What do Scientists Interested in Earth Rotation Need from the GGFC?

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Forcing Mechanisms

- External
  - Tidal potential
    - Body, ocean, atmosphere
- Surficial
  - Atmospheric winds and surface pressure
  - Oceanic currents and bottom pressure
  - Water stored on land (liquid, snow, ice)
- Internal
  - Earthquakes and tectonic motions
  - Mantle convection
  - Outer core dynamics / coupling with mantle
  - Inner core dynamics / coupling with outer core and mantle

Geodetic Measures of Earth’s Response

- Rotation
  - Angular momentum exchange (surface fluids, outer core)
  - Torques (external, surficial, internal)
  - Changes in inertia tensor (earthquakes, GIA)
- Gravity and geocenter
  - Mass distribution (static field)
  - Mass redistribution (time varying field)
  - Surface displacements (ground-based)
- Shape
  - Tidal displacements
  - Surface loading and unloading
  - Internal deformation
    - Earthquakes, core pressure => mantle deformation

Polar Motion

Conservation of angular momentum expressed within rotating, body-fixed reference frame

$$\frac{\partial \mathbf{L}}{\partial t} + \omega \times \mathbf{L} = \tau$$

where the angular momentum vector $\mathbf{L} = \mathbf{I} \; \omega + \mathbf{h}$

Assume rotation is small perturbation from state of uniform rotation at rate Ω. Keeping terms to first order results in long period Liouville equation

$$\mathbf{m}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{m}}{\partial t} = \psi(t) = \chi(t) - \frac{i}{\Omega} \frac{\partial \chi}{\partial t}$$

where: $\mathbf{m} \equiv (\omega_1 + i\, \omega_2)/\Omega$ (terrestrial location of rotation pole)

$\psi(t)$, $\chi(t)$ are the polar motion excitation functions

σ_{cw} is complex-valued frequency of Chandler wobble

Written in terms of reported polar motion parameters:

$$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{p}}{\partial t} = \chi(t) = \frac{1.61}{\Omega\,(C-A)} \left[\mathbf{h}(t) + \frac{\Omega\, \mathbf{c}(t)}{1.44} \right]$$

where $\mathbf{p}(t) = x_p(t)$ Š $i\, y_p(t)$
$\mathbf{c}(t) = c_{13}(t) + i\, c_{23}(t)$

Length of Day

Conservation of angular momentum expressed within rotating, body-fixed reference frame

$$\frac{\partial \mathbf{L}}{\partial t} + \omega \times \mathbf{L} = \tau$$

where the angular momentum vector $\mathbf{L} = \mathbf{I}\ \omega + \mathbf{h}$

Assume rotation is small perturbation from state of uniform rotation at rate Ω. Keeping terms to first order yields:

$$\dot{m}_3(t) = \dot{\psi}_3(t) = -\dot{\chi}_3(t)$$

where: $$m_3(t) \equiv (\omega_3(t) - \Omega)/\Omega = \frac{\mathrm{d(UT1-TAI)}}{\mathrm{d}t} = -\frac{\Delta\Lambda(t)}{\Lambda_o}$$

$$\chi_3(t) = \frac{1}{C_m\ \Omega}[h_3(t) + 0.756\ \Omega\, c_{33}(t)]$$

Changes in length-of-day $\Delta\Lambda(t)$ are caused by changes in axial component of relative angular momentum $h_3(t)$ and by changes in 3,3 component of the Earth's inertia tensor $c_{33}(t)$:

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m\ \Omega}[h_3(t) + 0.756\ \Omega\, c_{33}(t)]$$

where: Λ_o **is the nominal length of day (86400 seconds)**

C_m **is the greatest principal moment of inertia of the Earth's crust and mantle**

0.756 accounts for yielding of the crust and mantle

Oceanic Angular Momentum (OAM)

Angular momentum of oceans changes due to:

- Changes in strength and direction of oceanic currents
- Changes in mass distribution of oceans (changes in ocean-bottom pressure)

Under principle of conservation of angular momentum, the rotation of the solid Earth changes as OAM is exchanged with the solid Earth

OAM can be computed from products of OGCMs by:

- OAM due to ocean-bottom pressure changes (inertia tensor)

$$\mathbf{M}^p(t) = M_1^p(t) + i\, M_2^p(t) = -\int_{V_o} r^2\, \Omega\, \rho(\mathbf{r},t)\, \sin\phi\, \cos\phi\, (\cos\lambda + i\, \sin\lambda)\, dV$$

$$M_3^p(t) = \int_{V_o} r^2\, \Omega\, \rho(\mathbf{r},t)\, \cos^2\phi\, dV$$

- OAM due to current changes (relative angular momentum)

$$\mathbf{M}^c(t) = M_1^c(t) + i\, M_2^c(t) = -\int_{V_o} \rho(\mathbf{r},t)\, r\, [\sin\phi\, u(\mathbf{r},t) + i\, v(\mathbf{r},t)]\, (\cos\lambda + i\, \sin\lambda)\, dV$$

$$M_3^c(t) = \int_{V_o} \rho(\mathbf{r},t)\, r\, \cos\phi\, u(\mathbf{r},t)\, dV$$

OAM is related to Earth rotation excitation functions by:

$$\chi(t) = \chi_1(t) + i\, \chi_2(t) = \frac{1.61}{\Omega\,(C-A)} \left[\mathbf{M}^c(t) + \frac{\mathbf{M}^p(t)}{1.44} \right]$$

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m\, \Omega} [\, M_3^c(t) + 0.756\, M_3^p(t)\,]$$

Excitation Functions

$$\chi_x(t) = \frac{h_x(t) + \Omega[1+(k_2' + \Delta k_{an}')]\Delta I_{xz}(t)}{\left[C - A' + A_m' + \varepsilon_c A_c\right]\sigma_o}$$

$$\chi_y(t) = \frac{h_y(t) + \Omega[1+(k_2' + \Delta k_{an}')]\Delta I_{yz}(t)}{\left[C - A' + A_m' + \varepsilon_c A_c\right]\sigma_o}$$

$$\chi_z(t) = k_r \frac{h_z(t)+\Omega[1+\alpha_3(k_2' + \Delta k_{an}')]\Delta I_{zz}(t)}{C_m \Omega}$$

where: $k_r = \left\{ 1 + \left[n_o + \frac{4}{3}\left(k_2 + \Delta k_{ocn,s}\right)\right]\frac{a^5\Omega^2}{3G}\frac{1}{C_m}\right\}^{-1}$

Earth Rotation Community Needs

- Angular momenta
  - Motion and mass redistribution
  - For all geophysical fluids
    - Atmosphere, oceans, hydrosphere, mantle, core
  - On all observable time scales
    - Subdaily to decadal and longer
  - On global and regional spatial scales
- Torques
  - Tidal, gravitational, friction, mountain, viscous, electromagnetic
- Conventions and standards
  - Consistent computations
    - Theory of Earth rotation variations
  - Degree-2 Love numbers
    - Body and load
    - Elastic and anelastic