Combination of Loosely Constrained Solutions

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1 Introduction

The Center for Space Geodesy (CGS) of Matera, Italy, has implemented a combination strategy of site coordinates, velocities and Earth Orientation Parameters (EOP) that generates a combined set of geodetic parameters referred to a unique terrestrial reference system. The strategy has its strength in the automatic combination of solutions without the estimation and removal of the relative rotation between their reference frames and is based on the assumption of solutions with large uncertainties on the reference frame, i.e., loosely constrained solutions (Heflin et al., 1992; Blewitt, 1998; Davies and Blewitt, 2000).

Loosely constrained solutions can be combined regardless of the datum definition of each contributing solution. The solution reference frame is defined stochastically by the input data and is basically unknown but, on the other hand, there is no need to impose any external constraint for their combination. Since the datum is treated as a stochastic noise, it is not necessary to estimate or to apply relative rigid transformations (rotation-translation-scale) between reference frames and this naturally leads to a combined solution not distorted by any constraint or transformation.

2 Procedure

The input solutions are accepted as SINEX files containing parameters of the following categories: site coordinates, site velocities, EOP and EOP rates. In principle also tightly constrained solutions could be accepted; in this case, the *a priori* constraints should be given to allow their removal and a loosening transformation (Davies and Blewitt, 2000) should be applied in order to achieve the desired reference frame uncertainty.

Denoting with X_i the site coordinates of the i-th solution and with Y_i the EOP of the i-th solution at times t_{ij} and P being the time propagation operator, the following equations show the relationship between two different solutions:

$$\begin{pmatrix} \mathbf{X}_1(t_1) \\ \dot{\mathbf{X}}_1 \\ \mathbf{Y}_1(t_{1j}) \\ \dot{\mathbf{Y}}_1(t_{1j}) \end{pmatrix} = P \begin{pmatrix} \mathbf{X}_0(t_0) \\ \dot{\mathbf{X}}_0 \\ \mathbf{Y}_0(t_{0j}) \\ \dot{\mathbf{Y}}_0(t_{0j}) \end{pmatrix} = \begin{pmatrix} \mathbf{I} & (t_1 - t_0)\mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & (t_{1j} - t_{0j})\mathbf{I} \\ 0 & 0 & 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{X}_0(t_0) \\ \dot{\mathbf{X}}_0 \\ \mathbf{Y}_0(t_{0j}) \\ \dot{\mathbf{Y}}_0(t_{0j}) \end{pmatrix} \tag{1}$$

The combination of the solutions is obtained by solving in a least squares sense the above design equation. Denoting with X_c,Y_c respectively the coordinates and EOP of the combined product, and with C_i the i-th solution covariance matrix, the least squares problem can be solved in the following recursive formulation:

$$\begin{pmatrix} \mathbf{X}_c \\ \dot{\mathbf{X}}_c \\ \mathbf{Y}_c \\ \dot{\mathbf{Y}}_c \end{pmatrix} = \left(\mathbf{C}_0^{-1} + \mathbf{P}^T\mathbf{C}_1^{-1}\mathbf{P} + \cdots\right)^{-1} \left[\mathbf{C}_0^{-1} \begin{pmatrix} \mathbf{X}_0 \\ \dot{\mathbf{X}}_0 \\ \mathbf{Y}_0 \\ \dot{\mathbf{Y}}_0 \end{pmatrix} + \mathbf{P}^T\mathbf{C}_1^{-1} \begin{pmatrix} \mathbf{X}_1 \\ \dot{\mathbf{X}}_1 \\ \mathbf{Y}_1 \\ \dot{\mathbf{Y}}_1 \end{pmatrix} + \cdots \right] \quad (2)$$

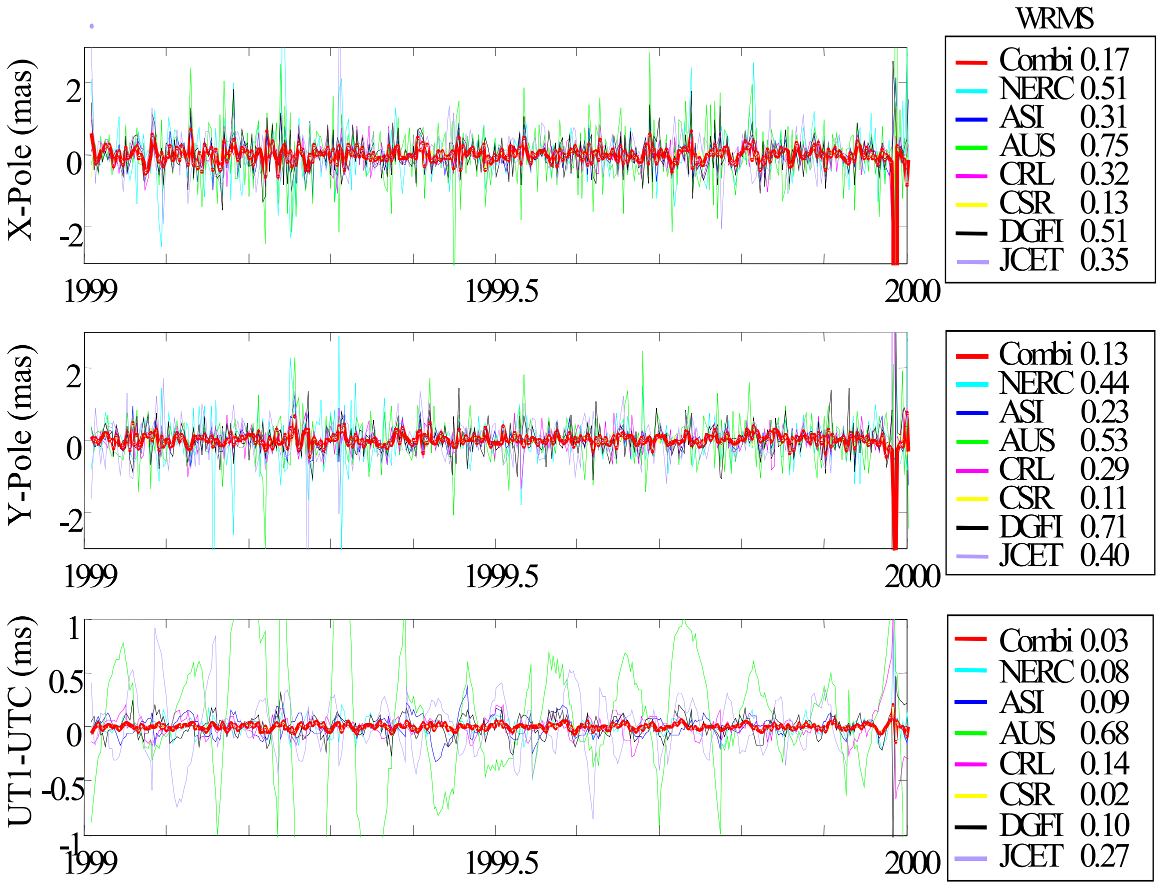

Fig. 1 EOP residuals with respect to IERS-C04 values for one year of SLR combination: the red line refers to the combined EOP. Large deviations among contributors are observed in the UT1–UTC parameter: this is a consequence of an intrinsic degeneracy in the SLR technique and therefore UT1 estimates cannot be interpreted and directly compared.

A scaling of the covariance matrix of the i-th solution is required because the relative weight of the contributing solutions may be incorrectly balanced. Imposing $\chi^2=1$ for the combination residuals and requiring that each contribution to the total χ^2 is equally balanced, one obtains the relative scaling factors (σ_i) iteratively with R_i being the solution residuals (with respect to the combined product) and Σ_i the solution covariance matrix, the imposed conditions are:

$$R_1^T\left(\sigma_1\Sigma_1\right)^{-1}R_1 = \cdots = R_i^T\left(\sigma_i\Sigma_i\right)^{-1}R_i \quad and \quad \chi^2 = R_1^T\Sigma_1^{-1}R_1 + \cdots + R_i^T\Sigma_i^{-1}R_i = 1$$

In the year 2000 the ILRS Analysis Working Group promoted a pilot project on the combination of SLR analysis products, namely a time series of 4-weekly site coordinate solutions and daily EOP over 1 year of LAGEOS-I and LAGEOS-II observations. The test solutions were provided by eight ILRS Associate Analysis Centers that adhered to the pilot project. The loosely constrained monthly solutions from each agency have been combined and compared to the official ITRF values. Figures 1 and 2 show the residuals of EOP and site coordinates with respect to the IERS-C04 time series and the ITRF2000 coordinates. The combined product shows the smoothest time series and the residuals are considerably smaller than each single solution. The EOP residuals have a weighted root-mean-square (WRMS) on the order of 130–170 µarcsec whereas the site coordinate residuals are on the order of a few mm (3-D) with rare exceptions in the range 10–20 mm. The global monthly WRMS of the coordinate residuals demonstrates a stable value between 5–10 mm throughout all the year.

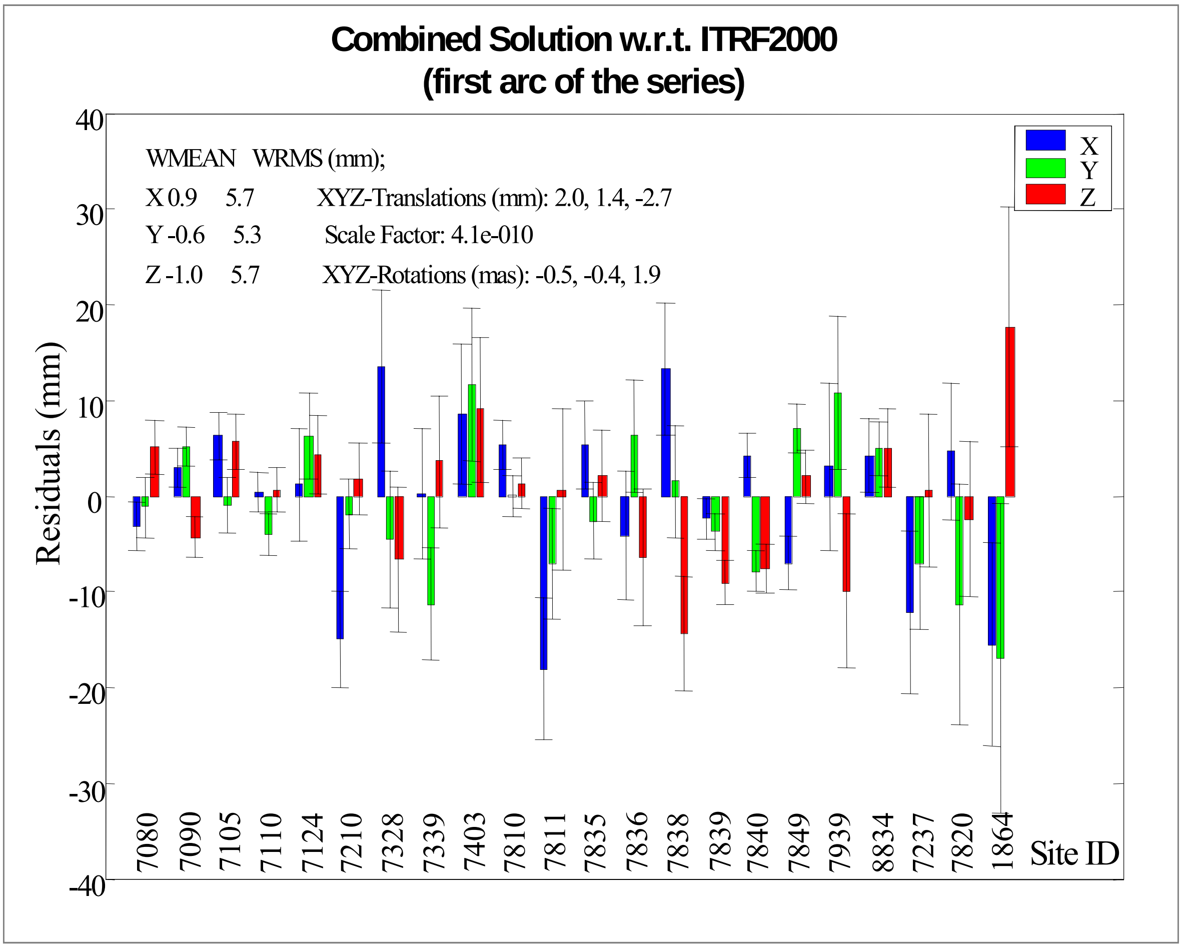

Fig. 2 Site coordinate residuals of a sample month (first arc) with respect to ITRF2000 values. The scatter of the site residuals is at the level of 5 mm in the three directions.

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