Lunar Laser Ranging: measurements, analysis, and contribution to the reference systems

J. Chapront and G. Francou

Observatoire de Paris - SYstème de Référence Temps-Espace (SYRTE) - UMR 8630/CNRS, 61 Av. de l'Observatoire, 75014, Paris, France

Abstract. On 21 July 1969, the first lunar reflector was placed on the Moon surface by Apollo 11 mission. Since this date 4 more reflectors have been deployed and, today, the Lunar Laser Ranging (LLR) is still operating in USA and France. The complete set of LLR observations covers a time interval longer than 30 years and the accuracy of measurements is regularly improved. This technique has brought remarkable results in lunar science, solar system dynamics, geophysics and fundamental physics. In 1998, the Paris Observatory Lunar Analysis Center (POLAC) has performed analysis of LLR observations. In this contribution we emphasize some results obtained in 2002 such as the orientation of the dynamical ecliptic, the correction to the IAU 1976 value of the precession constant in longitude and the improvement of the tidal acceleration of the Moon.

1 Lunar Laser Ranging measurements

Lunar laser telemetry consists in determining the round-trip travel time of the light between a transmitter on the Earth and a reflector on the Moon, which is an equivalent measurement of the distance between these two points. Neil Armstrong put down the first reflector array in the Sea of Tranquillity in July 1969 and a few weeks later McDonald Observatory succeeded in detecting photons returned from a laser pulse sent to the Moon.

Afterwards, more reflector arrays have been placed by two American Apollo missions in 1971 (Apollo 14 and Apollo 15), and two Soviet automatic missions, Luna 17 (1970) and Luna 21 (1973) which carried French-built reflectors, Lunakhod 1 and 2. Except Lunakhod 1 from which the signal has been lost, the 4 other reflectors are still operating normally (Fig. 1).

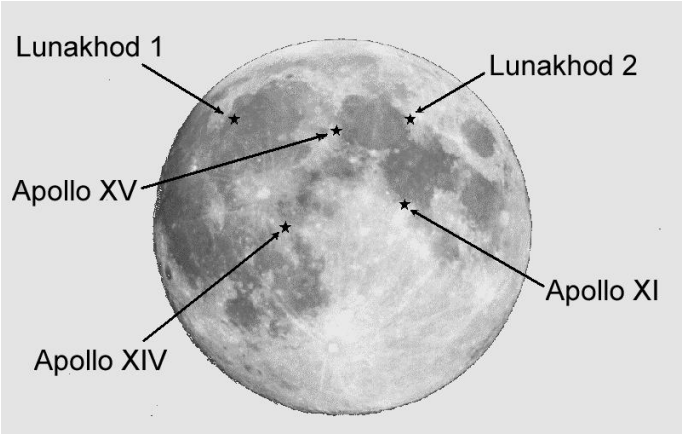

Fig. 1. Distribution of the reflectors on the lunar surface

The Apollo arrays consist of 3.8-cm diameter corner cubes mounted on an aluminium panel (100 for Apollo 11 and 14 and 300 for Apollo 15). The Lunakhod arrays have 14 triangularly faced corner cubes of 11-cm edge. The design of reflector arrays is straightforward: each corner cube reflects incident light back to its point of origin (Fig. 2).

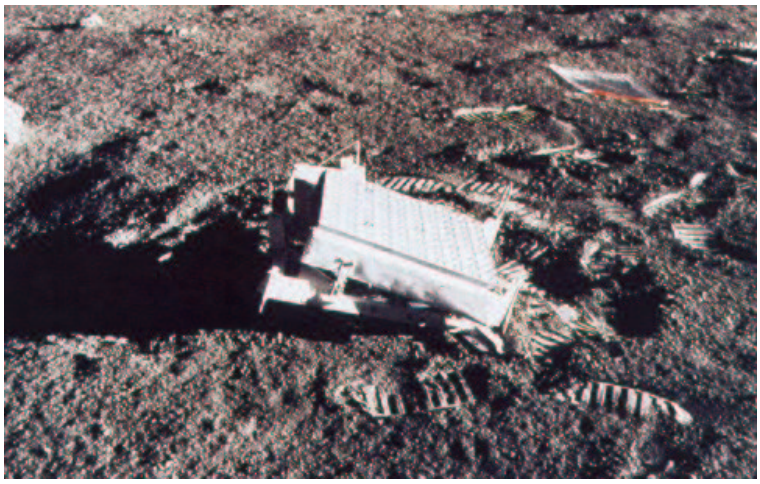

Fig. 2. Photography of the reflector array placed by Apollo 14

Fig. 3. McDonald Laser ranging station (Fort Davis, Texas)

Fig. 4. CERGA Laser ranging station (Plateau de Calern, Caussols, France)

Since 1970 there were few LLR observing stations. The first one was McDonald Observatory (2.7-m telescope) near Fort Davis, Texas (USA). It was fully dedicated to lunar ranging and ceased operation in 1985 after maintaining routine activities for more than 15 years. The transition was made in the mid-1980s to the McDonald Laser Ranging Station (0.76-m telescope) on two sites (Saddle and Mt. Fowlkes): MLRS1 (1983-1988) and MLRS2 (since 1988) which share lunar and artificial satellite ranging facilities (Fig. 3).

In the 1980s, two other stations have carried out Lunar Laser Ranging. The Haleakala Observatory on Maui, Hawaii (USA) produced high quality data over a few years around 1990. Since 1982, the CERGA station (Centre d'Etudes et de Recherche en Géodynamique et Astronomie) has been operating at the 'Observatoire de Côte d'Azur' (OCA) on the 'Plateau de Calern' near Grasse (France), with a 1.54-m Cassegrain telescope which replaced its Rubis laser by a YAG in 1987 (Fig. 4). Occasionally some other artificial-satellite stations have performed successfully LLR observations such as in Australia and in Germany, but all the data used for the analyses come from the 3 observatories: McDonald, Haleakala and CERGA. Today, just two observatories are still operating: McDonald (MLRS2) and CERGA.

The principle of Lunar Laser Ranging is to fire laser pulses towards the target reflectors on the Moon, to receive back localised and recognizable signals and to measure the duration of the round-trip travel of the light. If the concept is elementary, it is in fact a real technical challenge.

At first, the performances of the observations depend on the quality of the time measurements. On average the duration of the round-trip flight is 2.5 seconds varying according to the lunar distance, the mean distance Earth-Moon being 385000 km. If one aims at a precision of 1 cm for the separation between transmitter and reflector, one needs an accuracy of the order of 0.1 nanosecond (10^{-10} second) in the measurement of the round-trip travel duration. The measure of the time is based on a very stable high frequency signal generated by a caesium atomic clock whose the frequency accuracy is better than 10^{-12} yielding an uncertainty below 10 picoseconds (10^{-11} second) over the journey of the light. But several factors affect the precision of the measurements. The atmosphere induces a time delay which is difficult to estimate rigorously, probably between 50 and 100 picoseconds; it depends on temperature, pressure and humidity. The libration of the Moon makes an oscillation of the orientation of the array which produces, in worse cases, an unbiased scatter in distance of few centimeters (about 200 picoseconds in the time of flight). There are other sources of uncertainties such as the photodiodes connected to the start timer or the return detector. The CERGA team has tested several approaches and has converged towards widths of about 200 picoseconds. And to improve the chance to trapping the right return photon several kinds of filtering are performed to eliminate the noisy photons as much as possible. In these conditions, the precision of a measurement of the round-trip travel time of 'one photon' is of the order of 3 cm. With the most unfavourable librations this value can grow up to about 5 cm.

Nevertheless, the main problem stands elsewhere. The outgoing beam has a divergence of 3" to 4" after crossing the Earth atmosphere so that the size of the light spot on the Moon is about 7 km in diameter, which means that only one photon out of 10^9 impacts a reflector array. Besides the reflected beam has a significant angular divergence (12") due to the diffraction by the pupil of the reflector yielding a spot nearly 25 km

back on the Earth. With a 1-meter telescope, only a fraction 10^{-9} of the photons is intercepted.

Finally, allowing for other factors, just one photon is detected out of 10^{20} emitted in the initial pulse. With the YAG laser of the CERGA (since 1987), the energy of the beam is about 300 mJ per pulse at a wavelength of 532 nanometers; it is equivalent to 10^{18} photons. That gives an average of one return every 100 pulses, or every 10 seconds, the laser system producing pulses with a typical rate of 10 pulses per second.

2 Lunar Laser Ranging data

The actual observational data consists of 'Normal Points' built on statistics from the individual time returns. It is essentially a one parameter fitted on the round-trip travel time centered in the middle of the interval of the observations. Since the beginning of the Lunar Laser Ranging, the instrumental quality of the measurements has been noticeably improved. It was around 25 cm in the one-way distance 'station-reflector' in the 1970s and gradually getting better around 10-15 cm in the early 1980s. When the Rubis laser was replaced by a YAG device, after 1985, with much smaller pulse widths and a better timing system, the standard uncertainty in the measurements dropped to 3-cm level.

At the CERGA lunar laser station, a project aiming at sub-centimetric precision was initiated in 1992. Today, the 'Normal Points' provided by the CERGA have an 'instrumental internal precision' of 30-60 picoseconds in the round-trip travel time, which is equivalent to 5-10 mm in one-way distance.

Figure 5 shows the contribution of the 3 LLR stations: McDonald, Haleakala and CERGA since 1970. During the 15 early years, the McDonald Observatory was the only station to range the Moon with the 2.7-m telescope. Around 1990 a second American laser station, Haleakala on Maui, Hawaii, provided several hundreds of 'Normal Points'. Since the 1990s the CERGA station has produced 70% of lunar ranging data. Figure 6 gives the distribution of the Lunar Laser Ranging data between the several reflectors. The reflector Apollo 15 contributes to the major part of the observations (80%) compared with the other reflectors Apollo 11, Apollo 14 and Lunakhod 2.

Figure 7 shows the time distribution of 'Normal Points' for the MLRS2 (in light) and CERGA (in dark) between 1988 and 2001; the maximum of annual data for both stations is about 1000 observations (in 2000), the average being around 700 per year. Figure 8 gives the distribution of the number of returns per 'Normal Point' during the period 1988-2001. The average is respectively 50 returns per 'Normal Point' for CERGA and 15 for MLRS2 with a maximum around 20 for CERGA and 8 for MLRS2.

Figure 9 shows the distribution of observations within a lunar month. Observations are carried out about 25 days per lunation with no observation around New Moon. They are grouped around the First Quarter (evening observations) and the Third Quarter (morning observations). There is no observation at Full Moon because of heavy noise except during Moon eclipses as in 2000 at CERGA station. Figure 10 shows for both stations the distribution of the number of 'Normal Points' per night during the period 1988-2001. In average there are 8 'Normal Points' per night for CERGA and 3-4 for MLRS2. This information is useful in lunar

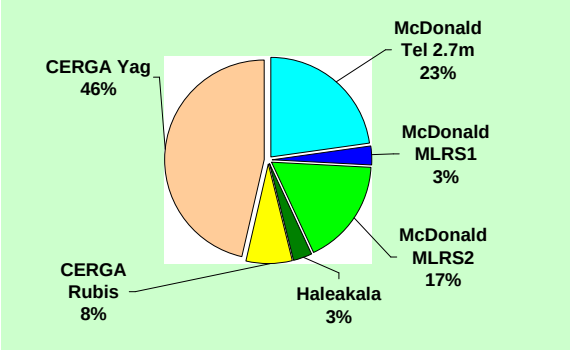

Fig. 5. Distribution of 'Normal Points' per LLR station

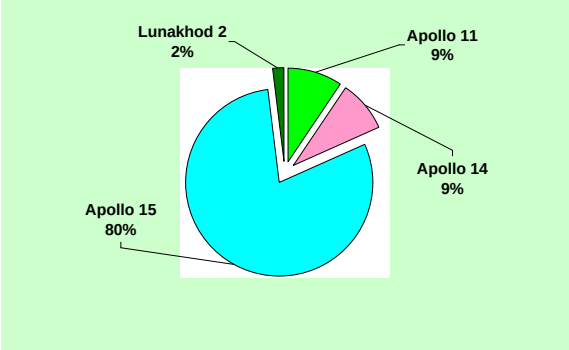

Fig. 6. Distribution of 'Normal Points' per Lunar Reflector

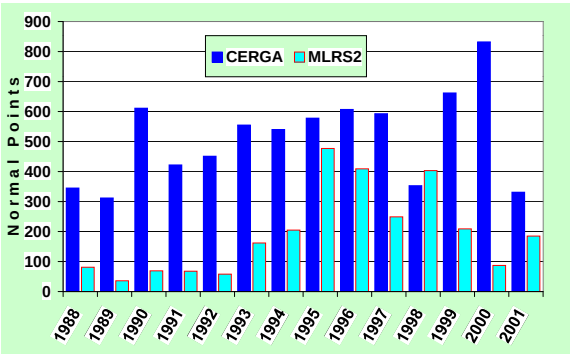

Fig. 7. Distribution of LLR 'Normal Points' over(1988-2001)

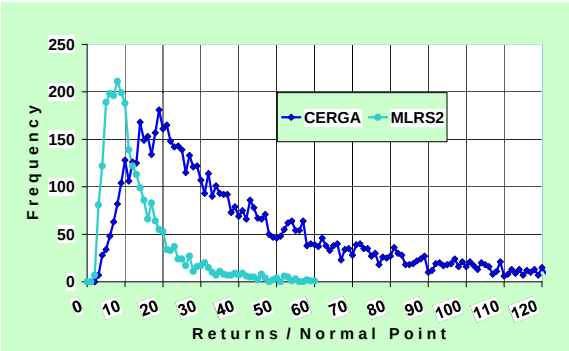

Fig. 8. Distribution of Returns per 'Normal Point'

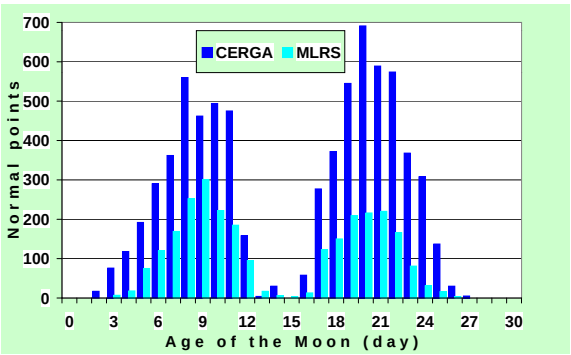

Fig. 9. Distribution of LLR 'Normal Points' within the lunar month

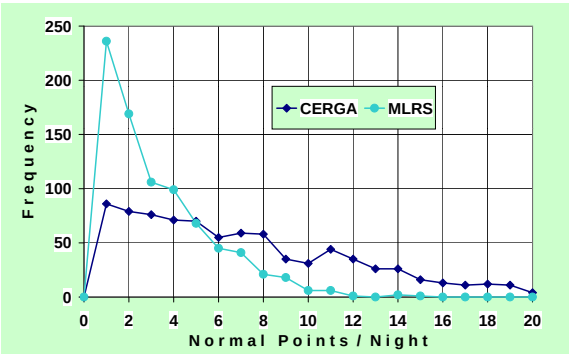

Fig. 10. Distribution of LLR 'Normal Points' per Night

analysis which provide estimations of the Earth orientation parameters (EOP) UT0-UTC and VOL (variation of latitude). Generally we need 3 or 4 'Normal Points' per night and per reflector to determine these parameters.

The thousands of Lunar Laser Ranging data collected since 1970 have given significant contributions to a large scientific domain. We enumerate below a non-exhaustive list of them.

- Ephemerides
  The improvement of the orbital motion of the Moon has followed the accuracy in the LLR measurements. Several numerical integrations (Development Ephemerides at JPL) and analytical solutions (ELP) have been constructed. Libration models (forced and free librations) have been elaborated in parallel.

- Gravitational physics
  It concerns the verification of the equivalence principle, a determination of the PPN parameters β and γ, and the geodetic precession. Correlatively, precise boundaries of the rate of change of G (the gravitational constant) and of the solar J_2 have been also derived. Since 1996, compared to previous determinations, the uncertainties in gravitational physics parameters have been reduced by a factor 2 or more. It is mainly due to the data (an improved range accuracy and a longer time span), some additional solutions (plate motion on the Earth and solid body tides on the Moon), a better determination of Earth Orientation Parameters, and finally, a modeling of the energy dissipation of the Moon.

- Geodynamics
  The LLR technique is also a main contributor to the improvement of various parameters of the Earth-Moon system. We shall mention in particular the determination of the lunar mass and the coefficients of the Earth potential when combined with SLR technique. It contributes, complementary to VLBI, to a better knowledge of the precession constant and of few long-periodic terms in nutation. A domain of excellence is a precise evaluation of the Earth tidal dissipation (Love numbers and time delay) due to the lunar action, which induces a secular acceleration of the lunar longitude.

- Lunar science and selenophysics
  The Moon in rotation is submitted to two different kinds of effects: forced and free librations. Through the analysis of the lunar parameters, one derives an estimate of the principal moment of inertia, low degree gravitational harmonics, Love number κ_2 and dissipation parameter Q. LLR data are combined with lunar orbiting satellites and result in a determination of the parameter C/MR^2 which is fundamental for the knowledge of the Moon's evolution. The free libration is implicit in the equations of motion. Once the total libration is known by the observations and the forced libration modeled, a frequency analysis of the difference: Total Libration - Forced Libration, exhibits 3 frequencies (and 3 modes of libration: a wobble mode, a precession mode and a longitude mode).

- Reference systems
  The lunar motion defines intrinsically a dynamical system. It allows in particular a positioning of the dynamical reference frame in the ICRS. This topic is discussed in the last section.

3 Lunar Laser Ranging data analysis: evaluation of the residuals

LLR data analysis has to be performed rigorously in the frame of General Relativity. We use below the common language of classical mechanics for sake of simplicity, without changing the nature of the physical problem.

The LLR stations provide the 'Normal Points', which are used as 'observed values'. A 'Normal Point' consists in Δt_o, the duration of the round trip travel of the light in atomic time (TAI, Temps Atomique International) and t_1, the date of the starting pulse in Universal Time Coordinated (UTC). The other data supplied are the wavelength of the pulse, the number of returns, the estimated uncertainty of measurements, the signal/noise rate and the temperature, pressure and humidity of the atmosphere.

The principle of the analysis is to compare the 'observed value' Δt_o with a 'computed value' Δt_c and to deduce the residual: $\rho = \Delta t_0 - \Delta t_c$.

The 'computed value' Δt_c is determined in two steps as follows: from the LLR station transmitter O at time t_1 to the lunar reflector R at time t_2, and then from R at time t_2 to O at time t_3.

The propagation time has to be corrected taking into account several effects such as the relativistic curvature of the light beam or the influence of the troposphere. Relativistic changes of time scales have to be performed in order to express the various components in the same time scale. In the frame of the General Relativity theory, the computed Δt_c depends on the barycentric positions of T, L and S, respectively the center of mass of the Earth, of the Moon and of the Sun; B is the barycenter of the solar system.

Hence, as shown on Figure 11, the computation involves the following coordinates in the same celestial barycentric reference system:

- the coordinates of TL (Earth-Moon) provided by a lunar ephemeris, and the barycentric coordinates of the Earth-Moon barycenter BG provided by a planetary ephemeris,
- the coordinates LR (lunar reflector) which requires an ephemeris of the lunar librations and the selenocentric coordinates of the reflectors,
- the coordinates of TO (observing station) which requires a precise knowledge of the Earth rotation (precession, nutation, polar motion, and Universal Time UT through the sidereal time), of the coordinates of the station in a terrestrial reference system, and of relativistic corrections.

The value of Δt_c is given by:

$$\Delta t_c = [t_3 - \Delta T_1(t_3)] - [t_1 - \Delta T_1(t_1)]$$

$$t_3 = t_2 + \frac{1}{c}\,|BR(t_2) - BO(t_3)| + \Delta T_3 + \Delta T_4$$

$$t_2 = t_1 + \frac{1}{c}\,|BR(t_2) - BO(t_1)| + \Delta T_3 + \Delta T_4$$

where c is the velocity of the light, ΔT_1 is a relativistic correction on the time scales, ΔT_3 is the time contribution due to the gravitational curvature of light beam and ΔT_4 is the atmospheric delay. The correction ΔT_2, not mentioned above is the difference between Barycentric Dynamical Time (TDB) and Terrestrial Time (TT).

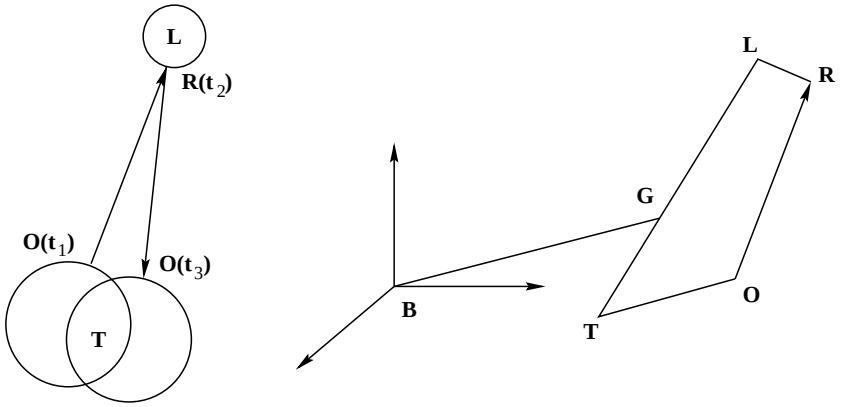

Fig. 11. Light beam OR (Observer-Reflector), Earth T, Earth-Moon barycenter G, Barycenter of the solar system B

Several coordinates transformations are needed in the computations of the barycentric coordinates of the station O and of the reflector R to relate the barycentric reference system (BRS) to the various systems in which the positions are provided: lunar and solar ephemerides, libration and Earth rotation. Actually, the complete model to represent at best the observations is complex and one has to consider a great variety of models: plate motions, atmospheric delay, solid tides of Earth and Moon, ... The ephemeris computations are performed in Barycentric Coordinate Time (TCB). Practically TCB is replaced by a linear function close to the former Barycentric Dynamical Time (TDB). Therefore there are the successive transformations:

$$UTC \rightarrow TAI \rightarrow TT = TAI + 32.184s \rightarrow TDB = TT + \Delta T_2$$

At the end of the computation Δt_c is tranformed from TDB to TAI by means of the relativistic correction ΔT_1.

The global formulation of Δt_c is:

$$\Delta t_c = \frac{1}{c}\,|BR(t_2) - BO(t_1)| + \frac{1}{c}\,|BR(t_2) - BO(t_3)|$$

$$+2\Delta T_3 + 2\Delta T_4 - \Delta T_1(t_3) - \Delta T_1(t_1)$$

with:

$$BR(t) = BG(t) + \frac{m_T}{m_T + m_L} TL(t) + LR(t)$$

$$BO(t) = BG(t) + \frac{m_L}{m_T + m_L} TL(t) + TO(t)$$

where m_T and m_L are respectively the Earth and Moon masses.

The ephemerides of the Moon TL and of the Earth-Moon barycentre BG are referred to the dynamical ecliptic frame of J2000.0 and depend on a large number of physical parameters. Many of them can be fitted to LLR observations: elements of the lunar and terrestrial orbits, coefficients of the gravity field, mass of the Earth-Moon system, tidal dissipation that produces the secular acceleration of the lunar longitude, and relativistic parameters. Indirectly the value of the astronomical unit and the planetary ephemerides play an important role on the lunar motion through various parameters, in particular planetary masses.

The ephemeris of the reflectors LR is dependent of the reflector coordinates in a selenocentric frame defined by the principal axes of inertia of the Moon. They also depend on the lunar rotation which is provided by a theory of the forced libration and the free libration constrained by the elastic deformation of the Moon. The Love numbers and the value of the moments of inertia are fitted parameters. It is worth noticing that a coupling effect exists between the lunar libration and circulation, which is very sensitive to the harmonic coefficients of the lunar potential, in particular in the secular motions of the node and of the perigee. The selenocentric frame is linked to the dynamical ecliptic frame of J2000.0 by the transformation:

$$R_z(-W_1 + 180^\circ) \times M(p_1, p_2, \tau)$$

where R_z is the rotation around the z-axis, W_1 is the mean lunar longitude and M is the matrix fonction of the libration variables p_1, p_2, τ at the time t_2.

The ephemeris of the station TO are primarily defined in the International Terrestrial Reference Frame (ITRF) and are subject to various corrections due to the Earth deformations: terrestrial and oceanic tides and pressure anomaly. For expressing the components of TO in the dynamical ecliptic frame of J2000.0 we have to apply the following transformations:

ITRF → equatorial frame of date
→ equatorial frame of J2000.0
→ dynamical ecliptic frame of J2000.0

The first transformation involves the Earth rotation parameters (EOP): the polar motion (x_p, y_p) and UT1-UTC used through the sidereal time θ as a function of UT1. The second transformation is obtained by the matrix of precession-nutation $P \times N$. The last transformation is performed for referring the station in the dynamical equinox and ecliptic frame defined by the lunar and planetary ephemerides with the position angles: ϵ (obliquity in J2000.0) and ϕ (arc separating the inertial equinox and the origin of the right ascensions on the equator of J2000.0) (see Fig. 13). Hence, the total transformation is:

$$R_x(\epsilon) \times R_z(\phi) \times P^{-1} \times N^{-1} \times R_z(-\theta) \times R_x(y_p) \times R_y(x_p)$$

where R_x, R_y and R_z are respectively the rotations around the x-axis, y-axis and z-axis.

4 Lunar Laser Ranging data analysis: dynamical model and fitted parameters

Two different approaches are possible to represent the planetary and lunar motions: the numerical integrations and the analytical theories.

The Jet Propulsion Laboratory (JPL) has produced very precise ephemerides of the Moon and planets by numerical integration: the Development Ephemerides such as DE200, DE403, or DE405. The most recent integration DE405/LE405 covers 6000 years. It has been fitted to observations and oriented onto the ICRF; it represents presently the most elaborated ephemeris, taking into account the most precise observations

(Radar, Mars spacecraft data, VLBI and LLR) and a very refined dynamical model in the frame of General Relativity, including asteroids. Its main characteristic is a simultaneous integration of the planets, the Moon and the libration.

At Paris Observatory, an analytical approach has been followed. The lunar analytical solution ELP, and its different versions elaborated since 1982, are not as precise as a numerical integration but contains explicitely a large number of parameters. On the short term, it is not adequate to reach the centimeter level required by LLR. Hence, it is completed with numerical complements obtained by difference with a JPL numerical integration as a reference. The numerical complements do not depend on the orbital constants; they improve solely the numerical precision of the analytical solution. ELP plus the numerical complements - fixed once for all - has the precision of a numerical integration and retains the advantages of an analytical solution. Then we can use the analytical form of the solution to fit numerous parameters since the partial derivatives are straightforward. But beyond this practical advantage, the main interest is to make possible an analysis of the perturbations having the same signature by putting aside the various components.

Three important examples can be given:

a) The secular acceleration of the lunar longitude is the sum of several components: Sun and planets, Earth's figure and tides. When observing the total phenomena in LLR residuals it is fundamental to separate the planetary effects from tides, in particular to be able to fit precisely the Love number and the timelag.

b) The libration is composed of a forced component arising from time-varying torques on the lunar figure due to attraction by Earth, Sun and planets and a free component which is a departure of the lunar angular position from an equilibrium state. The separation of the two modes is essential to isolate the free librations to understand the underlying excitation mechanism.

c) Important contributions due to Relativity exist in the sidereal mean motions of the lunar perigee and node, of about 2 arcseconds per century, which have to be separated from other contributions (planets, Earth and lunar figures, tides). Analytical expressions of these motions have been obtained as a function of the PPN parameters β and γ ($\beta = \gamma = 1$ in General Relativity), that can be then tested.

The last version of the analytical solution issued from ELP and fitted to LLR observations is called S2001.

Several improvements were introduced in S2001 with respect to the previous solutions: mainly in the libration model with numerical and analytical complements, but also in the program of reduction with an up-to-date nutation model and a realistic statistical treatment of the data, in particular an adequate distribution of weights among the various observing stations and periods of observations. The fits of the lunar orbital and rotational parameters in S2001 are listed hereunder; the orbital parameters are referred to J2000.0 under the formulation: $Q = Q^{(0)} + Q^{(1)}t + Q^{(2)}t^2 + Q^{(3)}t^3 + Q^{(4)}t^4$ where t is the time in Julian centuries since J2000.0.

- the geocentric lunar orbital parameters $W_1^{(0)}$, $W_2^{(0)}$, $W_3^{(0)}$, (constants of the mean longitude, mean longitudes of perigee and node), $\nu =$

$W_1^{(1)}$, Γ and E (sidereal mean motion, constants for inclination and eccentricity);

- the heliocentric orbital parameters of the Earth-Moon barycenter $T^{(0)}$, and $\varpi'^{(0)}$, (constants of the mean longitude and longitude of perihelion), n' and e' (sidereal mean motion and eccentricity);

- the bias parameters $W_1^{(2)}$, $W_2^{(1)}$, $W_3^{(1)}$, (observed corrections to the computed coefficient of the quadratic term of the lunar mean longitude, and the computed mean motions of perigee and node). $W_1^{(2)}$ yields an observed value of $W_1^{(2,T)}$, the tidal part of the coefficient of the quadratic term of the mean longitude of the Moon (half tidal secular acceleration);

- the 6 libration parameters which are the amplitudes of the 3 main free libration terms and their respective arguments.

Fits of other parameters have been also performed in S2001 such as the position angles of the dynamical ecliptic frame with respect to various systems of axes (ϵ, ϕ, ψ), the correction to the IAU 1976 precession constant Δp and the coordinates of the reflectors and stations. Note that a simultaneous fit of all the parameters produces much correlated values. Hence, the fits have been performed in several steps and tests have been made in order to check the stability of the results. Indeed, strong correlations that exist among some parameters may weaken the accuracy of our determinations; in particular, it is the case of the variables related to the reference frame and the positions of the stations. The correction to IAU 1976 precession constant Δp and the obliquity rate are correlated with the velocities of the stations. The principal nutation term and Δp are also difficult to separate. It is also the case for the selenocentric coordinates of the reflectors and the parameters of the free libration.

We have adopted the following strategy. First, we determine the whole set of parameters mentioned above except the positions and velocities of the stations. Then fixing the value ϕ (position angle, see Fig. 13), we add the positions of the stations to the whole set and make a new improvement. Next, we determine the velocities of the stations separately. Finally, fixing all the parameters, we perform a last analysis including Δp and the principal term of the nutation in longitude. At each step of the process we check the coherence of the determinations; for example we verify that the introduction of the fitted values of the station coordinates does not change significantly the value of ϕ when the first step is reiterated.

Table 1 shows the evolution of the residuals obtained with S2001 for the distance between the LLR stations and the lunar reflectors expressed in centimeters. It gives the time distribution of the root mean square of post-fit residuals obtained with the data provided by the different instruments used since 1972. We note an important gain of precision, between the earliest observations and the recent ones.

The evolution in the quality of the observations is also illustrated in Figure 12 that gives the time distribution of r.m.s. of the residuals obtained with S2001, retaining solely the data provided since1988 by the 2 modern instruments: MLRS2 for McDonald and YAG for the CERGA.

Table 1. LLR residuals: time distribution of the rms (unit:cm). N is the number of LLR 'Normal Points'.

OBSERVATORY *and instruments*	Time Interval	S2001 rms	N
McDONALD	1972-1986	43.5	1487
Telescope 2.70 m	1976-1979	27.7	1035
and MLRS1	1980-1986	29.1	990
CERGA *Rubis*	1984-1986	18.7	1165
HALEAKALA	1987-1990	6.3	451
McDONALD	1987-1991	5.8	232
MLRS1	1991-1995	4.6	586
and MLRS2	1995-2001	3.3	1669
CERGA	1987-1991	5.3	1574
Yag	1991-1995	3.9	2044
	1995-2001	3.0	3273

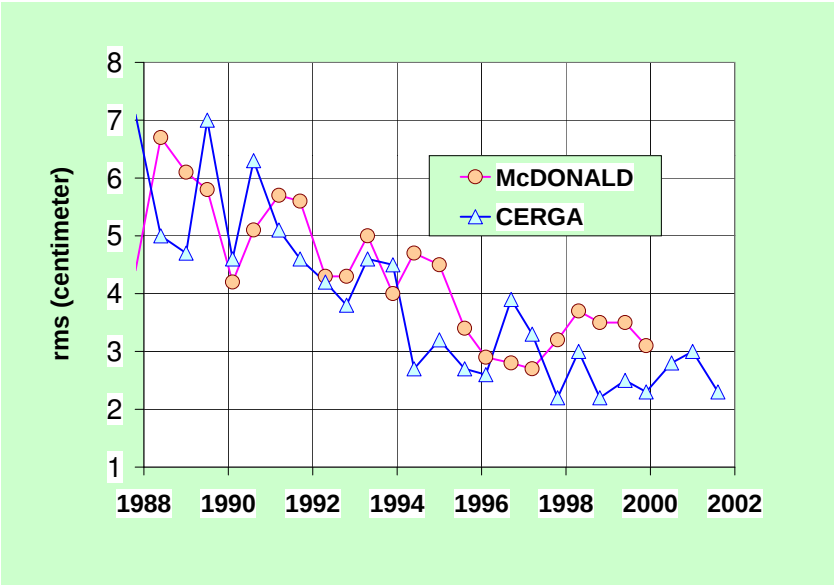

Fig. 12. Residuals of the observations of the 2 operating stations: McDONALD and CERGA

5 Lunar Laser Ranging data analysis: reference system

5.1 Positioning the inertial mean ecliptic of J2000.0

The lunar solution is referred to a dynamical system and introduces the inertial mean ecliptic of J2000.0. With LLR analysis, we can adjust the position of this plane with respect to various equatorial systems. Figure 13 presents the inertial mean ecliptic of J2000.0 and the various equatorial systems (R) involved in our study:

- ICRS: International Celestial Reference System,
- MCEP: Reference system linked to CEP (Celestial Ephemeris Pole),
- JPL: Reference system defined by a JPL numerical integration such as DE403 or DE405.

$\gamma^I_{2000}(R)$ is the ascending node of the inertial mean ecliptic of J2000.0 on the equator of (R), that is the 'inertial equinox of J2000.0', and $o(R)$ is the origin of right ascensions on the equator of (R). The position angles ϵ, ϕ and ψ are:

– $\epsilon(R)$, the inclination of the inertial mean ecliptic to (R),
– $\phi(R)$, the arc between $o(R)$ and $\gamma^I_{2000}(R)$ on (R),
– $\psi(R)$, the arc between $\gamma^I_{2000}(ICRS)$ and $\gamma^I_{2000}(MCEP)$ on the mean ecliptic of J2000.0.

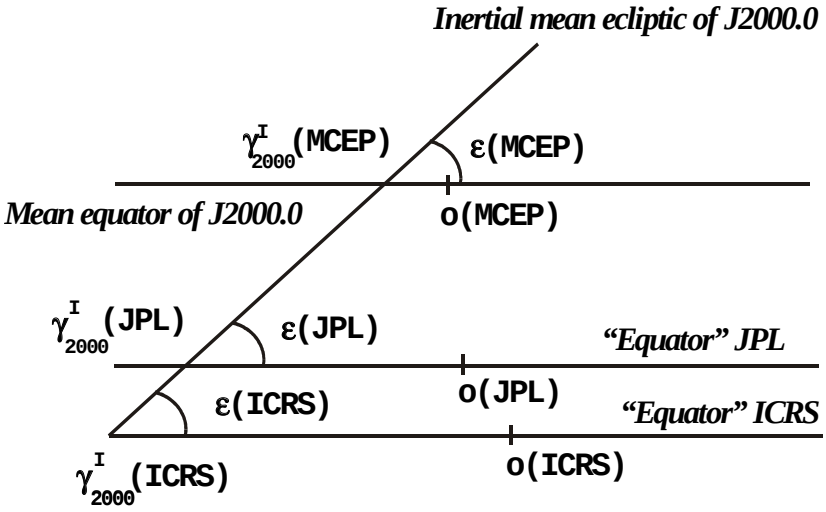

Fig. 13. Position angles of the inertial mean ecliptic of J2000.0

Concerning the ICRS and MCEP reference system, two solutions 'S2001' have been investigated. In both of them the position angles ϵ and ϕ are fitted, but the transformations of the terrestrial coordinates of the stations to celestial ones are different:

- in S2001(ICRS),
  the precession-nutation matrix $P \times N$ is computed via the conventional set of values recommended by the International Earth Rotation System (IERS), in particular the nutation corrections $\delta\epsilon$ and $\delta\psi$ of the series EOP(C04) produced by the Earth Orientation Center,

- in S2001(MCEP),
  the precession-nutation is provided by analytical solutions: polynomial expressions of the precession and theory of nutation; in this case, the correction to the IAU 1976 precession constant Δp is also fitted. In the same way, the lunar solution compared to any JPL ephemeris provides the angles referring the inertial mean ecliptic of J2000.0 with respect to the system defined by the corresponding JPL numerical integration.

Table 2 gives the different values of the position angles ϵ, ϕ and ψ determined for each equatorial reference system. The mean epoch mentioned in this table corresponds to the weighted mean date of LLR observations in the case of ICRS and MCEP. For DE403 and DE405, we have arbitrarily chosen the epoch in function of the dates mentioned in the JPL literature about the observations used in the corresponding numerical integrations.

Table 3 gives the offsets of the Celestial Ephemeris Pole (CEP) with respect to ICRS computed with LLR analysis and a recent value obtained by VLBI measurements; S1998 and S2000 are previous versions of S2001. The notations $\Delta\epsilon_0$ and $\psi\sin\epsilon_0$ are those used in IAU Report on Astronomical Constants.

The separation between the origin of right ascensions in ICRS, $o(ICRS)$, and the projection $o'(JPL)$ of the origin of right ascensions $o(JPL)$ on the ICRS equator (see Fig.13) can be determined using the values of Table 2:

$$o(ICRS)\ o'(DE403) \quad = \quad 1.9\ mas\ \ (Epoch,\ 1985\ Jan\ 1)$$

$$o(ICRS)\ o'(DE405) \quad = \quad 0.7\ mas\ \ (Epoch,\ 1990\ Jan\ 1)$$

Table 2. Position of the inertial mean ecliptic of J2000.0 with respect to various equatorial celestial systems R in S2001. (unit: arcsecond). The uncertainties are formal errors.

R	$\epsilon - 23°26'21"$	ϕ	ψ	Mean Epoch
ICRS	0.41100 ± 0.00005	-0.05542 ± 0.00011		Dec 1994
MCEP	0.40564 ± 0.00009	-0.01460 ± 0.00015	0.0445 ± 0.0003	Dec 1994
DE403	0.40928 ± 0.00000	-0.05294 ± 0.00001	0.0048 ± 0.0004	Jan 1985
DE405	0.40960 ± 0.00001	-0.05028 ± 0.00001	0.0064 ± 0.0003	Jan 1990

Table 3. Offsets of Celestial Ephemeris Pole at J2000.0 with respect to ICRS in S2001, $\Delta\epsilon_0$ and $\psi\sin\epsilon_0$ (unit: arcsecond). The uncertainties are formal errors.

Method	Source	$\Delta\epsilon_0$	$\psi\sin\epsilon_0$
LLR	POLAC S1998	-0.0056 ± 0.0002	-0.0183 ± 0.0004
LLR	POLAC S2000	-0.0054 ± 0.0002	-0.0173 ± 0.0004
LLR	POLAC S2001	-0.0054 ± 0.0002	-0.0177 ± 0.0004
VLBI	IAU2000 (*)	-0.0049 ± 0.0003	-0.0167 ± 0.0005

(*) Fukushima, 2000, Report on Astronomical Constants.

Note that the numerical integration DE405 uses very precise VLBI observations made between 1989 and 1994 and is oriented onto the ICRS.

The angle $\psi(R)$ is the separation between the inertial equinox of J2000.0 in ICRS, $\gamma^I_{2000}(ICRS)$, and the inertial equinox of J2000.0 in the reference system (R), $\gamma^I_{2000}(R)$. It depends of the expression of the lunar mean longitude of the Moon W_1 which has the following secular expansion:

$$W_1 = W_1^{(0)} + W_1^{(1)}t + W_1^{(2)}t^2 + W_1^{(3)}t^3 + W_1^{(4)}t^4$$

where t is the time in Julian centuries since J2000.0.
$W_1^{(0)}$ is the constant term (mean longitude at J2000.0), $W_1^{(1)}$ is the sidereal mean motion (also noted ν) and $W_1^{(2)}$ is the total half-secular acceleration of the Moon. Hence, $\psi(R)$ can be determined by:

$$\psi(R) = W_1(ICRS) - W_1(R) =$$

$$[W_1^{(0)}(ICRS) - W_1^{(0)}(R)] \ +$$

$$[W_1^{(1)}(ICRS) - W_1^{(1)}(R)]t \ +$$

$$[W_1^{(2)}(ICRS) - W_1^{(2)}(R)]t^2$$

where t is the time at a mean epoch, reckoned in centuries from J2000.0 (see Table 2).

5.2 Tidal acceleration

Figure 14 shows the evolution of the correction to the 3 components of W_1: $\Delta W_1^{(0)}$, $\Delta W_1^{(1)}$ and $\Delta W_1^{(2)}$ with the variation of the upper limit of the time span covered by the fit. In other words, the graph represents the different values of $\Delta W_1^{(i)}$ with: $i = 0, 1, 2$ for intermediate solutions in which the characteristics of the fit are the same as in the solution S2001, except the time intervals of the LLR observations that we have successively limited to equidistant dates between 1996 and 2001. We can notice in particular the convergence of the acceleration. This is mainly explained by the evolution of the fitted value of the tidal part of the acceleration when using more and more recent LLR observations.

The tidal component of the secular acceleration of the Moon's longitude is a fundamental parameter, which expresses the dissipation of energy in the Earth-Moon system. It is due to a misalignment of the bulge of the Earth relative to the Earth-Moon direction, which exerts a secular torque. It produces a secular negative acceleration in the lunar longitude of approximately $-25.8"/cy^2$ and correspondingly a decrease in the Earth's rotation rate (or an increase of the length of day). Another consequence is the displacement of the Moon that corresponds to an increase of the Earth-Moon distance of 3.8 $cm/year$.

Table 4 gives a list of determinations of the tidal secular acceleration of the lunar longitude since 1939 and provided by several types of observations: occultations, eclipses and LLR. The most recent values have been obtained with LLR observations. We note for this type of determination a significant improvement of the precision when increasing the number of observations and their accuracy. It is also worth noticing that the most recent determination around $-25.86"/cy^2$ comes closer to the value of Morrison & Ward (1975) obtained with optical observations over a very long time span.

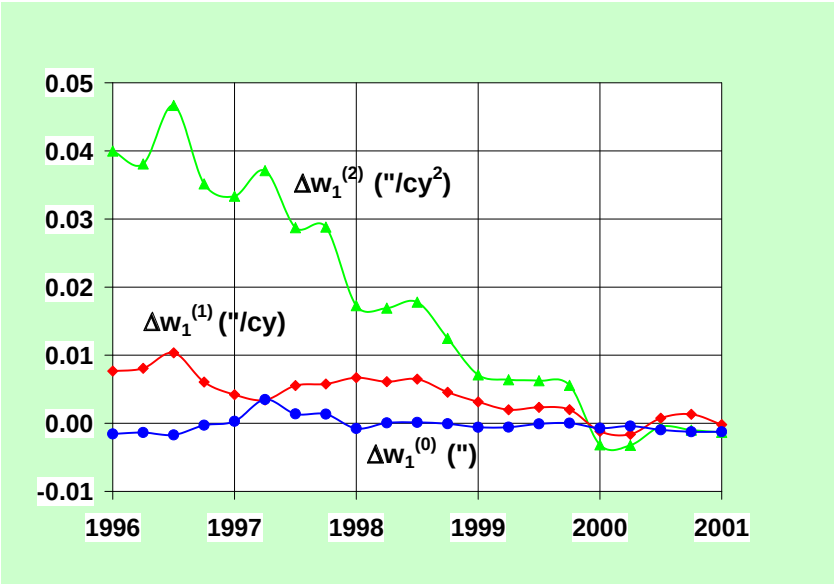

Fig. 14. Evolution of the corrections to the secular components of the mean lunar longitude

Table 4. Tidal acceleration of the lunar mean mean longitude. (unit: arcsecond/cy^2)

Authors	Value	Publication
Spencer Jones [a]	-22	1939
Oesterwinter & Cohen [a]	-38	1975
Morrison & Ward [a]	-26	1975
Muller [b]	-30	1976
Calame & Mulholland [c]	-24.6	1978
Ferrari et al. [d]	-23.8	1980
Dickey et al. [c]	-23.8	1982
Dickey and Williams [c]	-25.10	1982
Newhall et al. [c]	-24.90	1988
Dickey et al. [c]	-25.88	1994
Chapront Touzé et al. [c]	-25.62	1997
(Solution S1998) [c]	-25.78	1999
(Solution S2000) [c]	-25.836	2000
(Solution S2001) [c]	-25.858	2002

Type of observations:
a: Occultations, *b*: Eclipses,
c: LLR, *d*: LLR and Lunar orbiter

Table 5. Tidal acceleration of the lunar mean longitude in various JPL ephemerides (unit: arcsecond/cy^2) (Standish, 1982, 1995, 1998)

JPL ephemeris	Value	Publication
DE200	-23.895	1982
DE245	-25.625	1990
DE403	-25.580	1995
DE405	-25.826	1998

Table 5 gives the intrinsic values of the tidal acceleration in various JPL numerical integrations. The difference between the last JPL value (DE405) and our determination in S2001 gives an idea of the present uncertainty and allows to ensure nowadays a realistic precision of better than $0.03"/cy^2$ in the knowledge of this fundamental lunar parameter. Note that the values presented here as 'JPL values' do not appear explicitly in the list of parameters provided by JPL. We have deduced them from the comparisons of the solution S2001 to the JPL ephemerides.

5.3 Precession constant

In the solution S2001(MCEP), which is linked to the Celestial Ephemeris Pole (CEP), the precession constant has been fitted and the correction Δp to the IAU 1976 constant is equal to $-0.3364"/cy$. We observed that this correction was noticeably divergent from the values of this correction obtained by previous solutions such as S1998 or S2000.

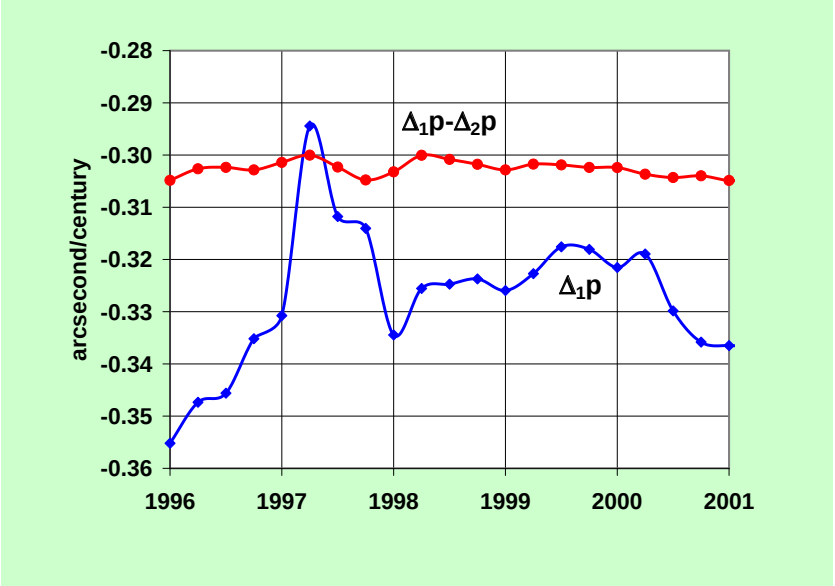

Fig. 15. Evolution of the correction to the IAU 1976 precession constant with the time span covered by the fit: $\Delta_1 p$ for S2001(MCEP) and $\Delta_2 p$ for S2001(ICRS)

Figure 15 illustrates the evolution of Δp with the variation of the upper limit of the time span covered by the fit with the same division in 'slices of time' as we did previously in Figure 14 for the components of the mean lunar longitude. We note $\Delta_1 p$ the evolution of Δp in solution S2001(MCEP). The 'accidental jump' that we observe for 1997 corresponds to an offset of 0.7 nanosecond in the CERGA measurements which has been partly corrected in our analysis. In fact, the relevance of this graph is to put in evidence a residual $\Delta_2 p$ which also arises when we put Δp in the list of the fitted parameters of the solution S2001(ICRS). We note in particular that the difference $\Delta_1 p - \Delta_2 p$ is almost constant on the time interval [1996-2001].

S2001(MCEP) and S2001(ICRS) use the same observations, which are the main source of errors, and the same models, except for the motion of the reference frame due to precession and nutation. If we assume that the series EOP(C04) $\delta\epsilon$ and $\delta\psi$ used in S2001(ICRS) and based on VLBI

observations, contributes ideally to the precession-nutation matrix, we can make the hypothesis that the residual $\Delta_2 p$ is also included in $\Delta_1 p$. Hence, the differences between the two corrections $\Delta p = \Delta_1 p - \Delta_2 p$ gives an better estimate of the corrections to the IAU 1976 precession constant than $\Delta_1 p$. This correction remains around the value $-0.302"/cy$.

Table 6 shows that the last value of the correction to IAU 1976 precession constant Δp obtained by LLR are very close to recent determinations obtained by VLBI analysis: the value presented in the IAU General Assembly in 2000 (Fukushima, 2000) or the value of the theory of nutation MHB2000 (Herring et al., 2002). LLR and VLBI determinations converge nicely with a separation smaller than 0.03 mas/year.

Table 6. Correction to the IAU 1976 precession constant Δp (unit: arcsecond/cy). The uncertainties are formal errors.

Method	Source		Δp
LLR	S2001	(Chapront, 2002)	-0.302 ± 0.003
VLBI	IAU2000	(Fukushima, 2000)	-0.297 ± 0.004
VLBI	MHB2000	(Herring, 2002)	-0.2997 ± 0.0008

6 Conclusion

The improvement of the LLR measurements during the 10 last years associated with a more refined Earth-Moon model have allowed to provide a better determination of the angles involved in the positioning of the dynamical ecliptic reference frame with respect to the ICRS and other equatorial reference systems. This gain in accuracy is also noted for the estimation of the correction to the precession constant and for the determination of the tidal secular acceleration of the lunar longitude. Presently, at the CERGA, an individual LLR measurement shows an error around 30-60 picoseconds. So, there is still a margin between the present 'instrumental' accuracy of the data and the values of the residuals obtained by the LLR analysis. The team of the Paris Observatory lunar analysis center works in cooperation with the LLR staff of the CERGA since 1998. Besides the LLR analysis, we assume also a role of control and validation of LLR data as soon they are provided by the CERGA. Nevertheless, the first goal of our analysis is to improve the analytical solution of the orbital and rotational motions of the Moon. A theory of the lunar motion including new planetary perturbations, ELP/MPP02, will replace the previous solution and in this version the constants will be fitted to LLR data taking into account the results presented here (Chapront and Francou, 2002).

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