Numerical evidence for the inconsistent separation of the ITRF-ICRF transformation into precession-nutation, diurnal rotation and polar motion

Athanasios Dermanis and Dimitrios Tsoulis

Aristotle University of Thessaloniki

IERS Workshop on Conventions, 20-21 September 2007, BIPM, Paris

A computation of the celestial pole direction as induced by geodetic observations and its comparison with the Celestial Intermediate Pole

Athanasios Dermanis and Dimitrios Tsoulis

Aristotle University of Thessaloniki

IERS Workshop on Conventions, 20-21 September 2007, BIPM, Paris

A geodesist's point of view

Do not include astronomical / geophysical hypotheses
in data analysis for the estimation of parameters
which can be determined by geodetic observations
in a hypothesis-free way

Then data analysis provides theory-independent parameters
appropriate for comparison with theoretical results

↓

Theory verification / Data validation

Comparing geodetic data with precession-nutation theory

- IAU2000 precession-nutation theory refers to the Celestial Intermediate Pole (CIP)

- The CIP is not "observable" (its position cannot be determined by observations) because it is defined by purely theoretical means in the framework of a particular solution and a particular mathematical representation

- The real observable is the 3-parameter rotation matrix **R** from the terrestrial to the celestial reference system

  From observed **R** it is possible to determine the direction (and modulus) of the instantaneous earth rotation vector
  and not the direction of the CIP

Attention !!!

Updating a theory-provided rotation matrix $\mathbf{R}_0$ from the left ($\delta\mathbf{R}_L$) and the right ($\delta\mathbf{R}_R$) using geodetic data, does not provide an update to precession-nutation ($\delta\mathbf{R}_L$) and update of LOD and estimates of polar motion ($\delta\mathbf{R}_R$), respectively.

ICRF

$$\mathbf{R} = \boxed{\delta\mathbf{R}_L}\;\mathbf{R}_0\;\boxed{\delta\mathbf{R}_R}$$

ITRF

not only an update
of precession-nutation

not only an update of LOD and
an estimate of polar motion

Attention !!!

Updating a theory-provided rotation matrix $\mathbf{R}_0$ from the left ($\delta\mathbf{R}_L$) and the right ($\delta\mathbf{R}_R$) using geodetic data, does not provide an update to precession-nutation ($\delta\mathbf{R}_L$) and update of LOD and estimates of polar motion ($\delta\mathbf{R}_R$), respectively.

ICRF

$$\mathbf{R} = \boxed{\delta\mathbf{R}_L}\ \mathbf{R}_0\ \boxed{\delta\mathbf{R}_R}$$

ITRF

They both contribute to
- precession-nutation,
- LOD
- polar motion

PROOF: A simple exercise in matrix algebra

Attention !!!

Updating a theory-provided rotation matrix $\mathbf{R}_0$ from the left ($\delta\mathbf{R}_L$) and the right ($\delta\mathbf{R}_R$) using geodetic data, does not provide an update to precession-nutation ($\delta\mathbf{R}_L$) and update of LOD and estimates of polar motion ($\delta\mathbf{R}_R$), respectively.

ICRF $\leftarrow$ $\mathbf{R} = \boxed{\delta\mathbf{R}_L}\ \mathbf{R}_0\ \boxed{\delta\mathbf{R}_R}$ — ITRF

$$\mathbf{R} = \boxed{\delta\mathbf{Q}(\delta X, \delta Y)}\ \mathbf{Q}_0(X, Y)\mathbf{R}_3(-\psi_0)\ \boxed{\mathbf{R}_3(\delta\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)}$$

Attention !!!

Updating a theory-provided rotation matrix $\mathbf{R}_0$ from the left ($\delta\mathbf{R}_L$) and the right ($\delta\mathbf{R}_R$) using geodetic data, does not provide an update to precession-nutation ($\delta\mathbf{R}_L$) and update of LOD and estimates of polar motion ($\delta\mathbf{R}_R$), respectively.

ICRF $\longleftarrow$ $\mathbf{R} = \boxed{\delta\mathbf{R}_L}\;\mathbf{R}_0\;\boxed{\delta\mathbf{R}_R}$ — ITRF

They both contribute to
- precession-nutation,
- LOD
- polar motion

Attention !!!

Updating a theory-provided rotation matrix $\mathbf{R}_0$ from the left ($\delta\mathbf{R}_L$) and the right ($\delta\mathbf{R}_R$) using geodetic data, does not provide an update to precession-nutation ($\delta\mathbf{R}_L$) and update of LOD and estimates of polar motion ($\delta\mathbf{R}_R$), respectively.

ICRF

$$\mathbf{R} = \boxed{\delta\mathbf{R}_L}\ \mathbf{R}_0\ \boxed{\delta\mathbf{R}_R}$$

ITRF

They both contribute to
- precession-nutation,
- LOD
- polar motion

Cannot be directly used for verifying precession-nutation theory

e.g. small δX, δY in $Q = \delta Q(\delta X,\delta Y)\ Q_{IERS}$ (IERS Conventions, Ch. 5)

do not compare directly IAU200 precession-nutation

OUR APPROACH

THEORY

Theory of precession-nutation provides direction of instantaneous rotation axis

“Removal” of selected precession-nutation theoretical components defines the Celestial Intemediate Pole (CIP)

OBSERVATION

Theory is updated by observational evidence to provide an “observed” rotation matrix $\mathbf{R}$ from terrestrial to celestial reference system

mathematical compatibility provides

an “observed” Compatible Celestial Pole (CCP)

COMPARISON

Computation of CCP – CIP differences

OUR APPROACH

THEORY

Theory of precession-nutation provides direction of instantaneous rotation axis

"Removal" of selected precession-nutation theoretical components defines the Celestial Intemediate Pole (CIP)

OBSERVATION

Theory is updated by observational evidence to provide an "observed" rotation matrix **R** from terrestrial to celestial reference system

mathematical
compatibility
provides

an "observed" Compatible Celestial Pole (CCP)

COMPARISON

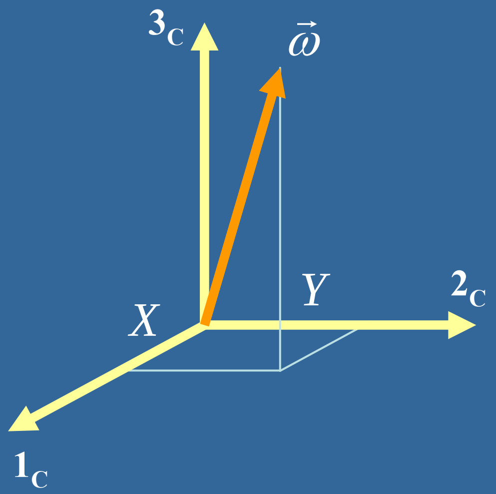
Computation of CCP – CIP differences

STOP

EARTH ROTATION COMPONENTS

Precession-Nutation

$$\mathbf{Q}(X,Y)\mathbf{R}_3(s)$$

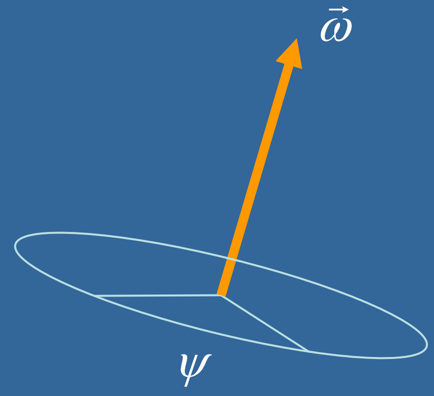


celestial reference system

1_C, 2_C, 3_C

Diurnal Rotation

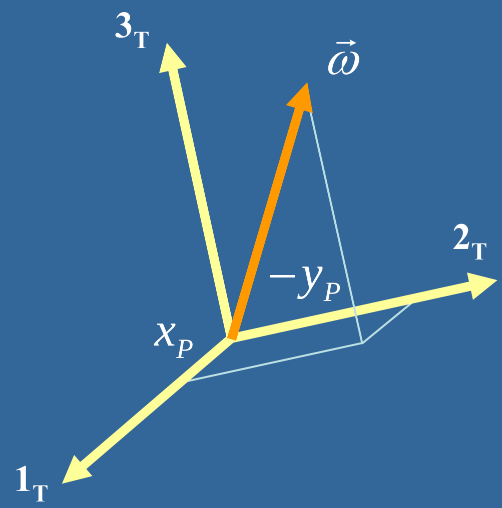
$$\mathbf{R}_3(-\theta)$$



$$\psi = -s + \theta + s'$$

Polar motion

$$\mathbf{R}_3(-s')\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$



terrestrial reference system

1_T, 2_T, 3_T

IERS earth rotation representation:

$$\mathbf{R} = \delta\mathbf{Q}(\delta X, \delta Y)\mathbf{Q}_0(X,Y)\mathbf{R}_3(-\psi)\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)$$

Separation by NRO conditions $\psi = -s(X,Y) + \theta + s'(x_P, y_P)$

THE CELESTIAL INTERMEDIATE POLE

IERS Representation: Diurnal rotation $\mathbf{R}_3(-\psi)$ around the Celestial Intermediate Pole (CIP)

CIP = Direction provided by theoretical earth rotation $\mathbf{Q}_0(X, Y)$ after removal of particular frequency terms

THE COMPATIBLE CELESTIAL POLE

IERS provided rotation matrix $\mathbf{R}$, as updated by observations, defines an estimate of the complete earth rotation and thus also a corresponding rotation vector estimate $\vec{\omega}$ by mathematical compatibility.

Compatible Celestial Pole (CCP) = direction of the rotation vector $\vec{\omega}$ mathematically compatible with the IERS provided rotation matrix $\mathbf{R}$

COMPATIBLE EARTH ROTATION REPRESENTATION

Diurnal rotation $\mathbf{R}_3(-\psi) = \mathbf{R}_3(-s - \theta + s')$ takes place around $\vec{\omega}$ and diurnal rotation angle θ satisfies: $d\theta / dt = \omega = |\vec{\omega}|$ (compatibility in direction and magnitude)

Mathematical separation of the rotation matrix **R** into precession-nutation, diurnal motion (LOD) and polar motion

$\vec{\omega}$ = rotation vector, with components $\boldsymbol{\omega}_C$ (celestial) and $\boldsymbol{\omega}_T$ (terrestrial)

$$[\boldsymbol{\omega}_C \times] = \frac{d\mathbf{R}}{dt}\mathbf{R}^T \qquad [\boldsymbol{\omega}_T \times] = \mathbf{R}^T \frac{d\mathbf{R}}{dt}$$

The mathematically induced **Compatible Celestial Pole** (CCP) has components

celestial

$$\mathbf{n}_C = \frac{1}{\omega}\boldsymbol{\omega}_C = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \sqrt{1-\tilde{X}^2-\tilde{Y}^2} \end{bmatrix}$$

terrestrial

$$\mathbf{n}_T = \frac{1}{\omega}\boldsymbol{\omega}_T = \begin{bmatrix} \tilde{x}_P \\ -\tilde{y}_P \\ \sqrt{1-\tilde{x}_P^2-\tilde{y}_P^2} \end{bmatrix}$$

$$\omega = \sqrt{\boldsymbol{\omega}_C^T \boldsymbol{\omega}_C} = \sqrt{\boldsymbol{\omega}_T^T \boldsymbol{\omega}_T}$$

COMPATIBLE EARTH ROTATION REPRESENTATION

$$\boxed{\mathbf{R} = \mathbf{Q}(\tilde{X},\tilde{Y})\mathbf{R}_3(-\tilde{\psi})\mathbf{R}_2(\tilde{x}_P)\mathbf{R}_1(\tilde{y}_P)}$$

where

$$\tilde{\psi} = \tilde{s}(\tilde{X},\tilde{Y}) - \tilde{\theta} - \tilde{s}'(\tilde{x}_P,\tilde{y}_P)$$

$$\tilde{\theta}(\tilde{\tau}) = A + B\cdot \mathrm{UT1} = A + B(\mathrm{UTC} - \tilde{\tau})$$

$$\tilde{\tau} = \mathrm{UTC} - \mathrm{UT1}$$

COMPUTATIONS

$$\mathbf{R}_3(-\tilde{\psi}) = \mathbf{Q}(\tilde{X},\tilde{Y})^{-1}\,\mathbf{R}\,\mathbf{R}_1(-\tilde{y}_P)\mathbf{R}_2(-\tilde{x}_P) \longrightarrow \tilde{\psi}$$

NRO conditions $\longrightarrow$ $\tilde{s}(\tilde{X},\tilde{Y})$ $\quad \tilde{s}'(\tilde{x}_P,\tilde{y}_P)$

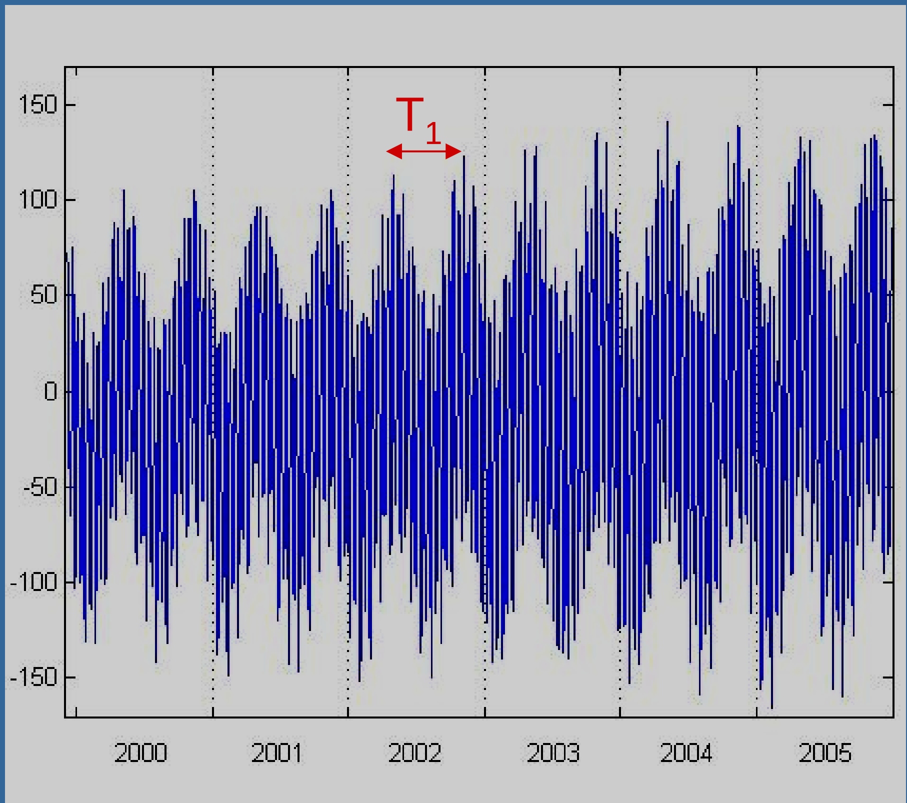
$$\tilde{\theta} = \tilde{s}(\tilde{X},\tilde{Y}) - \tilde{s}'(\tilde{x}_P,\tilde{y}_P) - \tilde{\psi} \longrightarrow \tilde{\theta}$$

$$\tilde{\tau} = \mathrm{UTC} - \frac{\tilde{\theta}(\tilde{\tau}) - A}{B} \longrightarrow \tilde{\tau} = \mathrm{UTC} - \mathrm{UT1}$$

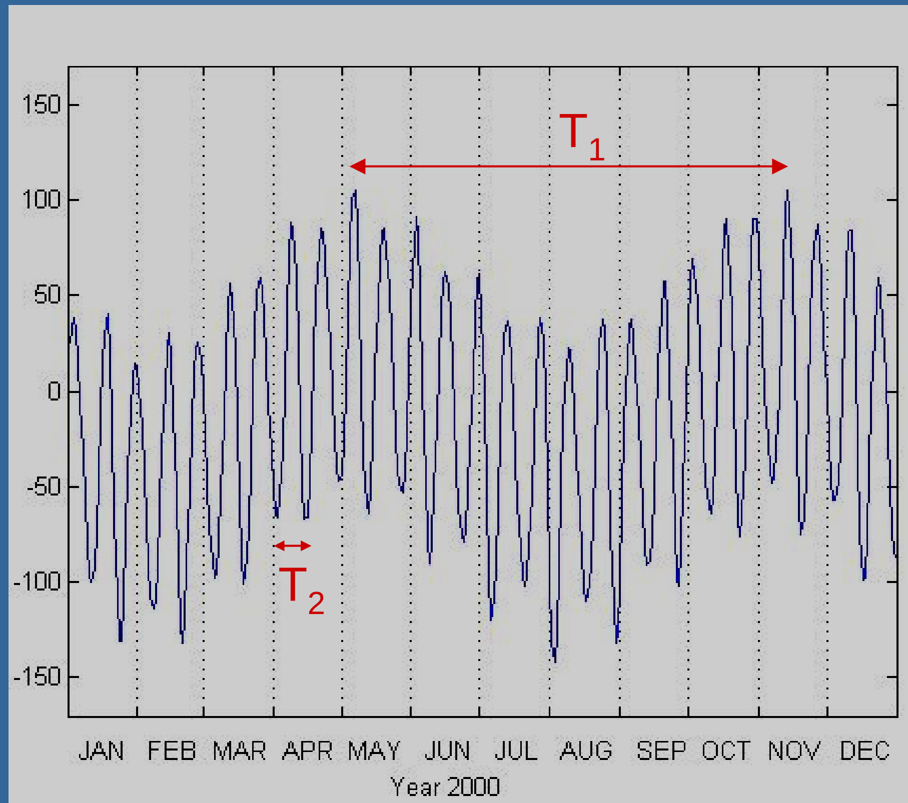
Comparison of the CCP with the Celestial Intermediate Pole (CIP)

Precession-nutation components

$\tilde{X} - X$



$\tilde{X} - X$



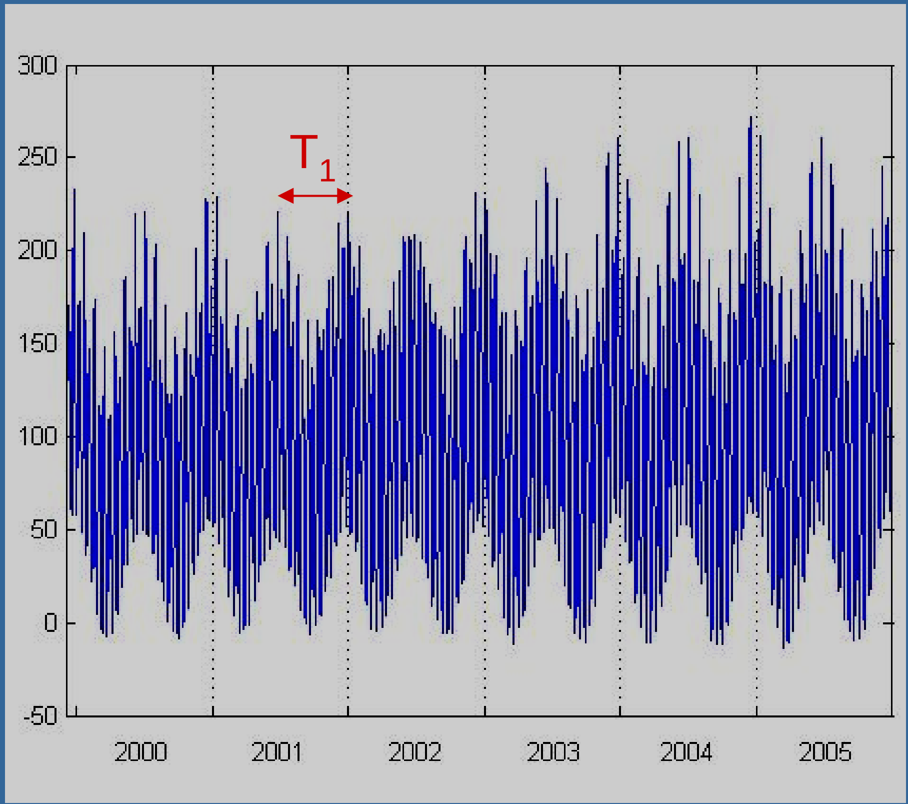
Units = meters on the earth surface (30 m ~ 1 arcsec)

Two dominant components with periods: T_1 = 186.1 days T_2 = 13.6 days

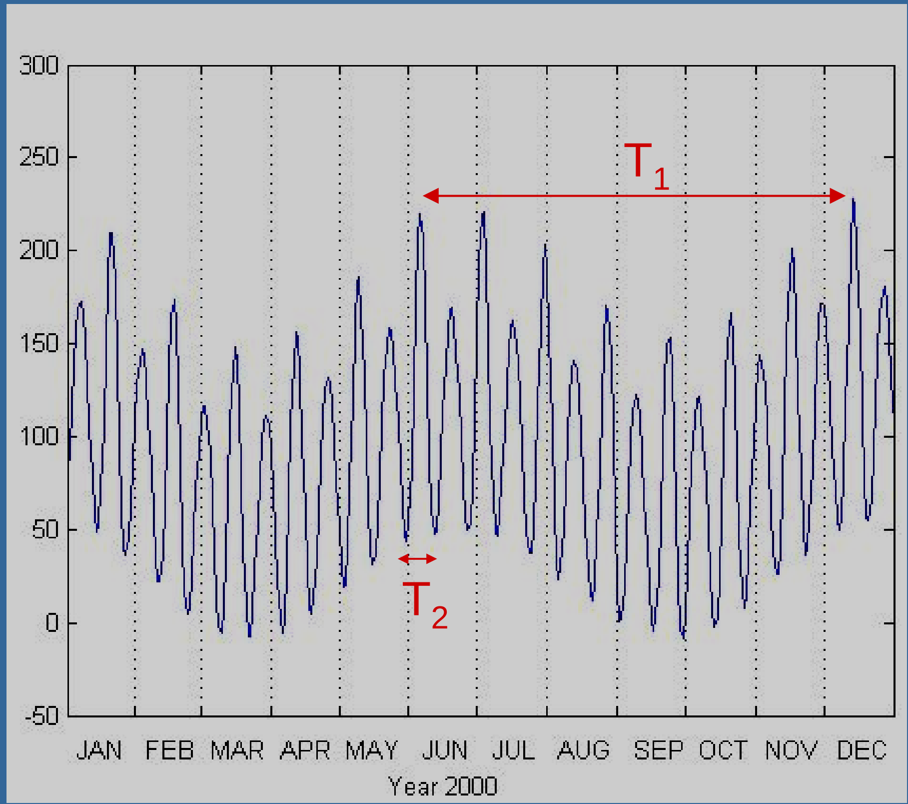
Comparison of the CCP with the Celestial Intermediate Pole (CIP)

Precession-nutation components

$\tilde{Y} - Y$



$\tilde{Y} - Y$



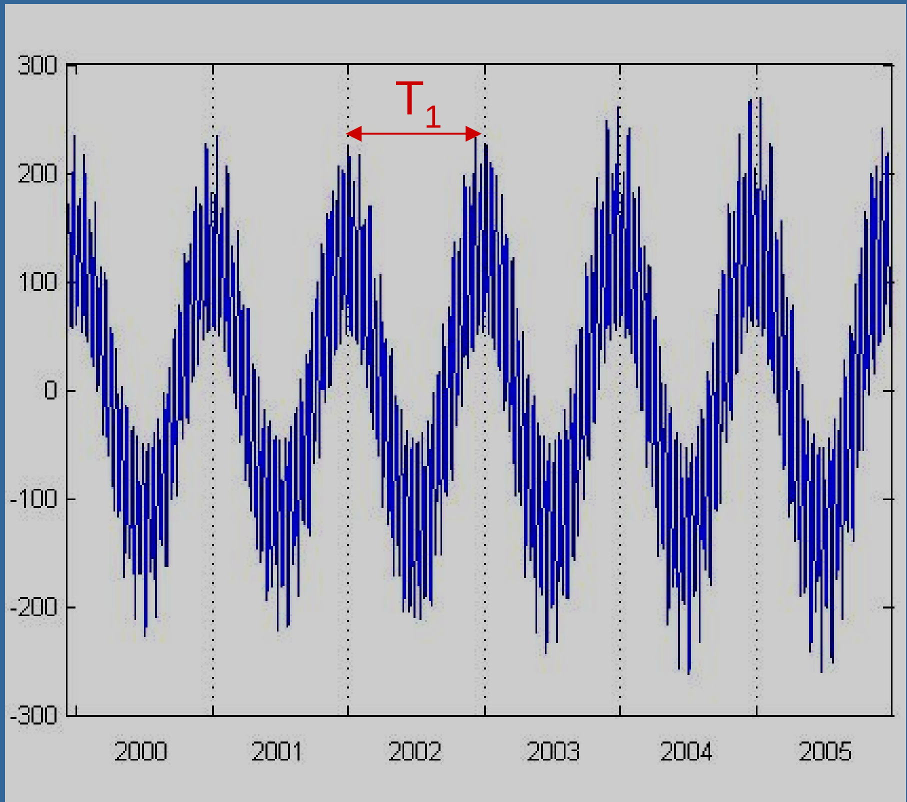
Units = meters on the earth surface (30 m ~ 1 arcsec)

Two dominant components with periods: T_1 = 186.2 days T_2 = 13.6 days

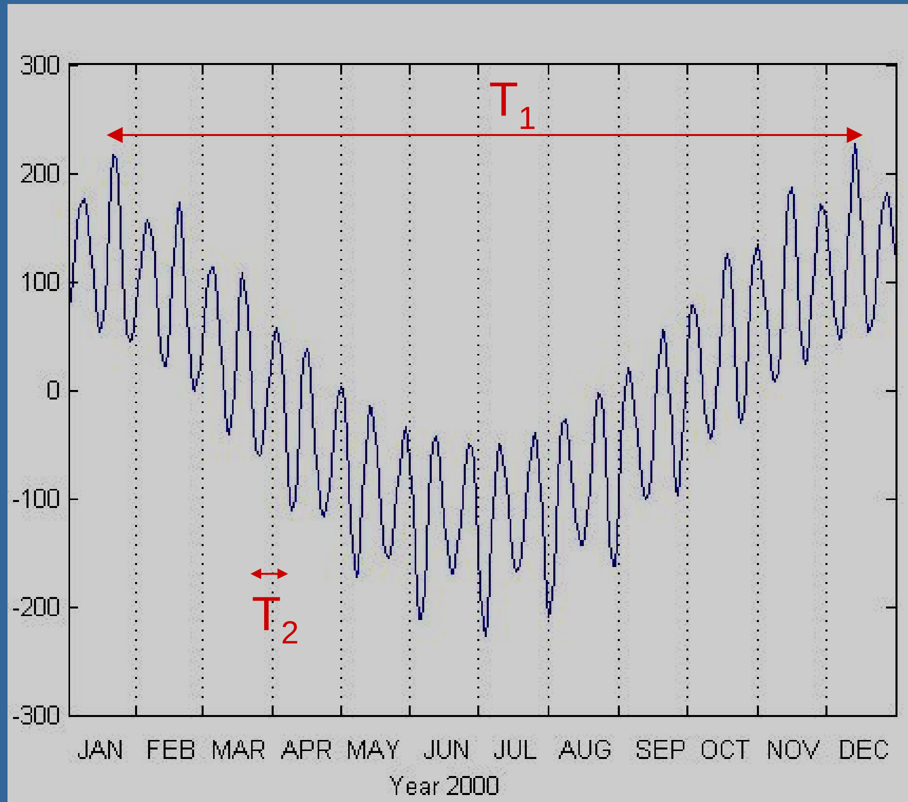
Comparison of the CCP with the Celestial Intermediate Pole (CIP)

Polar motion components

$\tilde{x}_P - x_P$



$\tilde{x}_P - x_P$



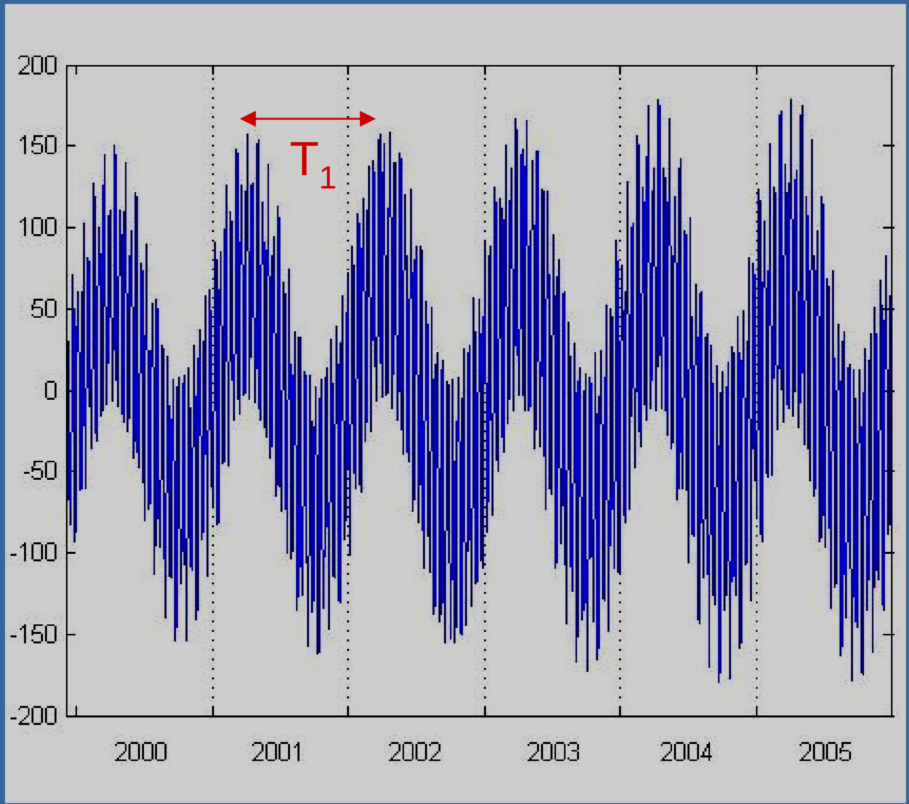
Units = meters on the earth surface (30 m ~ 1 arcsec)

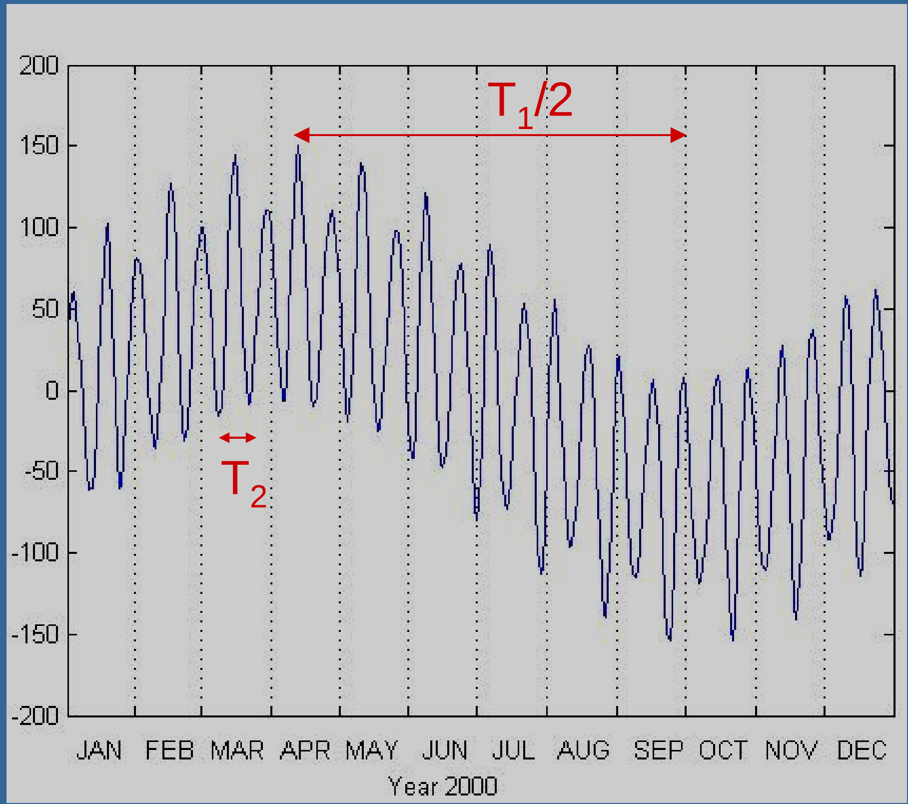
Two dominant components with periods: T_1 = 341.2 days T_2 = 14.2 days

Comparison of the CCP with the Celestial Intermediate Pole (CIP)

Polar motion components

$\tilde{y}_P - y_P$ $\tilde{y}_P - y_P$





Units = meters on the earth surface (30 m ~ 1 arcsec)

Two dominant components with periods: T_1 = 341.2 days T_2 = 14.2 days

VALIDATION OF RESULTS – PART 1

Computation with **4 different methods** from original IERS data:

$$\mathbf{R} = \mathbf{R}(\mathbf{p})$$

$$\mathbf{p} = \{X, Y, \delta X, \delta Y, \tau, x_P, y_P\}$$

1

NUMERICAL

$$\mathbf{p} \rightarrow \mathbf{R} \xrightarrow[\text{differentiation}]{\text{numerical}} \dot{\mathbf{R}}$$

$$\boldsymbol{\omega}_C = \boldsymbol{\omega}_C(\mathbf{R}, \dot{\mathbf{R}})$$

$$\boldsymbol{\omega}_T = \boldsymbol{\omega}_T(\mathbf{R}, \dot{\mathbf{R}})$$

2

ANALYTICAL

$$\mathbf{p} \xrightarrow[\text{differentiation}]{\text{numerical}} \dot{\mathbf{p}}$$

$$\boldsymbol{\omega}_C = \boldsymbol{\omega}_C(\mathbf{p}, \dot{\mathbf{p}})$$

$$\boldsymbol{\omega}_T = \boldsymbol{\omega}_T(\mathbf{p}, \dot{\mathbf{p}})$$

Separation in components

$$\mathbf{R} = [\delta\mathbf{Q}\,\mathbf{Q}_0]\,\mathbf{R}_3(-\psi)\,[\mathbf{R}_2(x_P)\mathbf{R}_1(y_P)] = \mathbf{QDP}$$

$$[\boldsymbol{\omega}_Q\times] = \dot{\mathbf{Q}}\mathbf{Q}^T \qquad [\boldsymbol{\omega}_D\times] = \dot{\mathbf{D}}\mathbf{D}^T \qquad [\boldsymbol{\omega}_P\times] = \dot{\mathbf{P}}\mathbf{P}^T$$

$$\boldsymbol{\omega}_C = \boldsymbol{\omega}_Q + \mathbf{Q}\boldsymbol{\omega}_D + \mathbf{QD}\boldsymbol{\omega}_P \qquad \boldsymbol{\omega}_T = \mathbf{P}^T\mathbf{D}^T\mathbf{Q}^T\boldsymbol{\omega}_C$$

3

NUMERICAL BY COMPONENTS

$\mathbf{p} \rightarrow$

$$\mathbf{Q},\mathbf{D},\mathbf{P} \xrightarrow[\text{differentiation}]{\text{numerical}} \dot{\mathbf{Q}},\dot{\mathbf{D}},\dot{\mathbf{P}}$$

$\rightarrow \boldsymbol{\omega}_Q,\boldsymbol{\omega}_D,\boldsymbol{\omega}_P \rightarrow \boldsymbol{\omega}_C,\boldsymbol{\omega}_T$

4

ANALYTICAL BY COMPONENTS

$$\mathbf{p} \xrightarrow[\text{differentiation}]{\text{numerical}} \dot{\mathbf{p}}$$

$$\boldsymbol{\omega}_Q = \boldsymbol{\omega}_Q(X, Y, \dot{X}, \dot{Y}, \delta X, \delta Y, \delta\dot{X}, \delta\dot{Y})$$

$$\boldsymbol{\omega}_D = \boldsymbol{\omega}_D(\psi, \dot{\psi})$$

$$\boldsymbol{\omega}_P = \boldsymbol{\omega}_P(x_P, y_P, \dot{x}_P, \dot{y}_P)$$

$\rightarrow \boldsymbol{\omega}_C,\boldsymbol{\omega}_T$

VALIDATION OF RESULTS – PART 2
Stability of numerical differentiation

Determination of derivative $$f_i' = \frac{df}{dt}(t_i)$$

from equidistant values: $$f_i = f(t_i) \qquad t_{i+1} = t_i + \Delta t$$

Use of 2k+1 values: $$f_{i-k}, \dots, f_{i-1}, f_i, f_{i+1}, \dots, f_{i+k}$$

“Moving” polynomial interpolation: $$P_{i,k}(t) = a_0 + a_1 t + \dots + a_{2k} t^{2k}$$

$$P_{i,k}(t_m) = f_m, \quad m = i-k, \dots, i+k$$

$$f_i' = \frac{dP_{i.k}}{dt}(t_i)$$

Various choices of *k* give essentially identical results!

VALIDATION OF RESULTS – PART 3
Effect of data noise

High frequencies in data errors may create large error values in computed derivatives

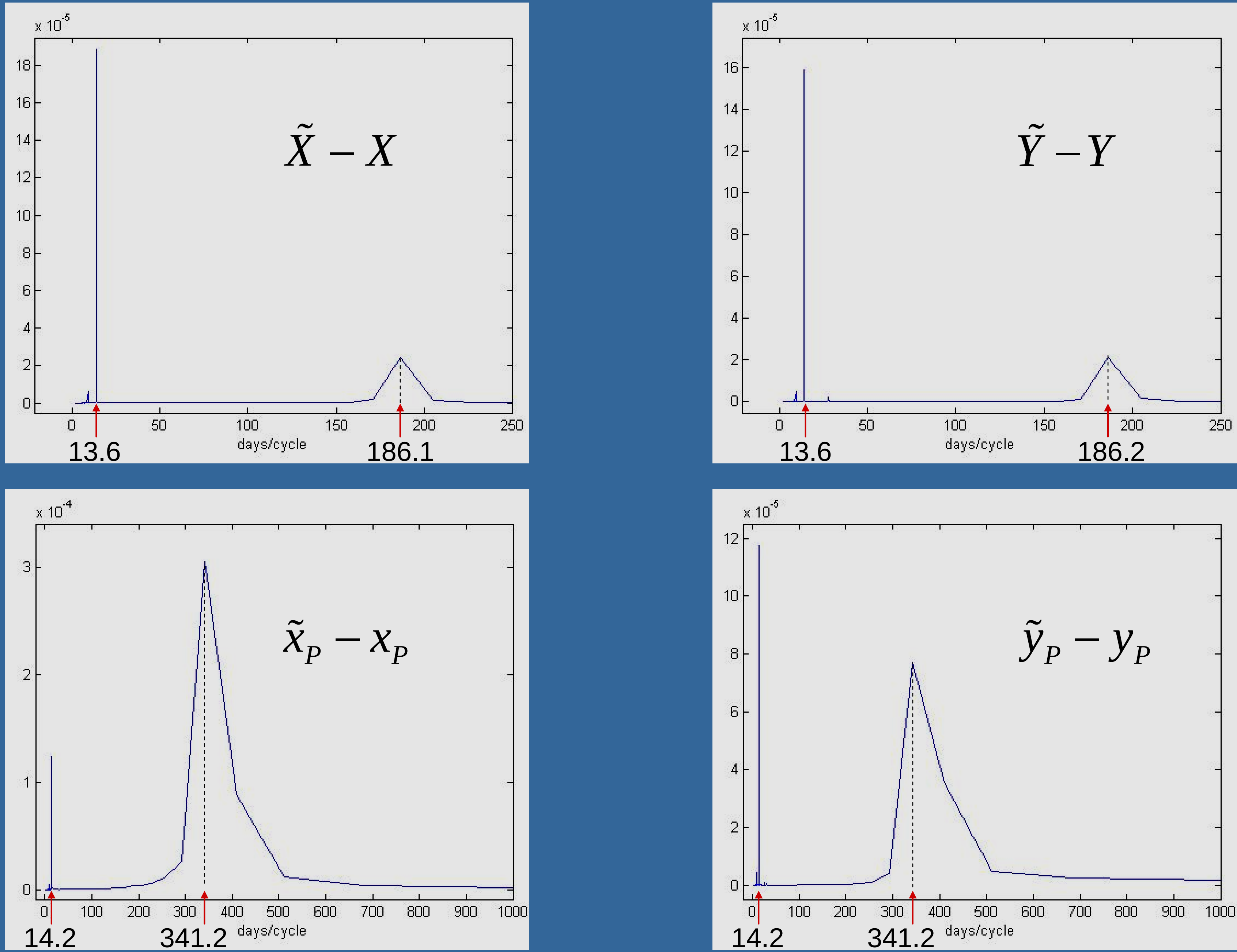
Treatment: Data smoothing by moving averages

Simple moving average:

$$\tilde{f}_i = \frac{1}{2k+1} \sum_{m=-k}^{k} f_{i+m}$$

Effect on final results: Somewhat smaller amplitudes for larger k
in computed differences between CCP & CIP parameters.
But 2 basic frequencies remain dominant !

SPECTRA OF DIFFERENCES BETWEEN CCP & CIP



CONCLUSIONS

Differences between the position of the Compatible Celestial Pole (CCP) and the position of the Celestial Intermediate Pole (CIP) are **significant**. The respective parameters referring to the celestial (X,Y) and the terrestrial reference system (polar motion x_P, y_P) demonstrate differences which vary in time **with two dominant terms**:

$\tilde{X} - X$	T_1 = **186.1** days	T_2 = **13.6** days
$\tilde{Y} - Y$	T_1 = **186.2** days	T_2 = **13.6** days
$\tilde{x}_P - x_P$	T_1 = **341.2** days	T_2 = **14.2** days
$\tilde{y}_P - y_P$	T_1 = **341.2** days	T_2 = **14.2** days

FUTURE WORK

Investigate theoretically the effect of biases & systematic errors in the rotation matrix **R**, on the CCP coordinates $\tilde{X}, \tilde{Y}$

Investigate theoretically the effect of aliasing on data with diurnal resolution. Higher resolution data available?

BEFORE

Comparing with CIP – Instantaneous Celestial Pole separation as defined by astronomical theory.