Rank Defect Analysis and Variance Component Estimation for Inter-Technique Combination

Rainer Kelm

Deutsches Geodätisches Forschungsinstitut, Munich, Germany

1 Introduction

The SINEX files, which are stored in the SINEX pool of the IERS SINEX Combination Campaign project, contain normal equation systems and/or solutions of individual space techniques. When the solution is given we try to reconstruct the normal equation system by using the constraints with which the solution was obtained. Before the combination the number and type of rank defect of each normal equation matrix should be determined in order to get an insight of the constraint characteristics. Theoretical considerations and first numerical results for this rank defect analysis are presented here. One of the problems in the inter-technique combination is the correct weighting of the input normal equation systems. The variance component estimation may be considered as a potential weighting method. First experiences with this method are reported here.

2 General combination model

The inter-technique combination is defined here as the computation of a unified solution with input solutions being stochastically independent from each other and may be the result of an arbitrary space technique (SLR, VLBI, GPS and/or DORIS). The estimated target parameters of each input solution k are station coordinates as well as EOP (pole and UT1) and their rates . Hence the parameter input vector for solution k may be defined as $\mathbf{p}_k$ and the total parameter input vector for n_f independent solutions as

$$\mathbf{p}_f^T = \left[\mathbf{p}_1^T \ \mathbf{p}_i^T \ \mathbf{p}_{n_f}^T \right] \tag{1}$$

Additionally, for the collocation site l local tie coordinates $\boldsymbol{x}_l$ are required and for n_s local ties the coordinate vectors are presented as

$$\mathbf{x}_s^T = \left[\mathbf{x}_1^T \ \mathbf{x}_i^T \ \mathbf{x}_{n_s}^T \right] \tag{2}$$

The target parameters to be solved for in the combined solution are station coordinates $\mathbf{x}$ and station velocities $\dot{\mathbf{x}}$ as well as EOP values $\mathbf{m}$ and their rates $\dot{\mathbf{m}}$ within a unique terrestrial reference frame. Hence the general deterministic combination model may be defined as a Gauss-Markov model:

$$E\begin{bmatrix} \mathbf{p}_f \\ \mathbf{x}_s \end{bmatrix} = f\left(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{m}, \dot{\mathbf{m}}\right) = f\left(\mathbf{p}\right) \tag{3}$$

and after linearization w.r.t. a priori parameters

$$E\begin{bmatrix} d\mathbf{p}_f \\ d\mathbf{x}_s \end{bmatrix} = \mathbf{A}\begin{bmatrix} d\mathbf{x} \\ d\dot{\mathbf{x}} \\ d\mathbf{m} \\ d\dot{\mathbf{m}} \end{bmatrix} \quad \text{or} \quad E\left[d\mathbf{r}_f \right] = \mathbf{A}\, d\mathbf{p} \tag{4}$$

Assuming that the input solutions as well as the local tie solutions are stochastically independent from each other, following normal equation system of the general combination model results:

$$\mathbf{A}^T\mathbf{N}\mathbf{A}\,d\mathbf{p}=\mathbf{A}^T\,d\mathbf{r}_f \quad \text{with} \quad \mathbf{N}=\begin{bmatrix}\mathbf{N}_{ff} & 0\\ 0 & \mathbf{N}_{ss}\end{bmatrix} \tag{5}$$

$\mathbf{N}_{ff}$ and $\mathbf{N}_{ss}$ consist of block diagonal normal equation matrices for each solution.

For the optimal case that all input solutions are based on free networks, $\mathbf{N}$ should be singular, because each block matrix should represent rank defects which are characteristically for the specific space or local tie technique. Hence, before applying the general combination model a rank defect analysis is required for each normal equation matrix .

3 Rank defect analysis

The objective of the rank defect analysis is to find the number and the type of rank defect in the normal equation matrices by numerical methods. Input normal equation matrices which are to be used for terrestrial reference frame combination should have following rank defect characteristics seen from the theoretical and modelling point of view: For SLR and DORIS there should be 3 rank defects w.r.t. the rotation of the coordinate reference system which may be slightly smeared because of dynamical modelling, and one UT1 offset because of the linear dependency on the right ascension of ascending node of the satellite orbit. In principal, the same holds for GPS. Depending on the modelling and the extension of the network, GPS may additionally have 3 translational rank defects. VLBI should have 3 rotational and 3 translational rank defects w.r.t. the terrestrial reference system when fixing the nutation parameters. A simulated 3D distance network is analysed for test purpose and because the rank defects are rigorously defined: 3 translational and 3 rotational defects. For the numerical analysis following input solutions are chosen: for VLBI 3 solutions on 04-10-1999, first GPS week in October 1999 for GPS (14 solutions) and DORIS (1 solution), for SLR 9 four-week solutions in October 1999.

4 Eigenvalue analysis

The rank defect may numerically be determined by computing the eigenvalues of the normal equation matrix. In theory, the number of zero eigenvalues equals the rank defect. As well known, in practice, the numerical eigenvalue computation is sensitive to algorithm and round-off error deficiencies. Hence smearing effects have to be expected from this point of view, also.

The eigenvalues of the simulated distance network are obtained as expected: The first six smallest eigenvalues are in the range of 10^{-11} following a 10^{15} jump to the next smallest eigenvalue. Concerning the 3 VLBI solutions, the "best" one yield 6 smallest eigenvalues in the range of 10^{7} with a jump of 10^{6}, the next "best" solution has 6 smallest eigenvalues in the range of 10^{-4} with a jump of 10^{4}. The eigenvalues of the third VLBI solution does not represent the rank defect behaviour as expected. 3 out of the 9 SLR solutions contain eigenvalues as expected. The interpretation of a significant jump between the third and fourth smallest value requires further investigations. Only one out of 14 GPS solutions contains 4 near zero eigenvalues with jumps similar to the SLR solutions. All other solutions seem to be over constrained, or the reconstruction of the normal equations was not successful. The one week DORIS solution presents a not-expected eigenvalue behaviour: The

first 14 eigenvalues scatter around zero, then a significant jump of 10^4 follows.

5 Rank defect type analysis

The rank defect type of a normal equation matrix **N** may be determined by computing the matrix $\mathbf{Z}=\mathbf{H}^T\mathbf{N}\mathbf{H}$ with **H** being the coefficient matrix of those similarity transformation parameters for which the rank defects are searched. In theory, those diagonals of **Z** which are zero signify the rank defect types.

The diagonals of **Z** for the simulated distance network are in the range of 10^{-8} for the 3 translations, 10^{-3} for the 3 rotations and 10^{13} for the scale – a result which may be expected. The computation of **Z** was not successful for all input solutions. Further investigations are required.

6 Variance component estimation

Because the normal equation matrices in (5) may be differently scaled and weighted, the respective scale factors should be estimated within the general combination model (4) additionally. This may be performed by variance component estimates as e.g. described in (Koch, 1999, chapter 3.6). The respective algorithms for model (4) have been derived for a rigorous and simplified version. The application of these algorithms for the input solutions has just started. First numerical results seem to be appropriate, but further research is needed in order to obtain a reliable solution.

7 Conclusions

The eigenvalue analysis is a sensitive tool to determine the number of rank defects in normal equation matrices. For GPS, SLR and VLBI, there are some input solutions in the SINEX pool which agree to theoretical considerations, and may be used directly in the general combination model. They are appropriate to determine the geodetic datum parameters of the combined network: scale (SLR, VLBI, may be GPS), origin (SLR, may be GPS), and UT1 offset (VLBI). The rotation of the combined network has to be fixed by applying minimal constraints. The other solutions have to be investigated in order to find out which contribution they may give to the geodetic datum of the combined solution. The investigations on the rank defect type analysis and the variance component estimation in model (4) have to be continued.

References

Koch, K. R., Parameter Estimation and Hypothesis Testing in Linear Models, Springer-Verlag, Berlin, 1999