Pulsars: timing reduction at Nançay's radio-telescope and construction of an astrometric catalog

J. Souchay[1] and I. Cognard[2]

[1] *Observatoire de Paris/CNRS UMR8630, 61 Av. de l'Observatoire, 75014, Paris, France*
[2] *CNRS/Laboratoire de Physique et Chimie de l'Environnement, 34 Avenue de la Rercherche Scientifique, 45071 Orléans, France*

Abstract. After summarizing the general physical properties of pulsars ans the basis of pulsar timing reduction we emphasize the possibility to construct an astrometric catalogue of pulsars starting from the pulsar observations at Nançay's radiotelescope, not sensitive to the ephemerides used for the reduction.

1 Introduction

Pulsars, discovered by radioastronomers in 1967, not only offer the opportunity to study fundamental physics in conditions not reachable on Earth, but also can play an essential role in modern astrometry. Indeed, pulsar timing analysis is directly dependent on the way by which the position of the Earth on its orbit is modelized. The arrival times of the pulses are shifted by the Doppler effect due to the relative speed of the Earth on its orbit, whose direction and amplitude vary along the year with respect to the line of sight. Therefore, pulsar timing provides precious information about the location of the ecliptic, i.e. a dynamical frame related to the revolution of the Earth, with respect to a frame related to the pulsars, as galactic objects.

Thus after recalling fundamental principles both related to pulsars physical properties and to pulsar timing reduction, we explain how radio pulses observations of pulsars carried out at Nançay's radiotelescope can participate actively to the construction of both a very accurate catalogue of the coordinates of pulsars, which should give birth to a Pulsar Reference Frame, and to a very accurate positioning of the ecliptic with respect to inertial directions. A set of about 20 pulsars are regularly observed at Nançay, among which PSR1937+21 and PSR1821-21 have been subject to a very intensive campaign for more than 15 years. We show the advantages of all these observations to accomplish the objectives above.

2 Physical characteristics of pulsars

Pulsars are radio-sources which emit radio pulses periodically. During each period, the information received from the pulsar covers a very limited time span. The period P of the pulsar can be defined as the time interval between two consecutive pulses. Presently more than 1500 pulsars have been discovered. Their period covers a range between $1\ ms\ <\ P\ <\ 8\ s$, with a bi-modal distribution (one peak at 4 ms and another one at 0.6 s) and with a slowing down typically at the level of $\dot{P} = 10^{-15} s\ /\ s$. The magnetic fields of the pulsars are the strongest ones observed among galactic sources. All the pulsars are emitting in radio frequencies, but some of them, as the Crab pulsar, are emitting in other frequencies, as in the optical, X, and γ bands.

2.1 Pulsar emission: the physical explanation.

The periodicity of the signal as well as the hypothesis of a rotational object strongly suggests that the emissive part of the beam is very small. The beam accompanying the pulsar in its rotation is crossing the line of sight of the observer located on Earth at each period of the pulsar, as it is a case for the light coming from a lighthgouse. The mechanism of emission of the pulsars is not still well understood, but the theoritical explanation involves a strong magnetic field around a neutron star. A scenario often considered is the following one: the magnetic field creates itself a strong electric field which brings the light particules towards the upper magnetosphere. The electrons are dragged away from the surface along the lines of force, thus creating a vacuum several hundreds of meters high above the surface. The difference of electric potential in this vacuum enables to accelerate the electrons along the lines of magnetic forces, with overwhelming energies leading to emissions of γ rays reaching a few Mev. These γ rays at their turn interact with the magnetic field to create electron-positron pairs. These pairs are themselves re-accelerated to produce new γ rays. This scenario re-produced in an unlimited way is at the origin of the coherent radio emission usually observed.

2.2 Evolution of a pulsar

The emission of the electro-magnetic radiations is responsible of a decreasing of the speed of rotation of the underlying neurtron star, i.e. an increasing of its period of rotation P. The decreasing law has been set already 30 years ago by Manchester and Taylor (1977):

$$\dot{P} = \frac{8\pi R^6 B_0^2 \sin^2 \alpha}{3Ic^3 P} \tag{1}$$

where :

- B_0 is the value of the magnetic field at the surface of the star
- R is the radius of the neutron star
- α is the angle between the axis of rotation and the magnetic axis
- I is the angular momentum of the star
- c is the speed of light
- P is the period of the pulsar

2.3 Magnetic field

Therefore it is possible to evaluate roughly the intensity of the magnetic field B_0 of a pulsar by measuring its period P and the derivative of its period $\dot{P}$ (Manchester et al., 1988; Rawley, 1986). B_0 is generally ranging between 10^4 T and 10^8 T. Such an amplitude is possible if the magnetic flux $\Phi = BR^2$ is conserved during the contraction of the initial star towards a neutron star. For instance, if the Sun radius should contract from $R_\odot = 7 \times 10^5 km$ to $R_\odot = 10km$, then the intensity of the magnetic field, initially of $B = 10^{-4}$ T should reach 10^6 T. According to the equation (1), the product $P\dot{P}$ gives an estimate of the value of the magnetic field, which currently degenerates in a time scale of 10^6 to 10^7 years.

As the strength of the pulsar magnetic field is decreasing, the period becomes longer and its derivative smaller. As the magnetic field is at the basis of the emission, it could happen that the field is not strong enough to create γ rays energetic enough to sustain the emission, and thus leads to the extinction of the pulsar.

2.4 Phase and period of a pulsar

The short periods observed for pulsars are explained by the conservation of the angular momentum. For instance, if the Sun, whose the period of rotation (at the equator) is roughly 26 days and the radius approximatively $7 \times 10^5 km$, would contract as a neutron star, then its period of rotation should be 4 ms.

The pulse of a pulsar has a mean feature characteristic for each pulsar which does not depend on time. Profiles can exhibit several peaks. The principal one is called "pulse", whereas the other ones are quoted as "interpulses". Each individual pulse is made itself of a stream of subpulses, themselves consisting in micro pulses, whose intensity and phase can vary. The study of these variations is precious for the understanding of the scenarios of emission mechanisms.

The emission of a pulsar is observed as a periodic sequence of pulses, associated with the periodicity of its rotation. Therefore it is natural to introduce the terminology of *phase* to describe the time along a sequence of pulses.

2.5 Variations of the period

The rotation of a pulsar is slowing down because of the conversion of its rotational energy into electro-magnetic emissions. This loss of energy lengthens the period in a stable way, so that the period can be modelled in the following way:

$$P(T) = P_0 + \dot{P}_0(T - T_0) \tag{2}$$

Where P_0 and $\dot{P}_0$ are the period and its derivative with respect to the time, at a given epoch T_0. Nevertheless, the period can undergo sudden and hasardous changes with large amplitudes. These changes, called *glitches*, recently observed for a millisecond pulsar by the second author (Cognard and Backer, 2004) are explained by a re-equilibrium of the internal forces of the pulsar consecutive to a "starquake", i.e. a quake of the external crust. One of the explanations for these starquakes are the changes provoked by the slowing down of the pulsar: the centrifugal forces leading to the flattening of the neutron star are decreasing and the lack of elasticity of the crust leads to a crunch whose the manifestation is a starquake.

3 A specific class of objects: the millisecond pulsars

The theory according which pulsars are neutron stars enables to consider pulsars with period of the order of 1 *millisecond*. In fact a lot of such objects have been discovered, following the detection of the first candidate, PSR1937+14, in 1982 (Backer et al.,1982). Their period is always larger than 1 *millisecond*, but the existence of sub-millisecond pulsars is presently investigated. Whereas the very small period of rotation of ms pulsars suggests that these objects are very young, the magnetic field calculated starting from the equation (1) as well as the lack of nebula generally surrounding the pulsar suggest that they are in fact very old objects.

The basic scenario to explain the presence of a millisecond pulsar is the following one: one of the member of a binary star is evolving on the main sequence more rapidly as its companion star. After it explodes as a supernova, it becomes a neutron star and forms a pulsar. Then its magnetic field is gradually decreasing whereas its period increases, thus leading to a complete extinction. At this stage a binary system is made of as star located on the main sequence together with an old and non emitting pulsar. When the star companion leaves the main sequence to reach the phase of expansion, its external material is undergoing an accretion from the dense pulsar. Therefore angular momentum is transferred, which re-accelerates the pulsar. This explains why this last object can be subject to a large angular speed of rotation mixed with a low magnetic field, properties which are generally found for the rapid pulsars observed.

The quantity of material being subject to accretion is limited, because of the pressure exerted by the pulsar radiation (Eddington, 1926) and its magnetic field. The maximum of accretion rate has been set to roughly $10^{-8}\ M\odot/y$. The minimum period under which the rotation of the pulsar is too fast to accrete some material has been given by Van der Heuvel (1984).

$$P = (2.4ms)B^{6/7}R^{15/7}M^{-\frac{5}{7}}\dot{m}^{-frac37} \tag{3}$$

where :

- B is the magnetic field of the pulsar in unit of $10^9 G$
- R is the radius in unit of 10^4 m
- M is the mass in unit of solar mass
- $\dot{m}$ is the accretion rate in units of $M_\odot/y$

Although the scenario above is generally accepted, some millisecond pulsars, as PSR1937+214 do not show the presence of any companion star. For these kinds of objects, alternative theoretical investigations are proposed, which are not in the scope of the present paper.

4 Pulsar timing and the de-dispersion technique at Nancay's radio-telescope

The major part of the informations concerning the physical properties of pulsars are obtained at radio wavelengths, between 10 MHz and 10 GHz. The radio signals emitted are affected by the ionizated interstellar medium according to the frequency of emission. The lower the frequency, the longer the time delay due to the presence of interstellar medium. The consequence is that the pulses are wide instead of showing a thin profile. Therefore, a specific technique consists in a de-dispersion of the signal in order to study the pulses properly. Recently, the N.R.T. (Nançay Radio Telescope) is equipped with a battery of 80 bi-processors and an instrumentation intended to catch the signal and to send it to a calculator for de-dispersion and synchronous integration with the period of rotation of the pulsar. The sensistivity of observations has been noticably improved by this new technical configuration.

5 Pulsar timing, astrometry and dynamics of the solar system

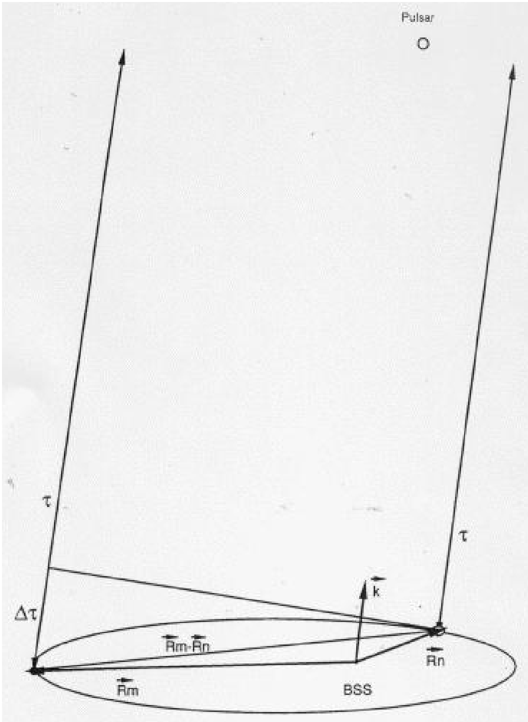

Fig. 1. Geometric configuration for pulsar timing

Analysis of pulsar timing data consists in comparing the observed arrival times of pulses with those obtained from the theory. These calculations involve a physical model describing the trajectory of an electro-magnetic signal from the pulsar (the emitter) to the observer (the receiver). For that purpose it is necessary to include the modeling of various astronomical motions (see Figure 1), as:

• The motion of the pulsar with respect to the Barycentric Reference System (BRS) centered on the barycenter of the solar system (this motion is generally considered as linear and uniform). Indeed, any pulsar is undergoing a motion around the center of the galaxy and consequently a relative motion with respect to the Earth, characterized by a radial velocity (along the line of sight) as well as a proper motion (perpendicular to the line of sight).

• The motion of the Earth barycenter with respect to the BRS, due to its annual revolution around the Sun and to the influence of the planets. Pulsar timing is affected by the position of the Earth with respect to the barycenter of the solar system. Therefore it is influenced by the small perturbations caused by the planets. For instance, the giant planets (Jupiter, Saturn, Uranus and Neptune) cause a displacement of the Sun and of the Earth with respect to the barycenter, this gravitational action having itself an influence on the motion of the Earth.

• The motion of the observer (i.e. the radiotelescope antenna) with respect to the Earth's barycenter, mainly due to the diurnal rotation of the Earth. Moreover the oscillations of the axis if figure of the Earth due to the combined effect of precession and nutation must also be taken into account.

5.1 Basic equation for pulsar timing

Let us call T_n the date of emission of a pulse and t_n the instant when the pulse reaches the observatory. We can write:

$$t_n = T_n + \tau \tag{4}$$

where τ is the interval of time necessary for the pulse to come from the pulsar to the observatory. For the analysis, t_n is measured and compared to the calculation of $T_n + \tau$. In fact $T_n + \tau$ cannot be directly calculated, because the date of emission of the pulse is unknown. Nevertheless, pulses are emitted periodically and as a consequence, if we denote T_m the date of emission of a pulse after T_n, we can write:

$$T_m = T_n + NP \tag{5}$$

P being the period of one pulse and N an integer. If t_m is the date of reception at the observatory of the pulse emitted at T_m, then:

$$t_m - t_n = (T_m + \tau + \delta\tau) - (T_n + \tau) = (T_m - T_n) + \frac{(\vec{R}_m - \vec{R}_n).\vec{k}}{c} \tag{6}$$

where c is the speed of light, $\vec{R}_m$ and $\vec{R}_n$ are the positions of the observatory respectively at t_m and t_n with respect to the barycenter of the solar system, and $\vec{k}$ is a unit vector directed towards the direction of the pulsar (cf. Figure 1).

6 A complete modeling of the arrival time

In order to calculate with a very high precision the propagation time of the pulses between the pulsar and the observer, it is necessary to elaborate a sophisticated modeling including the time delays due to the propagation of an electro magnetic signal through a ionized interstellar medium, as well as relativistic effects on the geometric space time due to the solar gravitational field. This model must be set at a precision of a few *nanoseconds* level in order to match the present accuracy of pulsar timing measurements.

6.1 Relativistic effects: geometric delay and time measurements

When adopting the Minkowski's metric, the trajectories of the photons coming from a pulsar are following the null geodesics corresponding to this metric and their coordinates must satisfy the following equation:

$$\mathrm{d}\tau^2 = (1 - 2\phi)c^2\mathrm{d}t^2 - (1 + 2\gamma\phi)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) = 0 \tag{7}$$

This can be reduced at the first order to:

$$c\mathrm{d}t = (1 + (1 + \gamma)\phi)[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2]^{1/2} \tag{8}$$

This equation gives the interval of time between the emission of the photon at T and its reception at the antenna at t, after covering a distance $(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)^{1/2}$. After replacing the gravitational potential ϕ by its expression, we find:

$$c(t - T) = \mid \vec{R}' - R \mid -(1 + \gamma) \sum_p \frac{GM_p}{c^2} ln \left[\frac{\vec{k}.\vec{r}^p + r^p}{\vec{k}.\vec{R}^p + R^p} \right] \tag{9}$$

where $\vec{R}'$ and R are the positions of the pulsar and of the station with respect to the origin (barycenter of the solar system), $\vec{r}^p$ and $\vec{R}^p$ respectively the positions of the pulsar and of the antenna with respect to the planet p, and $\vec{k}$ the unit vector towards the pulsar.

6.2 Proper time and coordinate-time

A transformation is necessary which enables to make the correspondence between the physical time τ in which the observations are measured, and the coordinate time t from which the equations are given. Relativity introduces a difference between these two time-scales. In order to make the correspondence, we can write the invariance of the element of distance $\mathrm{d}s^2$ in any change of coordinates:

$$\mathrm{d}s^2 = c^2\mathrm{d}\tau^2 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{10}$$

τ is the proper time of the station, and the x^i are the space-time coodinates. Therefore we obtain the differential equation which gives t as a function of τ:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = 1 - \left[U(\vec{r}_h) + \frac{1}{2}(\frac{v}{c})^2\right] \tag{11}$$

where $\vec{r}_h$ is the position of the clock insuring the pulsar timing, v its speed with respect to the barycenter of the solar system, and U is the gravitationnal potential.

6.3 Interstellar dispersion

The model used to express arrival time of the pulses must include the time delay $\mathrm{d}t$ due to the interstellar dispersion given by:

$$\mathrm{d}t = \frac{kDM}{\nu_b^2} \tag{12}$$

where ν is the frequency of the signal and:

$$k = \frac{e^2}{8\pi^2 c\varepsilon_0 m_e} = 4.14879 \times 10^3 cm^3 pc^{-1} MHz \tag{13}$$

with: $e = 1.60219{\times}10^{-19}C$ (electronic charge), $\varepsilon_0 = 8.85419{\times}10^{-12}AsV$ and: $m_e = 9.10956 \times 10^{-31}kg$ (electron mass). DM is the dispersion measure in pc/cm^3, corresponding to the integral of the electron density along the line of sight to the pulsar studied. ν_b is the frequency at the barycenter of the solar system, and consequently corresponds to the frequency ν_s of the signal at the station, corrected with the Doppler effect:

$$\nu_b = \frac{\nu_s}{1 + \frac{\vec{k}.\vec{v}}{c}} \tag{14}$$

6.4 Correction due to the curvature of the wavefront

The wavefront which reaches the solar system is not flat but curved. Some time delay $\delta\tau$ is due to this curvature (see Figure 2). The curvature is of course depending on the distance of the pulsar. As an example, for the pulsar PSR1937+214, which is located at a distance $\approx$ 2 kpc, the time delay of the signal is of the order of $1\mu s$, which is quite significant in view of the precision of the observations, which is at the level of the nanosecond.

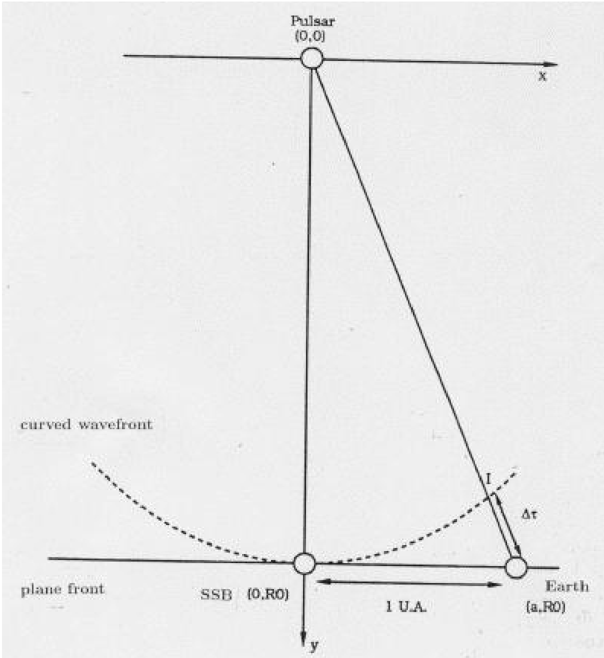

Fig. 2. Curvature of the wave front

6.5 Modeling of the arrival time of the pulse

Finally, after taking into account all the effects above, the initial equation (6) giving the difference of arrival of the pulses is becoming much more complicated, and has the following form:

$$t_n - t_0 = (T_n - T_0) + \frac{V_R}{c}(T_n - T_0) - \left(\frac{\vec{k}_n.\vec{R}_n}{c} - \frac{\vec{k}_0.\vec{R}_0}{c}\right)$$
$$+\left[\frac{r_n^2 - r_0^2 - (\vec{k}_n.\vec{r}_n)^2 + (\vec{k}_0.\vec{r}_0)^2}{2cR_0}\right] - \frac{(\vec{V}_T.\vec{r}_n)}{cR_0}(T_n - T_0) + \frac{V_T^2}{2cR_0}(T_n - T_0)^2$$
$$-\frac{(1+\gamma)}{c^3}\sum_p GM_p ln\left[\frac{\vec{k}\vec{r}_n^p + r_n^p}{\vec{k}.\vec{r}_0^p + r_0^p}\right] + 4.15 \times 10^3 DM(\nu_{b,n}^{-2} - \nu_{b,0}^{-2}) \quad (15)$$

In this equation all the parameters have already been defined previously except V_R and V_T which stand respectively for the radial and the tangential velocities of the pulsar with respect to the barycenter of the solar system. Notice that the first term $(T_n - T_0)$ at the right hand side of the equation is known: indeed the pulsar is emitting a number of pulses which is an integer between the initial date T_0 and the date T_n, as it was shown from (5).

7 Towards a Pulsar reference Frame constructed at Nancay's radiotelescope

Since 1988, the NRT (Nancay Radio Telescope, see Figure 3) has been largely involved in the detection of millisecond pulsars as well as in a huge and regular specific monitoring concerning some of them. High precision mesurements have accumulated since more than 10 years for various studies among which one of the most important were those concerning the interstellar medium. More particularly the very long and consistent data concerning the two millisecond pulsars PSR1821-24 and 1937+21 to which we added the long Arecibo PSR1855+09 data and the two shorter PSR13-02 and PSR1643-12 data lead us to begin an analysis devoted to the very accurate determination of the reciprocical position of the pulsars, as well as the position of the ecliptic, i.e. the trajectory of the Earth, with respect to them.

7.1 Influence of the ephemerides of the Earth orbital motion

As it is clearly shown by equation (15) the pulse coming from a given pulsar is directly dependent on the model chosen for the position of the Earth, through the intermediary of the vectors $\vec{r}_0$ and $\vec{r}_n$ which represent the radius vectors of the Earth with respect to the BRS (Barycenter Reference System) centered at the barycenter of the solar system. Moreover it is also dependent on the way by which the position of the pulsar is determined through the intermediary of $\vec{k}_0$ and $\vec{k}_n$ which are the unit radius vectors from the barycenter of the solar system to the pulsar at dates T_0 and T_n respectively. Therefore the combination of the adoption of any given ephemerides of the orbital motion of the Earth with respect to the barycenter together with datation of pulses arrival times lead to a specific determination of the celestial coordinates of the pulsar.

Thus we can naturally point out the following questions: what is the uncertainty of the positioning of a given pulsar in the celestial sphere due to the uncertainty of the orbital ephemerides of the Earth used for the calculations ? Even if the coordinates are subject to changes according to the ephemerides, what about the angular distance between two pulsars ? At which extent does it depend of the ephemerides used ?

8 Tests with a small set of pulsars.

In order to give a preliminary answer to the questions above, we have used 3 different ephemerides from the JPL (Jet Propulsion Laboratory), i.e. DE200, DE405 and DE407 (Standish, 1982; Standish, 1990) in order to reduce the pulsar timing observations.

In Table 1 we list our determinations of the coordinates for each pulsar selected above according to the ephemerides used, whereas in table 2 we give our determinations for the angular distances between the pulsars chosen. We can remark that these angular distances are poorly affected by the ephemerides used: the difference according to the ephemerides used does not exceed 3 milliarcseconds (Table 2). On the opposite the differences in the determination of the equatorial coordinates (α, δ) of the pulsars (Table 1) can reach roughly 15 millisarseconds.

These results are a clear support for the possible construction of a celestial reference frame with optimal accuracy based on the intrinsic relative position of pulsars rather than using an intermediate modelling of the position of the ecliptic which should worsen the quality of the astrometric measurements. Such an application has been done with success for the construction of the ICRF (International Celestial reference Frame) starting from the relative positions of quasars (IERS, 2001).

Table 1. List of the pulsars used for the astrometric analysis, together with their equatorial coordinates for J2000.0 obtained from 3 different ephemerides for the orbital motion of the Earth.

Name of pulsar	α DE200	δ DE200	$\Delta\alpha$ DE405 - DE200	$\Delta\alpha$ DE407 - DE200	$\Delta\delta$ DE405 - DE200	$\Delta\delta$ DE407 - DE200
	h mn s	° ' "	10^{-3} "	10^{-3} "	10^{-3} "	10^{-3} "
PSR1821-24	18 24 32.007721	- 24 52 10.803114	7.470	7.095	-2.772	- 4.702
	± 0.020	± 5.44	± 0.42	± 0.42	± 7.61	± 7.61
PSR1937+21	19 39 38.560135	+ 21 34 59.139705	16.650	14.205	-6.069	- 6.577
	± 0.002	± 0.04	± 0.04	± 0.04	± 0.06	± 0.06
PSR1855+09	18 57 36.393499	+ 09 43 17.323676	12.660	10.860	-4.144	- 5.096
	± 0.014	± 0.40	± 0.29	± 0.29	± 0.56	± 0.56
PSR0613-02	06 13 43.972992	- 02 00 47.110967	14.025	12.480	-1.386	- 2.781
	± 0.024	± 0.72	± 0.49	± 0.49	± 1.00	± 1.00
PSR1643-12	16 43 38.157313	- 12 24 58.711054	10.980	9.855	-3.611	- 1.531
	± 0.024	± 1.98	± 0.50	± 0.50	± 2.78	± 2.78

Table 2. Determination of the angular distances between the pulsars listed in Table 1, with respect to the ephemerides used for the orbital motion of the Earth.

Pulsar	Pulsar	arc inter-pulsar (degree)	Δ arc DE405-DE200 (mas)	Δ arc DE407-DE200 (mas)
1	2	49.90547948	0.2005	0.7722
1	3	35.51839847	-0.2010	0.4275
1	4	152.98996546	-0.7606	-1.4015
1	5	26.87642586	0.0214	0.3235
2	3	15.57641855	1.1884	1.1476
2	4	151.36653030	1.0796	1.0650
2	5	54.96006368	-1.2473	-1.2708
3	4	166.64788112	2.6505	2.5836
3	5	39.963920275	-2.7859	-2.7557
4	5	153.380677482	0.0979	0.1354

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