Impact of IAU Resolutions on Earth Rotation Parameters

Nicole Capitaine[1], Dennis D. McCarthy[2]

[1] *Observatoire de Paris, Syrte, Paris, France*
[2] *U.S. Naval Observatory, Washington D.C., USA*

1 Introduction

The implementation of the IAU 2000 Resolutions for all the applications related to Earth's rotation is described in Chapter 5 of the IERS Conventions entitled "Transformation between the celestial and terrestrial reference systems". Successive revisions of this Chapter have been made to include the models, expressions and numerical tables as soon as they became available for the implementation of the resolutions and to introduce the most recent developments and improvements. A major revision was made recently in order (i) to follow the actions recommended by the IERS Directing Board after the IERS Workshop in Paris (April 2002), (ii) to take into account the latest comments and remarks from various colleagues, (iii) to introduce the definitive and full forms of the numerical expressions and tables, and (iv) to provide a software package for the practical implementation. This has benefited from specific contributions from several experts for the following sections: "Motion of the Celestial Intermediate Pole (CIP) in the International Terrestrial Reference System (ITRS)" (A. Brzezinski, S. Mathews, Ch. Bizouard), "Prograde and Retrograde Nutation Amplitudes" (P. Defraigne, S. Mathews, Ch. Bizouard, A. Brzezinski), "Procedures and IERS Routines for Transformations between GCRS (Geocentric Celestial Reference System) and ITRS" (P. Wallace) and from general comments (Burghard Richter). Numerical Tables and Fortran subroutines have been added to those already available and now allow the users to implement the resolutions in several ways according to their choice and needs.

The updated version was posted on November 2002 on the webpage of the IERS Conventions Center together with the corresponding numerical tables and software. This version is supposed to be the final version, except for slight corrections which can still be applied before the Conventions are published as an IERS Technical Note in order to take into account the remarks that will be received on the posted version. This paper reports on the latest improvements that have been applied in this Chapter since the previous version that was presented at the IERS Workshop in Paris.

2 Update of Chapter 5 of the IERS Conventions

The major revisions in the text of Chapter 5 of the IERS Conventions 2000 presented here concern the sections on the motion of the CIP in the ITRS and GCRS, the expressions for precession and Greenwich Sidereal Time compatible with the IAU2000 model and the procedures and IERS routines for the Fortran subroutines implementing the IAU 2000 celestial-to-terrestrial transformations.

2.1 Progress in the implementation of the IAU Resolutions

The implementation of the IAU Resolutions in the transformation between the celestial and terrestrial reference systems concerns (see McCarthy & Capitaine, 2002) IAU Resolution B1.3 (*Definition of BCRS and GCRS*), IAU

Resolution B1.6 (*IAU 2000 Precession-Nutation Model*), IAU Resolution B1.7 (*Definition of Celestial Intermediate Pole*) and IAU Resolution B1.8 (*Definition and use of Celestial and Terrestrial Ephemeris Origins*). This introduces a "new paradigm" for the transformation between the celestial and terrestrial reference systems based on the use of the CEO, TEO, Earth Rotation Angle (ERA) and the x, y coordinates of the CIP unit vector in the GCRS, instead of the "classical paradigm" which refers to the equinox.

The implementation of Resolution B1.3 is realized in Chapter 5 using the correct time scales and the correct realization of the GCRS with the IAU 2000 precession-nutation (Mathews et al., 2002) including both the geodesic precession and nutation so that the GCRS and BCRS are without any time-dependent rotations. The implementation of Resolution B1.6 is realized by providing expressions for the model of the motion of the CIP in the GCRS which are consistent with the IAU 2000 precession-nutation model, either for the new or classical paradigms.

The implementation of Resolution B1.7 (which sharpens the definition of the current CEP in the high frequency domain) and B1.8 requires the use of the IAU 2000A expressions for the position of the CIP and the CEO in the GCRS, of the TEO in the ITRS and models for improving the realization of the pole in the high frequency domain.

2.2 Implementation of IAU Resolution B1.7

The standard pole coordinates to be used for the parameters x_p and y_p, if not estimated from the observations, are those published by the IERS with additional components to account for the effects of ocean tides $(\Delta x, \Delta y)_{tidal}$ and for nutation terms with periods less than two days $(\Delta x, \Delta y)_{nutation}$. Corrections for the diurnal and sub-diurnal variations in polar motion caused by ocean tides can be computed using a routine available on the website of the IERS Conventions (see Chapter 8). Table 8.2 (from Ch. Bizouard), the basis for this routine, provides the amplitudes and arguments of these variations for the 71 tidal constituents considered in the model. These subdaily variations are not part of the polar motion values reported to and distributed by the IERS and have therefore to be added after interpolation.

In order to realize the CIP as recommended by Resolution B1.7, nutations with periods less than two days are to be considered using a model for the corresponding motion of the pole in the ITRS. The prograde diurnal nutations correspond to prograde and retrograde long periodic variations in polar motion and the prograde semidiurnal nutations correspond to prograde diurnal variations in polar motion. A table for operational use of the model for these variations $(\Delta x, \Delta y)_{nutation}$ in polar motion for a nonrigid Earth has been provided by an *ad hoc* Working Group (Brzezinski, 2002) based on nonrigid Earth models and developments of the tidal potential (Brzezinski, 2001; Brzezinski & Capitaine, 2002; Mathews & Bretagnon, 2002). The amplitudes of the diurnal terms are in very good agreement with those estimated by Getino et al. (2001). The diurnal components of these variations should be considered similarly to the diurnal and semidiurnal variations due to ocean tides. They are not part of the polar motion values reported to the IERS and distributed by the IERS and should therefore be added after interpolation. The long-periodic terms, as well as the secular variation, are already contained in the observed polar motion and need not be added to the reported values.

2.3 Implementation of IAU Resolution B1.8

2.3.1 Positioning the CIP and the CEO in the GCRS

Definitive and full forms of the expressions for X, Y and s have been provided (Capitaine et al., 2003a) for the parameters to be used in the new trans-

formation, based on the IAU 2000A precession-nutation model and the IAU2000 definition of UT1. They include the effect of precession, bias, nutation and cross terms between precession and nutation.

2.3.2 Positioning the TEO in the ITRS

The expression for s' for use in both new and classical transformations for positioning the TEO in the ITRS, has been derived from the current mean amplitudes for the Chandlerian and annual wobbles (Lambert & Bizouard, 2002).

2.3.3 Classical paradigm a the microarcsecond level

The classical transformation, R, to transform from the ITRS to the GCRS, should be written in its rigorous form as (Wallace, 2002) R = B P N T, where B is the matrix transformation from the CRS to the mean J2000, P, the Precession matrix, N, the Nutation matrix, T, the Earth's rotation matrix using GST.

These matrices are such that:

Bias: $B = R_3(-d\alpha_0)\cdot R_2(-d\xi_0)\cdot R_2(d\eta_0)$,
Precession: $P = R_1(-\varepsilon_0)\cdot R_3(\psi_A)\cdot R_1(\omega_A)\cdot R_3(-\chi_A)$,
Nutation: $N = R_1(-\varepsilon_A)\cdot R_2(\Delta\psi)\cdot R_1(\varepsilon_A+\Delta\varepsilon)$,
Earth Rotation: $T = R_3(-GST)$.

The IAU 2000 expressions for the quantities to be used in the classical transformation have been provided. These include, in addition to the MHB 2000 (Mathews et al., 2002) nutation $\Delta\psi$ and $\Delta\varepsilon$, and precession quantities ψ_A, ω_A, ε_A, (obtained directly by addition of the precession rate corrections to the Lieske et al. (1977) precession), the expressions for:

- the equatorial precession quantities z_A, ζ_A, θ_A, derived from the IAU2000 A expressions for ψ_A, ω_A, so that their use is equivalent to the use of the former, at a microarcsecond level for a century (Capitaine et al., 2003c), and
- the Sidereal Time GST, which is the sum of the Earth Rotation Angle (ERA) as a linear function of UT1 and of the expression for the accumulated precession and nutation in right ascension (*i.e.* the sum of the polynomial part of the expression, denoted $GMST_{2000}$, and the periodic part, which itself is the sum of the classical equation of the equinoxes and of the *complementary terms in the equation of equinoxes)*. This can be written as

$$GST = \theta(UT1) + \int_{t_0}^{t} (\psi_A + \Delta\psi_1)\cos(\omega_A + \Delta\varepsilon_1)\,dt - \chi_A + \Delta\psi\cos\varepsilon_A - \Delta\psi_1\cos\omega_A$$

The definitive expressions for GST and *s* (Capitaine *et al.* 2003b) have been obtained by ensuring that there is equivalence between CEO-based and equinox-based z-rotations, and that there is continuity in UT1 on 1 January 2003 0 h TT with the previous relationship $GMST_{1982}$(UT1) and the current UT1 estimate.

2.3.4 Consequences of the frame bias

The effect of the celestial pole offsets ξ_0, η_0 is to introduce a constant offset in X, Y and a specific frame bias rotation, B, in the classical transformation. The effect of the equinox offset $d\alpha_0$ (in μas) between the inertial dynamical mean equinox at J2000 and the x-axis of the GCRS, is to introduce terms of the form: –142 t in Y and +1.6 t^2 in X and similar effects in the classical quantities $\Delta\psi \sin(\varepsilon)$ and $\Delta\varepsilon$ respectively.

Current VLBI procedures use corrections for biases and precession-nutation quantities, and this introduces secular and cubic discrepancies with respect to the rigorous transformation of the order of a few 100 μas/c.

3 Expressions and Tables for the implementation of IAU Resolutions

The motion of the CIP in the ITRS is provided in Chapter 5 by Table 5.1 for the components arising from the nutation terms with periods less than two days, $(\Delta x, \Delta y)_{nutation}$. In addition, the component due to the oceanic tidal effects, $(\Delta x, \Delta y)_{tidal}$ is provided in Chapter 8 (Table 8.b).

The definitive and full expressions for the other quantities are:

Motion of the TEO in the ITRS

$s' = -47\ \mu as\ t.$

Earth Rotation angle

$\theta(UT1) = 2\pi(0.7790572732640 + 1.00273781191135448 \times (\text{Julian UT1date} - 2451545.0).$

Greenwich Sidereal Time:

Expression (35), the electronic table, and Table 5.4, include the following polynomial and periodic parts:

$GMST_{2000} = 0".014506 + \theta + 4612".15739966\ t + 1".39667721\ t^2 - 0".00009344\ t^3 + 0".0000188\ t^4.$

$EE_{2000} = + \Delta\psi \cos \varepsilon_A - \Sigma_k C_k \sin \alpha_k - 0.87\ \mu as\ t \sin \Omega.$

Note that the two largest terms of the complementary terms in the equation of the equinoxes were already included in the current expression for GST (McCarthy 1996).

Motion of the CIP in the GCRS

$$X = -0."01661699 + 2004."19174288\ t - 0."4272190\ t^2 - 0."19862054\ t^3 - 0."00004605\ t^4 + 0."00000598\ t^5 + \Sigma i\ [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] + \Sigma_i\ [(a_{s,1})_i\ t \sin(\text{ARGUMENT}) + (a_{c,1})_i\ t \cos(\text{ARGUMENT})] + \Sigma i\ [(a_{s,2})_i\ t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i\ t^2 \cos(\text{ARGUMENT})] + \ldots$$

$$Y = -0."00695078 - 0."02538199\ t - 22."40725099\ t^2 + 0."00184228\ t^3 + 0."00111306\ t^4 + 0."00000099\ t^5 + \Sigma_i\ [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] + \Sigma_i\ [(b_{c,1})_i\ t \cos(\text{ARGUMENT}) + (b_{s,1})_i\ t \sin(\text{ARGUMENT})] + \Sigma_i\ [(b_{c,2})_i\ t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i\ t^2 \sin(\text{ARGUMENT})] + \ldots$$

Tables 5.2a (X), 5.2b (Y) and 5.2c (s) provide extracts of the full tables for the periodic components of the coordinates *X* and *Y* and for s, which are provided in an electronic form as tab5.2a and 5.2b and 5.2c. Tables 5.3a (luni-solar) and 5.3b (planetary) provide extracts of the full tables for the IAU2000A nutation, which are provided in an electronic form as tab5.3a and b.

IAU2000 precession developments are given by:

- expressions (27), (30), (31) for the updated expressions of Lieske et al. (1977) based on the MHB 2000 correction to precession rates, and
- expressions (32), (33) for the updated developments consistent with MHB2000.

The IAU 2000 precession quantities are

$$\psi_A = 5038."47875\ t - 1."07259\ t^2 - 0."001147\ t^3.$$
$$\omega_A = \varepsilon_0 - 0."02524\ t + 0."05127\ t^2 - 0."007726\ t^3.$$
$$\varepsilon_A = \varepsilon_0 - 46."84024\ t - 0."00059\ t^2 + 0."001813\ t^3.$$
$$\chi_A = 10."5526\ t - 2."38064\ t^2 - 0."001125\ t^3.$$
$$\varsigma_A = 2."5976176 + 2306."0809506\ t + 0."3019015\ t^2 + 0."0179663\ t^3 - 0."0000327\ t^4 - 0."00000002\ t^5.$$
$$z_A = -2."5976176 + 2306."0803226\ t + 1."0947790\ t^2 + 0."0182273\ t^3 + 0."0000470\ t^4 - 0."00000003\ t^5.$$
$$\theta_A = 2004."1917476t - 0."4269353t^2 - 0."0418251t^3 - 0."0000601t^4 - 0."00000001t^5.$$

4 IERS Software package for the implementation of the IAU Resolutions

The IERS software package at ftp://maia.usno.navy.mil/conv2000/chapter5/ includes the subroutines corresponding to the different options for implementing the IAU resolutions. The IAU2000A (T. Herring) and IAU2000B (D. D. McCarthy and B. Luzum) subroutines, compute the "total" nutation angles (including nutation, plus precession corrections, plus the constant celestial offsets at J2000) corresponding to the IAU2000A and IAU2000B precession-nutation models, respectively. The other subroutines provided by P. Wallace includes the following Fortran routines for implementing the IAU Resolutions, based on the rigorous method in the transformation between the celestial and terrestrial systems. Other software will in due course be released through the SOFA (Wallace, 2000) website.

Method (1):

- X, Y, s transformation consistent with the IAU2000A precession-nutation model
- ERA_{2000}: Earth Rotation angle at date t
- XYS_{2000A}: provides X, Y (GCRS coordinates of the CIP), s (position of the CEO on the equator of the CIP) at date t, consistent with IAU2000A precession-nutation
- BPN_{2000}: CEO-based precession-nutation matrix (based on the two routines above)

Method (2A): classical transformation using the IAU2000A precession-nutation model

- $GMST_{2000}$: Mean Sidereal Time consistent with IAU 2000 precession-nutation and new definition of UT1.
- $EECT_{2000}$: complementary terms in the equation of the equinoxes
- EE_{2000}: Equation of the equinoxes as the sum of the classical part plus $EECT_{2000}$
- GST_{2000}: Sidereal Time as GMST + EE_{2000}
- NU_{2000A}: IAU2000A nutation model
- $CBPN_{2000}$: classical bias-precession-nutation matrix

Method (2B): classical transformation using IAU2000B

- NU_{2000B}: IAU2000B nutation model

For all the methods :

- T2C2000: Forms the TRS-to-CRS matrix
- SP2000: Produces the position of the TEO on the equator of the CIP
- POM2000: Forms the matrix of polar motion

5 The IERS Products in agreement with the IAU Resolutions

The IERS products for the implementation of the IAU Resolutions are provided both for the CEO-based and equinox-based transformations. The new paradigm requires the use of the expressions for X, Y and s consistent with the IAU2000 precession and nutation model. In this case, the IERS products are the estimates for UT1, and the corrections dX and dY. The numerical values for the GCRS CIP coordinates X and Y at the date t are then provided by $X=X_{IAU2000} + dX$, $Y=Y_{IAU2000} + dY$.

The classical paradigm requires the use of the IAU2000 expressions for the classical quantities for precession, nutation and GST. In this case, the IERS products are the estimates for UT1, and the corrections $d\psi$ and $d\varepsilon$ in longitude and obliquity to the IAU2000 precession-nutation model. The IERS products will also continue to provide the corrections $d\psi$ and $d\varepsilon$ to the previous precession and nutation models.

References

Aoki, S., Guinot, B., Kaplan, G. H., Kinoshita, H., McCarthy, D. D., and Seidelmann, P. K., 1982, "The New Definition of Universal Time," *Astron. Astrophys.*, **105**, pp. 359–361.

Bretagnon, P., Rocher, P., and Simon, J.-L., 1997. "Theory of the rotation of the rigid Earth," *Astron. Astrophys*., **319**, pp. 305–317.

Brzezinski, A., 2001, "Diurnal and subdiurnal terms of nutation: a simple theoretical model for a nonrigid Earth," in *Proceedings of the Journées Systèmes de Référence Spatio-temporels 2000*, N. Capitaine (ed.), Observatoire de Paris, pp. 243–251.

Brzezinski, A., July 2002, Circular 2, IAU Commission 19 WG "Precession-nutation".

Brzezinski, A., and Capitaine N., 2002, "Lunisolar perturbations in Earth rotation due to the triaxial figure of the Earth: geophysical aspects," in *Proceedings of the Journées Systèmes de Référence Spatio-temporels 2001*, N. Capitaine (ed.), Observatoire de Paris, pp. 51–58.

Capitaine, N., Chapront, J., Lambert, S., and Wallace, P. T., 2003a, "Expressions for the Celestial Intermediate Pole and Celestial Ephemeris Origin consistent with the IAU 2000A precession-nutation model," *Astron. Astrophys.*, **400**, pp. 1145–1154.

Capitaine, N., Wallace, P. T., and McCarthy, D. D., 2003b, "Expressions to implement the IAU 2000 Definition of UT1," *Astron. Astrophys.*, **406**, pp. 1135–1149.

Capitaine, N., Wallace, P. T., and Chapront, J., and 2003c, "Expressions for IAU 2000 precession quantities," *Astron. Astrophys*., in press.

Getino, J., Ferrandiz, J. M. and Escapa, A., 2001, "Hamiltonian theory for the non-rigid Earth: semi-diurnal terms," *Astron. Astrophys.*, **370**, pp. 330–341.

IERS Technical Note No. 29: Proceedings of the IERS Workshop on the Implementation of the New IAU Resolutions, 2002, N. Capitaine, D.

Gambis, D. D. McCarthy, G. Petit, J. Ray, B. Richter, M. Rothacher, M. Standish, and J. Vondrak (eds.). (http://www.iers.org/iers/publications/tn/tn29/)

Lambert, S., Bizouard, C., 2002, "Positioning the Terrestrial Ephemeris Origin in the International Terrestrial Frame," *Astron. Astrophys.*, **394**, pp. 317–321.

Lieske, J. H., Lederle, T., Fricke, W., and Morando, B., 1977, "Expressions for the Precession Quantities Based upon the IAU (1976) System of Astronomical Constants," *Astron. Astrophys*., **58**, pp. 1–16.

Mathews, P.M. and Bretagnon P., 2002, "High frequency nutation," in the Proceedings of *Journées Systèmes de Référence Spatio-temporels 2001*, N. Capitaine (ed.), Observatoire de Paris, pp. 28–33.

McCarthy, D. D., 1996, *IERS Conventions 1996*, Observatoire de Paris

McCarthy, D. D. and Luzum, B. J., 2003, "An Abridged Model of the Precession-Nutation of the Celestial Pole," *Celest. Mech. Dyn. Astr.*, **85**, 37–49

McCarthy, D. D. and Capitaine, N., 2002, in IERS Technical Note No. 29, N. Capitaine *et al.*, eds., Verlag des Bundesamts fur Kartographie und Geodasie, Frankfurt am Main, pp. 9–17.

Wallace, P. T., 2002, in IERS Technical Note No. 29, N. Capitaine *et al.*, eds, Verlag des Bundesamts fur Kartographie und Geodasie, Frankfurt am Main, pp. 65–69.

Wallace, P. T., 2000, in *Towards Models and Constants for Sub-Microarsecond Astrometry*, K. Johnston, D. D. McCarthy, B. Luzum and G. Kaplan (eds), US Naval Observatory, Washington, D.C., pp. 353–362.