

Electromagnetic Core-Mantle Coupling

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Introduction

We discuss electromagnetic core-mantle coupling computations at decadal and diurnal time-scales. This coupling mechanism can lead to decadal length of day variations and can also influence certain nutations to an observable level. If electromagnetic coupling is the only important coupling between core and mantle, insight into physical properties of Earth's deep interior can be provided.

Electromagnetic core-mantle coupling at decadal time scale

The torque of the core on the mantle generated by the Lorentz force in the mantle is (Holme, 1998):

$$\vec{\Gamma} = \int_M \vec{r} \times (\vec{j} \times \vec{B}) dV,$$

where the integral is over the volume of the mantle, $\vec{B}$ is the magnetic field and $\vec{j}$ is the current given by Ampère's law:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j},$$

where μ_0 is the magnetic permeability of the vacuum. Applying the divergence theorem, the z-component of the electromagnetic torque can be converted to a surface integral over the radius of the core-mantle boundary:

$$\Gamma_z = \frac{r_{CMB}}{\mu_0} \oint_{CMB} B_\phi B_r \sin\theta dS$$

where B_r and B_ϕ are the radial and longitudinal components of the magnetic field. The radial component B_r is purely poloidal whereas the longitudinal component B_ϕ has both toroidal and poloidal components. If the mantle is insulating, the radial magnetic field in the mantle can be computed by downward continuation of the measured field at the surface of the Earth. The above torque can be decomposed into a poloidal, an advective and a leakage torque. The poloidal torque is associated with the poloidal magnetic field. Given a time-dependent model of the geomagnetic field and of the mantle conductivity, the poloidal torque can be computed directly (Stix and Roberts, 1984). The advective torque is induced by the advection of the radial magnetic field; assumptions on the velocity field are necessary to compute it. The leakage torque is associated with the diffusion of the magnetic field at the top of the core, it cannot be determined from surface magnetic field measurements. Often, the leakage torque is considered to be unimportant on decadal time scales (Jault and Le Mouél, 1991), (see however Van Hoolst et al., this volume). The flow inducing the advective electromagnetic torque cannot be determined uniquely from the observations and further assumptions concerning the flow are necessary. Since the electromagnetic coupling can therefore not be accurately determined, Holme (1998) looked for flows consistent with the secular variation of the magnetic field and producing an electromagnetic coupling that leads to the observed length-of-day (LOD) variations. To compute the core flow from the observed magnetic secular variation, the frozen-flux hypothesis (the advection of the magnetic field dominates its diffusion) and the radial component of the diffusionless induction equation are used:

$$\frac{\partial B_r}{\partial t} + \nabla_H \cdot (B_r \vec{u}) = 0.$$

This equation contains two unknowns: the two horizontal components of the velocity field. Several approximations concerning the flow are used in the literature: steady flow / steady flow in a drifting frame (drift of the flow with respect to the mantle), toroidal flow and tangential geostrophic flow. The toroidal flow at the CMB has only a horizontal component which is assumed to be toroidal. The tangential geostrophic flow is governed by the balance between the (tangential components) of the Coriolis force and the pressure gradient at the core-mantle boundary. This assumption is most often used. To test these assumptions, the LOD observations are compared with the core angular momentum computed using these different assumptions for the flow. The figure shows the angular momentum predictions for three geostrophic flows (Jault et al., 1988; Jackson et al., 1993; Pais and Hulot, 2000) using different inversion parameters and a drifting flow (Holme and Whaler, 2001). From around 1920, the agreement between these curves is good but less good before this date (although the agreement for the drifting flow is perhaps slightly better than for the geostrophic flows).

The computations of the electromagnetic coupling have shown that an efficient electromagnetic torque between the core and mantle can exist only if a conductive layer of 10^8 S is present at the base of the mantle (Holme, 1998).

Electromagnetic core-mantle coupling at diurnal time scale

When no dissipative coupling is considered, discrepancies appear between the observed and modeled out of phase 18.6-year retrograde, annual and semi-annual prograde nutations. A dissipative torque such as the electromagnetic interaction takes place between the fluid outer core and the mantle and between the solid inner core and the fluid outer core. At diurnal time scale, the relative motion of the outer core with respect to the mantle (resp. the inner core) is essentially a rigid body rotation (toroidal motion of order 1, degree 1). This nutation motion of the fluid core causes an advection of the magnetic

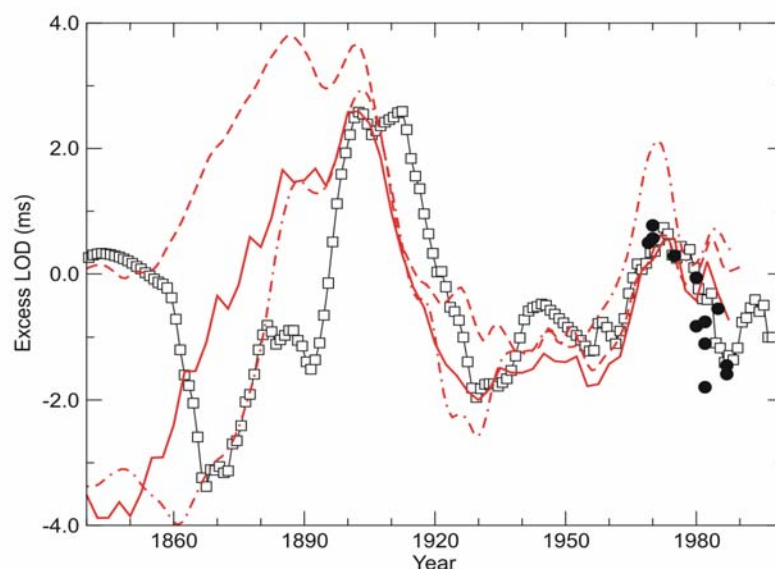


Fig. 1 Comparison of LOD observations (squares) with predictions from the tangential geostrophic flow obtained when using the inversion parameters of Pais and Hulot (2000) (dashed line), Jackson, Bloxham and Gubbins (1993) (solid line), Holme and Whaler (2001) (dot dashed line), Jault et al. (1988) (circles).

field at the CMB that induces a perturbation of the magnetic field. At first order, the electromagnetic torque associated to the Lorentz force can be expressed as (Rochester, 1960):

$$\vec{\Gamma} = \frac{1}{\mu} \int_S (\vec{r} \times \vec{b})(\vec{B} \cdot \vec{n}) dS$$

where $\vec{B}$ is the main magnetic field at the boundary, and $\vec{b}$ is the perturbation of the magnetic field associated to the advection of the main magnetic field by nutations. Using this expression, Buffett (2002) expresses the electromagnetic torque as a function of coupling constants and the differential rotation of the core with respect to the mantle. Mathews et al. (2002) compute the equatorial angular momentum budget of the Earth considering the electromagnetic coupling at the CMB and ICB. The coupling constants, which depend on the conductivities of the lower mantle, the inner core, the fluid outer core and the amplitude of the magnetic field at the outer core boundaries, are evaluated through the adjustment of a set of parameters to VLBI nutation amplitude observations so that the theoretical amplitudes of the nutations agree with the observed amplitudes. Assuming that the electromagnetic torque is the only one acting on nutations, it results (Buffett, 2002) that the conductivity of the lower mantle must be $5 \cdot 10^5 \text{ Sm}^{-1}$ over a 200 m thick layer, the magnetic field at the CMB has a radial dipolar component of 4.6 gauss and a "uniform" component so that the radial field at the CMB has an amplitude of 6.9 gauss, and the radial magnetic field at the ICB is 72 gauss. Note that the conductance of the lower mantle provided by these two studies is consistent (10^8 S).

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