Remarks on Some Problems in the Combination of Station Coordinate and Velocity Solutions

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1 Introduction

The present realizations of the IERS Terrestrial Reference Frame (ITRF) are based on the combination of individual technique specific solutions for station coordinates and velocities provided by several analysis centers. There are some problems in the combination procedure which shall briefly be discussed in the following.

2 Datum characteristics of individual solutions

The individual solutions may be unconstrained, partly constrained, minimum constrained, loosely constrained, or over-constrained with respect to the realization of the datum parameters, i.e., the definition of the necessary 7 parameters in case of stationary three-dimensional coordinates (3 translations, 3 rotations, 1 scale) or 14 parameters for kinematic coordinates (the same for the velocities).

- If the solutions are unconstrained, then the normal equation matrix (N) has to be singular with a rank defect of 7 or 14, respectively.
- If they are partly constrained, then some of the datum parameters are introduced into the adjustment procedure, e.g., the scale by fixing the speed of light. The rank defect is then reduced by these given datum parameters (e.g., from 7 to 6).
- If they are minimum constrained only a maximum of 7 or 14 datum parameters may be introduced into the processing (e.g., fixed).
- If they are loosely constrained, then the solution (adjustment) of coordinates (and velocities, respectively) must result in very large r.m.s. errors.
- If they are over-constrained, then they may deform the total network by contradictions or tensions in the constraints, and it will be very difficult (or impossible) to combine them with other solutions.

The problem in these classifications is that the constraints are often not clear.

- Solutions may be directly constrained by fixing a number of parameters (e.g., station coordinates or velocities). This will normally be known to the provider of the solution.
- They may also implicitly be constrained without the perception of the provider. This may happen by the applied models in the processing, e.g., by the used gravity field that fixes the origin of the coordinates by means of the lower harmonic coefficients (C_{10}, C_{11}, S_{11}).
- There are also hidden constraints by the uncontrolled introduction of conditions, e.g., by fixing velocities to zero in case of short time observations or to a certain model, e.g., NNR NUVEL-1A, or even by setting velocities identical for different stations.

A conclusion of these statements is, that individual solutions may **only** be combined by 7 (or 14) parameter similarity (Helmert) transformation, if they are **not** deformed by introducing any constraints. If some constraints are in-

troduced, the transformation has to be reduced to the remaining free parameters. If more than 7 (or 14) constraints are introduced, the solution cannot be used at all. The proof of such constraints can preferably be done by analyzing the **normal equations** (N) of the individual solutions.

3 Combination of individual solutions

The individual solutions of the same or different techniques may be combined on the level of observation equations, normal equations or solutions. The most appropriate approach is the combination of observation equations. This requires, however, sophisticated software packages and does not allow the distributed processing as it is done by the services.

The combination of normal equations is equivalent to the observation equation approach if all the used models and required parameters are identically introduced. Even parameters not estimable from individual techniques (e.g., geocentre variations from VLBI) may be included.

The combination of solutions with their variance-covariance matrix does not allow the analysis of constraints or the inclusion of additional parameters (e.g., atmospheric delay).

When combining individual technique specific solutions or normal equations one has to consider that the standards, models, or solved parameters may **not** be identical (e.g., time scale TDT or TCG, gravity field model, identical site velocities) or not consistent. Several services are performing benchmark projects for comparing different software of the Analysis Centers for this purpose.

3.1 Intra-technique combination (solutions using technique specific data)

Individual solutions of one technique are **not** independent of each other because

- they all use the identical observation data,
- some combined solutions include already other individual solutions (e.g., IGS solutions),
- regional solutions include parts of the global solutions.

The consequence is that the weighting (estimation of variance-covariance components) of individual solutions is problematic. If various solutions of the same technique are combined with other techniques' solutions they may not be introduced as independent. A conclusion is that the IERS technique centers should provide a unique combined solution for the ITRF.

3.2 Inter-technique combination (different techniques' solutions)

In the combination of solutions of different techniques co-location sites and the eccentricities (local ties) play the most important role. Are they sufficiently well determined to introduce them as a constraint?

Some estimated parameters of the different solutions should or have to be identical, e.g., the atmospheric parameters. Were identical models and standards used? Other parameters are supposed to be identical but they aren't, e.g., station velocities at co-location sites are not necessarily identical.

The weighting of individual solutions has to be done by a sophisticated a posteriori variance-covariance estimation including all the correlations between them. Individual solutions of the same technique are highly correlated. Identical observations may only enter once into the adjustment.

The conclusion of these statements is that inter-technique combination should be based on the unique solutions provided by the Technique Centers (Services) instead of individual solutions.

4 Estimated parameters in the combined solution

Identical effects in different observation techniques must be modeled or estimated by identical parameters (e.g., atmospheric refraction, atmospheric/oceanic loading, ...). The variation (time dependency) of parameters has to be analyzed (e.g., is the variation of coordinates linear, i.e., are there constant velocities?) The identity of parameters has to be proven, e.g., in case of identical site velocities the local ties have to be time independent. Repeated surveys are necessary to prove that, or local ties may be adjusted in the combination procedure.

The consistency of parameters of different reference systems has to be guaranteed, e.g., between the ICRF, EOP and ITRF. This leads to the requirement of a common adjustment of all components of the reference frames.

5 Effects of the ITRF network design on combination results

The network design may affect the results of the ITRF combination. Deformations or tensions may be caused by using

- too old observations in abandoned stations (e.g., mobile SLR and VLBI stations of the 80's).
- too young stations (e.g., newly installed stations with observations less than one year).
- too bad observations (e.g., observations with inaccurate instruments).

The quality and reliability of stations and observations is more important than the quantity. One should do a combination in hierarchic levels (e.g., reliable global network and densification by additional stations). Co-location conditions (identical parameters) should only be introduced for parallel (simultaneous) observations.

6 Recommendations

As a conclusion of the above discussion we give the following main recommendations:

- Techniques' combined solutions should be provided by the Technique Centers as unconstrained normal equations. The ITRF combination should be based on these normal equations.
- Parameters common to different techniques (affecting the observations, e.g., atmosphere **or** affecting the reference frame, e.g., EOPs) must be included in the normal equations.
- The combined station coordinates and velocities network should include only reliable sites and observations (emphasis to be given to quality rather than to quantity).