Recent Work on Theoretical Modeling of Nutation

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Abstract: In this note we make a brief sketch about the current state of the modeling of the Earth nutation, focusing on high frequency nutations. In particular, numerical nutations series are reported accounting for the harmonics (2,2), (3,1), (3,2) and (3,3) of the solid Earth potential, as well as for the case of the indirect effect due to the zonal harmonic of the second degree.

1 Introduction

In the last decade remarkable advances have taken place in the construction of theories that model the Earth rotational motion, aimed at improving the accuracy of the predictions of the Earth rotation. Several models have been developed both for rigid and non-rigid Earth nutations. The last theories involved with the rigid models claim to reach an accuracy about 0.1 μas (Souchay et al., 1999; Bretagnon et al., 1998; Roosbeck and Dehant, 1999; Navarro et al. 2002a; etc.). With respect to the non-rigid Earth models we would also like to underline the important progresses achieved in the last years, reflected in papers as those of Getino and Ferrándiz (2000), Shirai and Fukushima (2000), Huang et al. (2001), Mathews, Herring and Buffet (2002), etc. More details can be found in Johnston et al. (2000).

In order to select a nutation theory to replace the IAU 1980 theory of nutation, at IAU Colloquium 180 (Washington, 2000) three theories were selected as candidates to study for IERS 2000 or IAU 2000 nutations models: a) MHB2000 (Mathews, 2000); b) SF (Shirai and Fukushima, 2000); c) GF2000 (Getino and Ferrándiz, 2000). The first one, MHB, relies on an improved transfer function approach, based on a generalization of SOS equations. The SF theory uses a numerical convolution in the time domain, applied to Herring's transfer function. Finally, GF is a Hamiltonian, analytical theory for the Earth rotation, extending Kinoshita's one. Two theories reached an accuracy about 0.180 mas in Celestial Ephemeris Pole (CEP) offsets after being fitted to the official test series provided by USNO, namely: MHB2000 (two enhanced versions) and FGNH (GF theory plus planetary non-rigid series, Ferrándiz et al., 2000, plus oceanic corrections, Huang et al., 2001). At 24th General Assembly of the IAU, held in August 2000, MHB2000 was preferred by geophysical reasons (Dehant, 2000) and selected as IAU2000 nutation model, entering in force in 2003. There is also an agreement with respect to NASA/GSFC series about 150 μas, also using a model for FCN nutation. As the uncertainty of one daily value of CEP offsets is 0.3 mas (Gambis et al., 2001) we can conclude that there is a good agreement between theory and observation.

However, the constant improvement in the precision of the astrometric and geodetic techniques forces to examine the sources that may limit the present accuracy, both of the nutation models and the observational data. In our opinion there are two main obstacles that could stop the advances in Earth rotation studies. First, the use of "a priori" incomplete models in the observational data analysis and second, the existence of some difficulties in the fun-

damentals of the theories and inconsistencies among theories and practical procedures to reduce data. To overcome the first difficulty it should be necessary to improve the current nutation models by including the oceanic and second order effects as an intrinsic part of the theory, not as simple corrections. Besides, it should be expedient to take into account some effects previously disregarded, such as the case of the high frequency nutations (see next section). In order to avoid the problems related with the second obstacle, it should be necessary to re-examine comprehensively the construction procedure carried out in the elaboration of the theories themselves, and the way in which these theories are implemented to obtain practical procedures to reduce observational data. In this respect, the advances in Global Geophysical Fluids monitoring and International Terrestrial Reference Frame realization would help to solve possible inconsistencies (see reports by Chao and Altamimi et al. in this volume).

2 High frequency nutations

The Earth equatorial inertia moments A and B are not exactly equal but slightly different. As a consequence of this fact, known as triaxiality of the Earth, tesseral and sectorial harmonics appear in the expansion of the Earth potential, often referred to as tidal potential. These new terms give rise to a high frequency actual motion of the Earth figure axis in space, that is to say, to high frequency nutations (from now on denoted as HFN). HFN are due to both solid and ocean tides. The contribution of ocean tides can be estimated by using Topex/Poseidon based models (e. g. Ray et al., 1994), being nine times larger than that due to the tidal potential (Chao et al., 1996).

Empirical amplitudes of HFN have been obtained with the concourse of VLBI (Gipson, 1996) or GPS (Rothacher et al., 2001) observational data. Therefore, there is a real need of developing theoretical models to predict HFN amplitudes, as it was already claimed in Getino and Ferrándiz (2000). In this note we will pay attention to HFN of the non-rigid solid Earth arising from the tidal potential. The numerical contributions for the different kind of effects are reported in Tables 1, 2 and 4. In Table 1 we have kept the representation of the solution as nutations in longitude and obliquity. We think that this representation is the most adequate since it is the most natural for the physical motion that takes places. Nevertheless, B1.7 resolution adopted at the IAU XXIVth General Assembly recommends the inclusion of high frequency variations in the motion of the CIP in the International Terrestrial Reference System as polar motion (PM), what has motivated the inclusion in the tables of its corresponding X and Y components. Anyway, we have used the terminology polar libration, introduced in the pioneering work of Chao et al. (1991), in order to avoid confusion with the actual PM. For example, in Mechanics polar motion means the motion of the angular velocity vector with respect to a frame fixed in a body (the Earth, in this case). The remaining arguments appearing in the tables, P_{SF} and P_{TF}, denote the period of each lunisolar term, represented by a linear combination of Delaunay variables and the angle Φ, in the Space and Terrestrial Frames, respectively, in mean solar days. These results have been obtained from analytical formulae based on the Hamiltonian formalism of the non-rigid Earth developed by Getino and Ferrándiz. Specifically, we have considered a two-layer Earth model with mantle and core elasticity (Getino and Ferrándiz, 2001). The expansions of tidal potential are of an astronomical nature and have been worked out with the aid of the symbolic processor by Navarro (Navarro and Ferrándiz, 2002), using ELP and VSOP solutions.

Tables 1 and 2 are concerned with the **direct effect**, that is to say with the contribution due to the existence of (2,2), (3,1), (3,2) and (3,3) harmonics in

the tidal potential expansion. A more detailed discussion about the way of obtaining these results can be found in Getino et al. (2002), for the (2,2) harmonic, and in Navarro et al. (2002b) for the third degree harmonics.

Table 1 Direct effect in μas : harmonic (2,2)

Delaunay arguments							P_{SF}	P_{TF}	Polar libration (in μas)	
Φ_{SF}	Φ_{TF}	l_M	l_S	F	D	Ω	msd	msd	$X_{sin}=Y_{sin}$	$X_{cos}=-Y_{sin}$
2	1	0	0	0	0	0	.498634	.997269683	14.298	–8.207
2	1	0	0	0	0	–1	.498598	.997123413	1.938	–1.112
2	1	0	0	–2	2	–2	.500000	1.002745577	–4.768	2.737
2	1	0	0	–2	2	–2	.500000	1.002745577	–4.768	2.737
2	1	–1	0	0	0	0	.507826	1.034725222	.845	–.485
2	1	–1	0	0	0	0	.507826	1.034725222	.845	–.485
2	1	0	0	–2	0	–2	.517526	1.075810945	–11.385	6.536
2	1	0	0	–2	0	–1	.517569	1.075997464	–2.148	1.233
2	1	–1	0	–2	0	–2	.527441	1.119559097	–2.312	1.327

In addition to this direct effect, the triaxiality of the Earth couples with the zonal harmonics causing also HFN. This last effect is called **indirect effect** (Escapa et al., 2002a, b) and reflects the fact that Andoyer variables are not an action-angle set for a triaxial Earth. These contributions are shown in Table 4 for the case of the coupling between the triaxiality of the Earth and the second degree zonal harmonic. The expressions for these amplitudes have been computed using the formulae derived in Escapa et al. (2002b). They depend on the triaxiality of the whole Earth and on the triaxiality of the core, which provides a larger contribution. We have displayed some values derived for this parameter in Table 3 ($2d_c = 1 - A_c/B_c$). From these results it is clear that the core is not symmetrical and that there is some discrepancy in the values of d_c. The contribution of this effect for some terms is about 1 μas, so we think that to ignore core triaxiality and disregard this contribution is not a reasonable option. Therefore, our proposal is to consider a mean value of the triaxiality of the core ($d_c = 5.7010^{-6}$) as the best estimate available and incorporate the amplitudes obtained for this model (AV in Table 4) in HFN.

Table 2 Direct effect in μas: harmonics (3,1), (3,2), (3,3)

Type	Delaunay Arguments							P_{SF}	P_{TF}	Polar Libration	
	Φ_{SF}	Φ_{TF}	l_M	l_S	F	D	Ω	msd	msd	Y_{sin}	Y_{cos}
3 1	0	0	–3	0	–3	1	1	1.120	–9.107	.090	.734
3 1	–1	0	–1	0	–1	1	1	1.075	–13.719	.164	1.336
3 1	0	0	–1	0	0	1	1	1.035	–27.212	.304	2.476
3 1	0	0	–1	0	–1	1	1	1.035	–27.322	1.901	15.467
3 1	0	0	–1	0	–2	1	1	1.035	–27.432	–.105	–.854
3 1	–1	0	–1	2	–1	1	1	1.002	–193.560	.099	.803
3 1	1	0	–1	0	0	1	1	.998	–2.190.350	–.229	1.861
3 1	1	0	–1	0	–1	1	1	.998	–3.231.496	1.507	12.258
3 1	1	0	–1	0	–2	1	1	.997	–6.159.135	–.083	–.674
3 1	–1	0	1	0	2	1	1	.997	6.159.135	–.095	.773
3 1	–1	0	1	0	1	1	1	.997	3.231.496	1.973	–16.054
3 1	–1	0	1	0	0	1	1	.997	2.190.350	–.341	–2.775
3 1	1	1	–1	0	0	1	1	.995	438.360	.098	–.794
3 1	1	1	–1	0	–1	1	1	.995	411.807	.123	.997
3 1	1	0	1	–2	1	1	1	.992	193.560	–.257	2.089
3 1	0	0	1	0	2	1	1	.962	27.432	.107	–.866
3 1	0	0	1	0	1	1	1	.962	27.322	–2.048	16.660
3 1	0	0	1	0	0	1	1	.962	27.212	–.321	2.608
3 1	1	0	1	0	1	1	1	.930	13.719	–.158	1.282
3 2	2	1	–3	0	–3	2	2	.52752	1.119902988	–.254	–.174
3 3	3	2	–1	0	–1	3	3	.33652	.5079043940	–.224	–.114

Finally, let us point out that there are another two models that deal with HFN (Brezinski and Capitaine, 2001; Mathews and Bretagnon, 2001). These works are constructed by using different theoretical approaches and different tidal expansions and no reference is made to the indirect effect.

Table 3 Triaxiality of the core (unit 10^{-6})

Model	d_c	Citation Source	Author
GG	8.94	González & Getino 1997	Defraigne & Dehant 1997
BC	4.92	Brzezinski & Capitaine 2002	Morelli & Dziewonski 1987
SX	3.25	Szeto & Xu 1997	Mathews, Buffet, Herring & Shapiro 1991

Table 4 Indirect effect in μas

						Polar libration: X-component							
Delaunay variables						Model SX		Model BC		Model GG		Model AV	
Φ_{TF}	l_M	l_S	F	D	Ω	sin	cos	sin	cos	sin	cos	sin	cos
1	0	0	–2	0	–2	.101	–.058	.109	–.063	.130	–.075	.113	–.065
1	0	0	–2	2	–2	.097	–.056	.134	–.077	.224	–.129	.152	–.087
1	0	0	0	0	0	–.735	.422	–1.113	.639	–2.022	1.161	–1.289	.741
1	0	0	0	0	–1	–.105	.061	–.160	.092	–.292	.167	–.186	.107
1	0	1	0	0	0	.026	–.015	.043	–.024	.082	–.047	.050	–.029

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