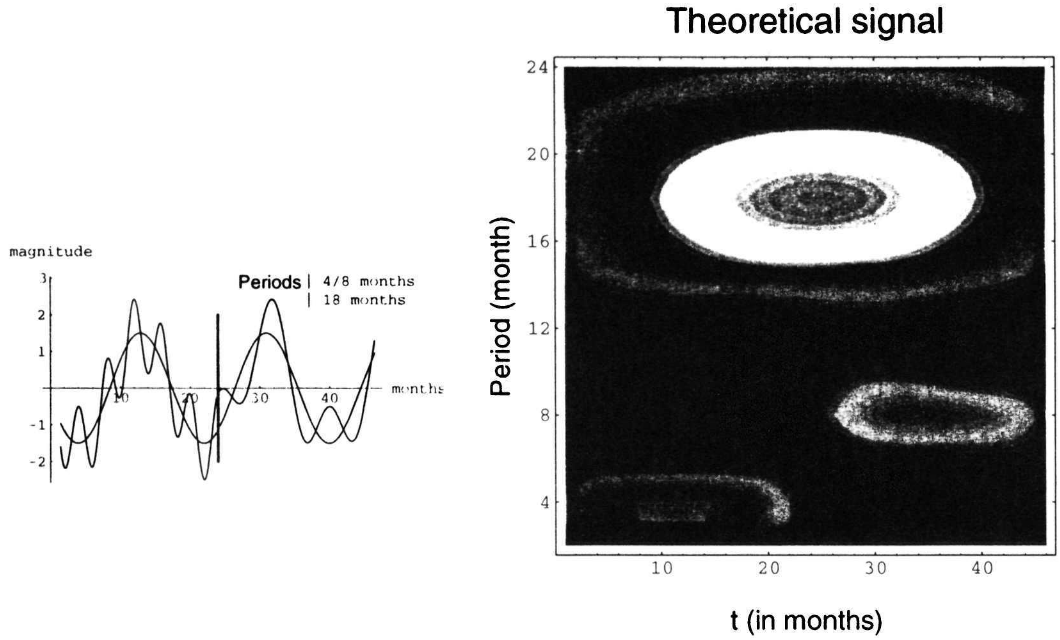


Figure 2: Wavelet analysis of a signal with frequency change

## 4 THE SETS OF DATA AND REGRESSION ANALYSIS

This section is dedicated to the analysis of the time series that have been used for this study. Some come directly from the web site of GEOC, and some others are deduced from sets of coordinates combination made at IGN. For all the time series of geocenter positions, a linear regression has been performed in agreement with Eq. 3. The results of this regression have been analyzed in amplitude and phase.

| Time series | $A_1$ *mm* | $\varphi_1$ *deg.* | $A_{1/2}$ *mm* | $\varphi_{1/2}$ *deg.* |
|---|---|---|---|---|
| LASER-CSR | 2.1 | $-58.8$ | 1.2 | 77.2 |
| | $\pm0.64$ | $\pm8.9$ | $\pm0.64$ | $\pm16.$ |
| DORIS-GRGS | 3.2 | $-153$ | 1.6 | 117 |
| | $\pm1.0$ | $\pm9$ | $\pm1.0$ | $\pm18$ |
| DORIS-IGN | 5.6 | 79.7 | 4.2 | $-160.$ |
| | $\pm5.$ | $\pm25.$ | $\pm4.8$ | $\pm34.$ |
| GPS-JPL | 5.3 | $-15.0$ | 7.3 | $-98.3$ |
| | $\pm4.8$ | $\pm23.$ | $\pm4.1$ | $\pm19.$ |
| Combined GPS-JPL | 2.4 | $-54.$ | 2.9 | $-81.3$ |
| + DORIS-IGN | $\pm3.2$ | $\pm40.$ | $\pm3.$ | $\pm33.$ |

Table 2: Annual and semi-annual components in X

### *4.1 Satellite Laser Ranging*

This technique is the one which has the greatest potential since it is sensitive to variations of geocenter positions : the dynamic of the satellite is well known and allows a very deep analyse of gravitational effects among which are geocenter variations. The results presented in Tab. 2, 3, 4 and Fig. 3 have been obtained by the Center of Space Research (CSR) and submitted during IERS geocenter campaign (Eanes et al., 1997). One can see that the uncertainties over regression parameters are the smallest that have been obtained, which confirm the sensitivity of this technique to gravity field. This also justify the interest of this technique in the field of geocenter motion. It can also be seen that the Laser Ranging geocenter variations have an amplitude which is compatible with theoretical prediction (Dong et al., 1997). SLR seems to be the only technique compatible with theory. To a smaller degree, DORIS seems to be able to confirm this magnitude (see §4.2).

| Time series | $A_1$ mm | $\varphi_1$ deg. | $A_{1/2}$ mm | $\varphi_{1/2}$ deg. |
|---|---|---|---|---|
| LASER-CSR | 3.2 | 59.5 | 0.68 | 111. |
| | ±0.73 | ±6.8 | ±0.74 | ±31. |
| DORIS-GRGS | 4.2 | 60 | 0.9 | −77 |
| | ±1.3 | ±9 | ±1.3 | ±42 |
| DORIS-IGN | 9.3 | −39.7 | 1.9 | −123. |
| | ±2.2 | ±6.7 | ±2.2 | ±32. |
| GPS-JPL | 8.6 | 174 | 1.5 | 120. |
| | ±2.5 | ±7.2 | ±2.2 | ±46. |
| Combined GPS-JPL | 2.4 | −54. | 2.9 | −81.3 |
| + DORIS-IGN | ±3.2 | ±40. | ±3. | ±33. |

Table 3: Annual and semi-annual components in Y

| Time series | $A_1$ mm | $\varphi_1$ deg. | $A_{1/2}$ mm | $\varphi_{1/2}$ deg. |
|---|---|---|---|---|
| LASER-CSR | 2.9 | −44.1 | 0.37 | −64.2 |
| | ±1.6 | ±16. | ±1.6 | ±130. |
| DORIS-GRGS | 5.6 | 51 | 0.9 | −48 |
| | ±5.2 | ±47 | ±5.2 | ±159 |
| DORIS-IGN | 19. | 168. | 11. | −63.4 |
| | ±5.1 | ±8.4 | ±5.4 | ±14. |
| GPS-JPL | 19. | −32.0 | 17. | −30.2 |
| | ±6.7 | ±9.9 | ±7.3 | ±11. |
| Combined GPS-JPL | 12. | 41.7 | 14. | −62.7 |
| + DORIS-IGN | ±10. | ±26. | ±9.7 | ±22. |

Table 4: Annual and semi-annual components in Z

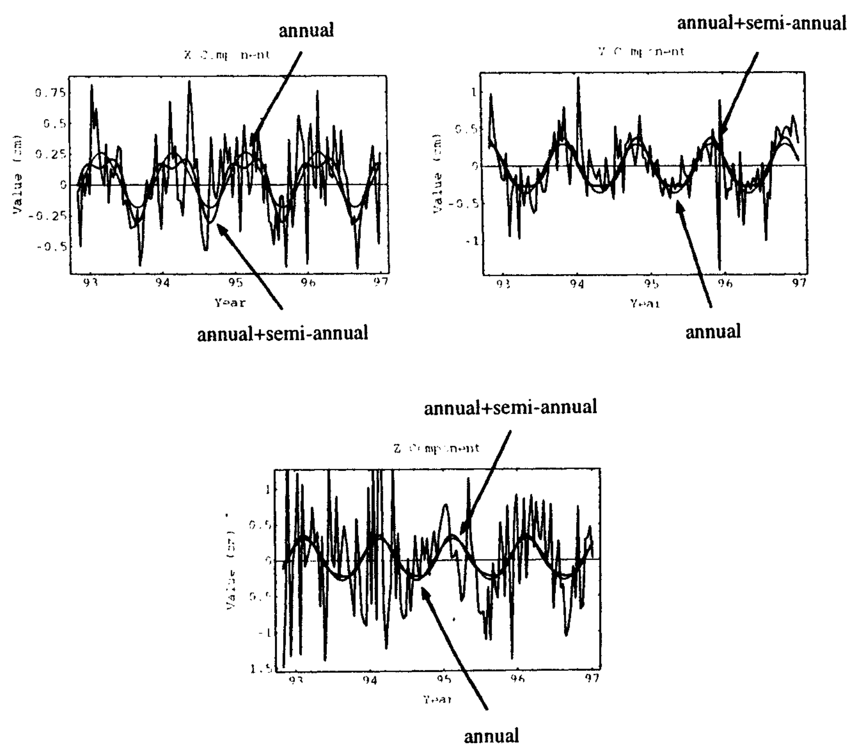


Figure 3: LASER geocenter components

### 4.2 *DORIS*

Two analysis centers provided data during IERS geocenter campaign : the Groupe de Recherche en Géodésie Spatiale (GRGS - Toulouse) and the Institut Géographique National (IGN). The results of the linear regression can be found in Tab. 2, 3 and 4. The DORIS-GRGS time series comes from a set of transformation parameters identified to geocenter position (according to section 2.1) (Bouillé and Cazenave, 1997). The DORIS-IGN time series comes from a section 2.2-type combination where transformation parameters between all the individual sets and the combined one have been estimated in the same run (Sillard et al., 1997; Willis et al., 1997).

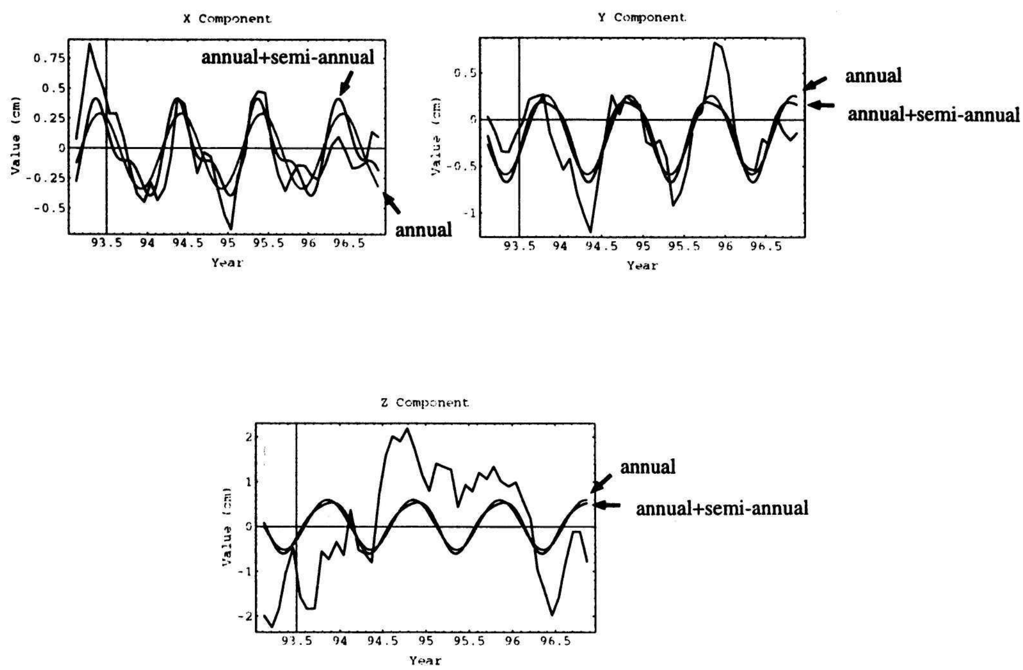


Figure 4: DORIS-GRGS geocenter components

The conclusion that can be drawn from the results is that the determinations are not really coherent : the amplitudes of IGN signals are systematically overestimated with respect to GRGS results. The phases are not more coherent. One of the main conclusion, apart from the previous one, could be that, in no case, the semi-annual signal seems to be significant. Finally, it is necessary to be careful on these results which should suggest that DORIS is not really capable to determine geocenter variations at the moment.

### 4.3 *GPS*

The GPS technique has a disadvantage with respect to previous techniques : the altitude of GPS satellites is much higher than DORIS or SLR, which leads to the fact that GPS is less sensitive than other techniques to the details of the gravity field. Nevertheless, it is interesting to study the use that can be done of GPS measurements to improve other techniques.

The GPS geocenter time series has been obtained through a combination (see §2.2) of weekly time series of sets of station coordinates computed by Jet Propulsion Laboratory (JPL) and provided to International GPS Service (IGS) (Heflin, 1997; Sillard et al., 1997).

One can see that the results are slightly different from other results, neither more nor less coherent. Fig. 6 shows the original curve together with adjusted annual and semi-annual curves.

### 4.4 *Combined DORIS and GPS determination*

Finally, and it is an advantage of the method described in section 2.2, a simultaneous combination from GPS-JPL and DORIS-IGN sets of station coordinates has been carried out. The transformation

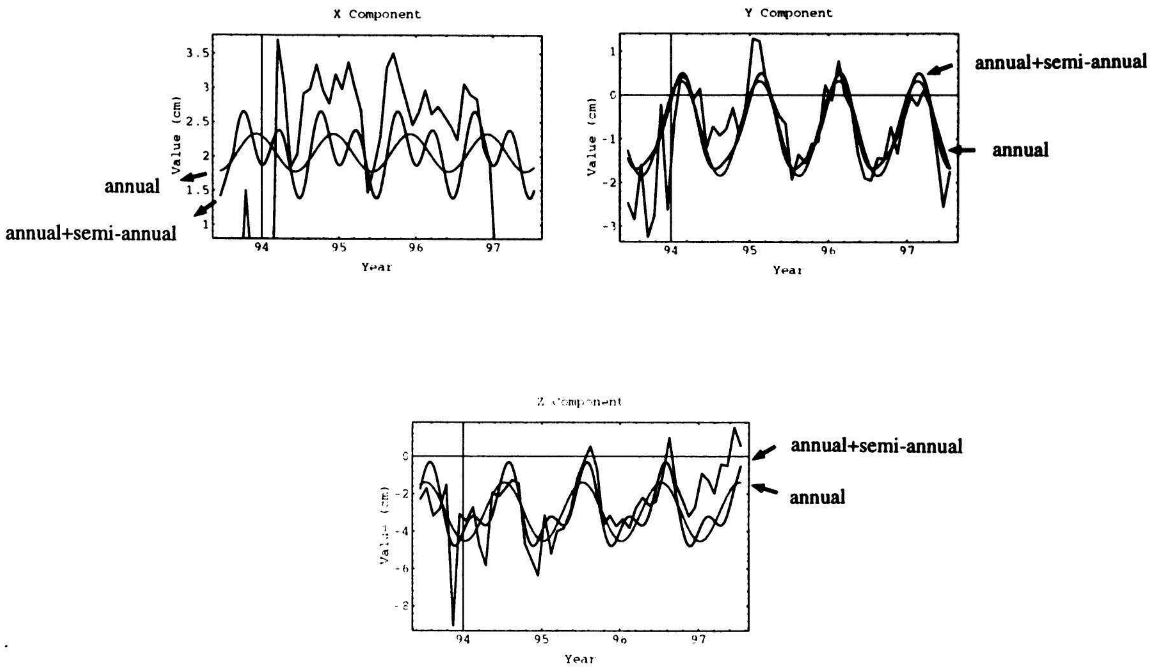


Figure 5: DORIS-IGN geocenter components (red : annual ; blue : annual + semi-annual)

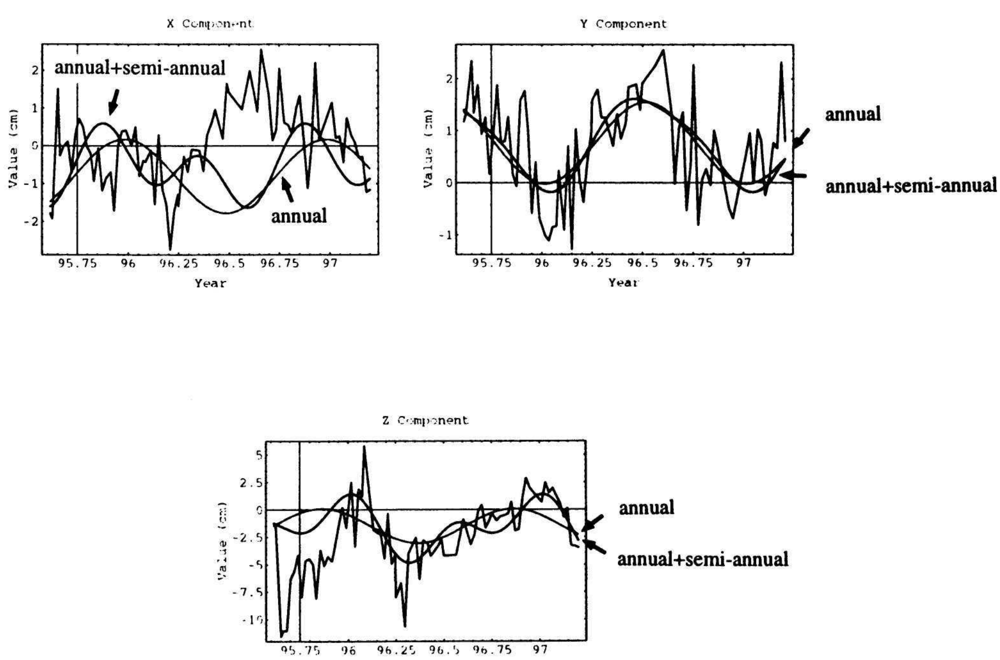


Figure 6: GPS-JPL geocenter components

parameters between the combined solution on one hand, and the individual DORIS and GPS sets on the other hand, were constrained to be equal at the same dates. The combination is based upon local eccentricities used for ITRF96 computation between DORIS and GPS collocated stations. Fig. 7 shows the plots of the resulting geocenter component time series (Sillard et al., 1997).

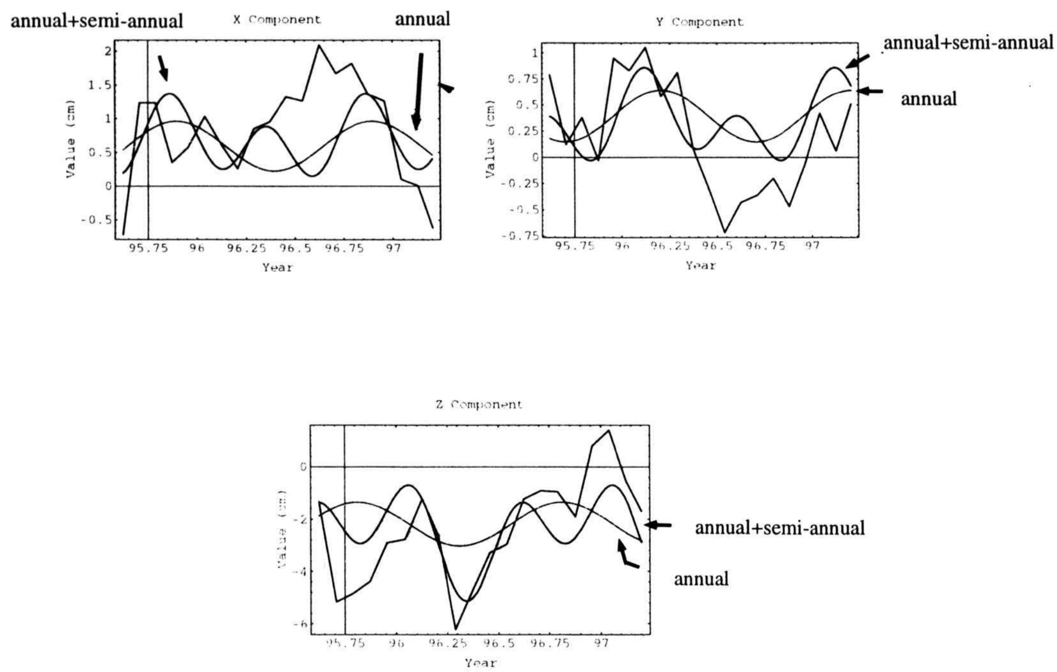


Figure 7: Combined DORIS-IGN and JPL-GPS geocenter components

As expected, since the DORIS and GPS time series are not really coherent, the combined signal has a smaller amplitude than original ones (DORIS and GPS). The combined signal is closer to the theoretically expected level and therefore more satisfying than individual solutions, even if the resulting signal is poorly significant. This technique could be used routinely in the view of geocenter motion monitoring, especially if SLR technique was able to produce time series of set of station coordinates.

## 5 COMPARISON OF THE REGRESSION RESULTS AND WAVELET ANALYSIS

In order to summarize the regression estimates, it is interesting to study the results in the phase space. It will then be easy to estimate the level of consistency between adjusted signals. Fig. 8 shows the graphics for annual analysis, and Fig. 9 shows the graphics for semi-annual analysis. The scale of the graphics is the millimetre, and the phase is measured with respect to X-axis.

A few elements can be noticed from Fig. 8 and 9 :

1. There is no semi-annual signal in the data that have been analyzed.
2. The amplitudes of the annual signals are generally larger than what is theoretically expected. Apparently, LASER-CSR and DORIS-GRGS are the only time series that could fit the theory in terms of amplitude of the annual signal. And to a smaller extend, the GPS-JPL and DORIS-IGN combined solution is in much better agreement with theory than original series.
3. The large distribution of phases (Fig. 8) suggests to keep a cautious eye on the presented results.
4. The discrepancies in the Z-component has to be related to the Z-component of Space Geodesy reference frame realization which is noisier than any other direction, since tracking networks are non-uniformly distributed over the Earth.

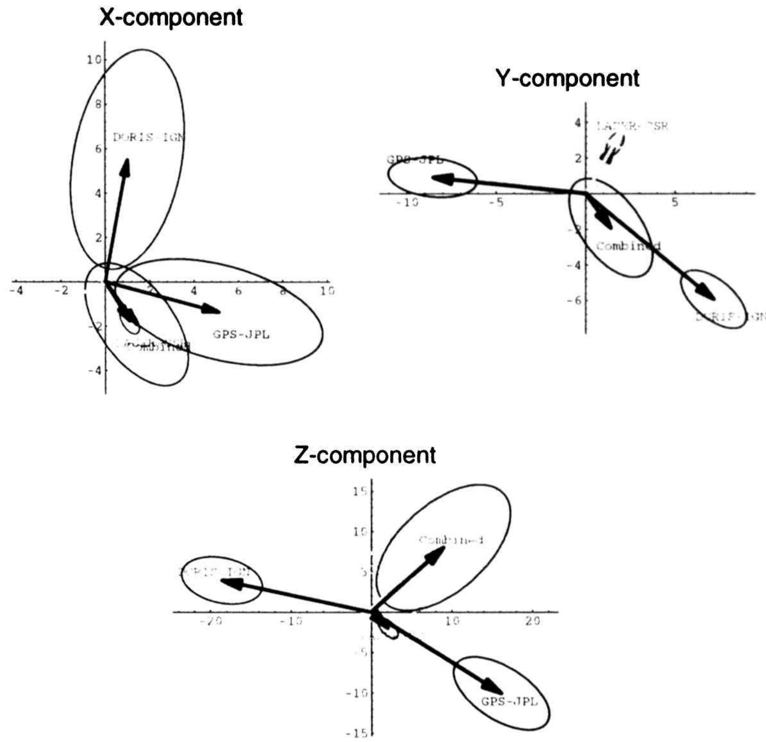


Figure 8: Annual signals

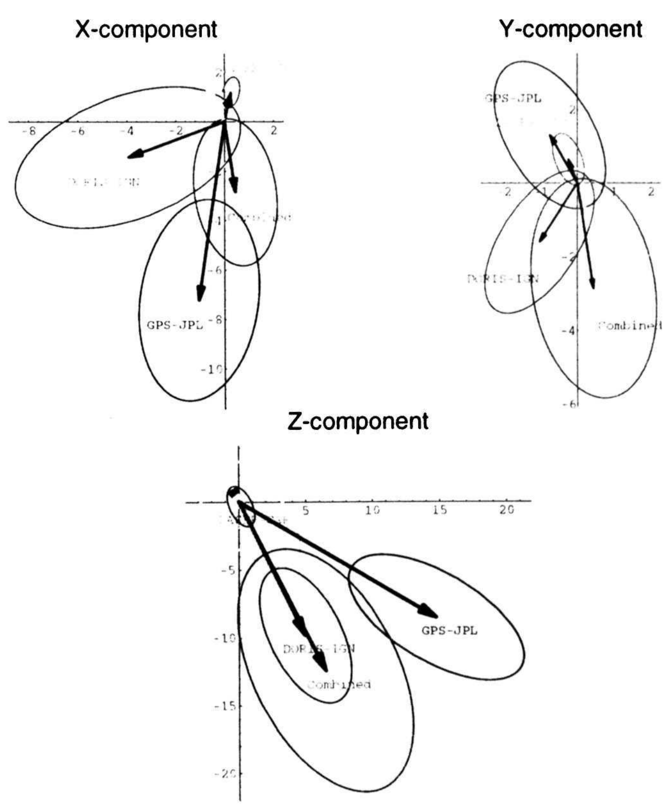


Figure 9: Semi-annual signals

5. There is still a doubt about the convention used by analysis centers : in which direction the position vector of geocenter is? For sets of coordinates combination, the convention is as the following:

$$\begin{pmatrix} x_G \\ y_G \\ z_G \end{pmatrix} = \overrightarrow{OG} \tag{4}$$

where $(x_G, y_G, z_G)$ are the components of geocenter in the given time series, $O$ is the origin of the frame of reference, and $G$ is the instantaneous geocenter.

Until now, the time series were studied in a very narrow part of their spectrum. A wavelet analysis has then been carried out in order to check if some different signals were detectable. The analysis was restricted to the Y-component, because it is the one which seems to be most promising and the results are quite coherent with what has already been explained.

Wavelet transform in amplitude is presented in Fig. 10. It is remarkable to see that apart from the annual signal, everything else is nothing but noise in all this time series. This fact has to be moderated for the GPS-JPL series, as the apparent period seems to be about 8 months. Nevertheless, there are only 20 months of data which gives an idea of the difficulty to find a 12-months period. It must also be emphasized that the combination of DORIS-IGN and JPL-GPS is again a 12-month signal which shows the interest of such a combination and emphazise the relevance of the method.

Fig. 11 shows, as an example of problems that still remain, the wavelet transform of LASER-CSR X and Z-components. As can be seen, the Z-component is no longer a one year period signal.

## 6 CONCLUSION

The results obtained by Space Geodesy techniques in terms of geocenter monitoring seems to be encouraging : an annual signal is clearly visible in all the time series. Nevertheless, the amplitudes an phases are not coherent enough to conclude that the detected signal is a common real physical signal. The theory predicts (Dong et al., 1997; Chao and O'Connor, 1988) an amplitude of the annual signal much smaller than the centimeter level. This theory is based upon atmosphere, ground and ocean water redistributions analysis. Space techniques have demonstrated the potential they have in determining geocenter motion (Watkins and Eanes, 1993; Vigue et al., 1992). This capability has to be crosscheck and until now, there remain some inconsistencies between individual determinations that might be due to the use of inconsistent physical models : as a matter of fact, many of the models used to reduce Space Geodesy observations might have an annual signature (Ocean and Earth tide models, troposphere, ... ), and before asking whether or not Space Geodesy is capable to detect Geocenter motion, one should study to which extend the discrepancies between the physical models used to reduce the data can disturb the geocenter determinations. The results presented here are promising, but the main task that has to be carried out is to unify the physical models and then, the convergence of the results should derive.

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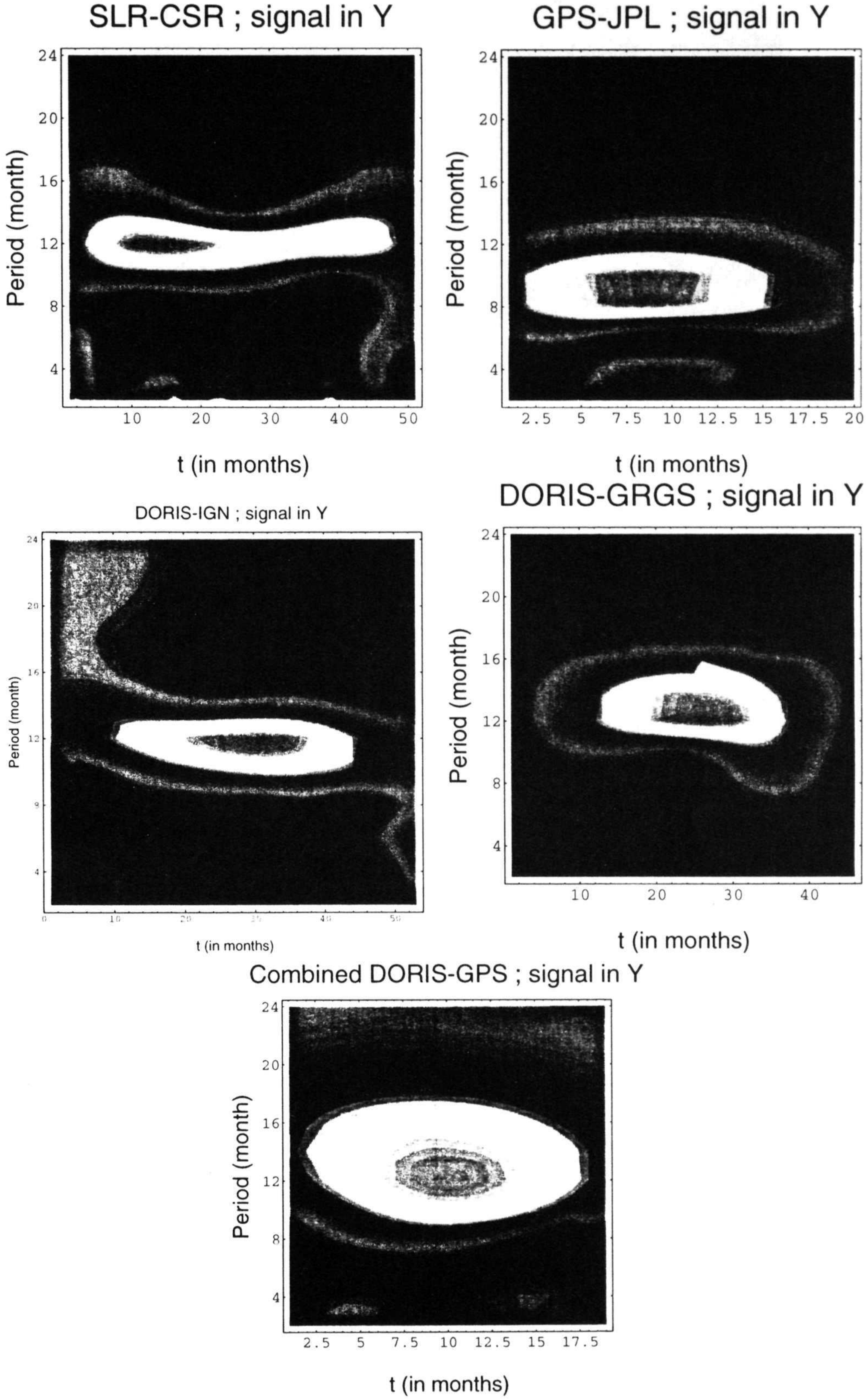


**Figure 10: Y-component wavelet analysis**

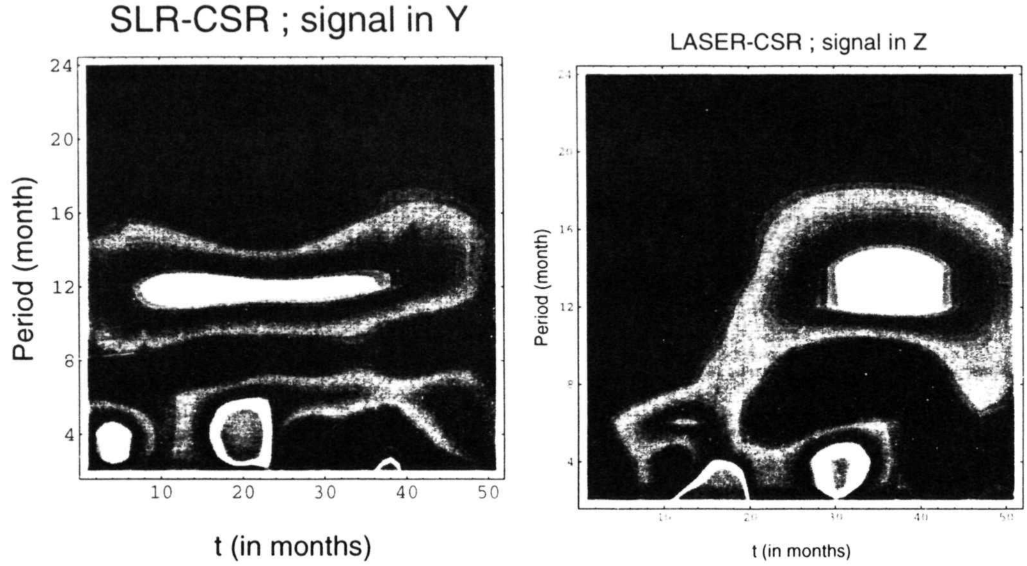


Figure 11: X and Z component of LASER-CSR geocenter

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