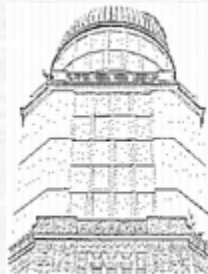


(Some) Relativistic Aspects of the IERS Conventions

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Section 1.2 Numerical Standards: “units”

Confusing wording:

“TDB units” $\neq$ “TT units” $\neq$ “SI units”

What are “units”? non-SI second?

Suggestion:

* _TDB --- TDB-compatible value

* _TT --- TT-compatible values

* _TCG --- TCG-compatible values

* _TCB --- TCB-compatible values

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Table 1.1 listing numerical standards is organized into 5 columns: item, value, uncertainty, reference, comment. Most of the values are given in terms of SI units (*Le Système International d’Unités (SI)*, 1998), *i.e.* they are consistent with the use of Geocentric Coordinate Time TCG as a time coordinate for the geocentric system, and of Barycentric Coordinate Time TCB for the barycentric system. The values of τ_A , $c\tau_A$, and ψ_1 , however, are given in so-called “TDB” units, having been determined previously using Barycentric Dynamical Time TDB as a time coordinate for the barycentric system. In this book some quantities are also given in so-called “TT” units, having been determined using Terrestrial Time TT as a time coordinate for the geocentric system. See Chapter 10 for further details on the transformations between time scales and Chapter 3 for a discussion of the time scale used in the ephemerides.

TDB and TCB units of time, t , and length, ℓ , may be easily related by the expressions (Seidelmann and Fukushima, 1992)

$$t_{TDB} = t_{TCB}/(1 - L_B), \quad \ell_{TDB} = \ell_{TCB}/(1 - L_B),$$

where L_B is given in Table 1.1. Therefore a quantity X with the dimension of time or length has a numerical value x_{TCB} when using “TCB” (SI) units which differs from its value x_{TDB} when using “TDB” units by

$$x_{TDB} = x_{TCB} \times (1 - L_B).$$

Similarly, the numerical value x_{TCG} when using “TCG” (SI) units differs from the numerical value x_{TT} when using “TT” units by

$$x_{TT} = x_{TCG} \times (1 - L_G)$$

where L_G is given in Table 1.1.

TDB-compatible quantities

- Three scalings in the BCRS are necessary to keep the equations of motion and light propagation invariant:

- time

$$TDB = (1 - L_B) \cdot TCB + const$$

- spatial coordinates

$$\mathbf{x}_{TDB} = (1 - L_B) \cdot \mathbf{x}_{TCB}$$

- masses of each body

$$\mu_{TDB} = (1 - L_B) \cdot \mu_{TCB}, \quad \mu = GM$$

TT-compatible quantities

- Three scalings are necessary in the GCRS to keep the equations of motion and light propagation invariant:

- time

$$TT = (1 - L_G) \cdot TCG + \text{const}$$

- spatial coordinates

$$\mathbf{x}_{TT} = (1 - L_G) \cdot \mathbf{x}_{TCG}$$

- masses of each body

$$\mu_{TT} = (1 - L_G) \cdot \mu_{TCG},$$

$$\mu_{TCG} = \mu_{TCB}$$

Concise overview of the background and a collection of reasoning:

Klioner, 2007, A&A, accepted

IAU Commission 52

“Relativity in Fundamental Astronomy”

- Created by the IAU in 2006
- Present Organizing Committee:

Sergei A. Klioner, *President*

Gérard Petit, *Vice President*

Victor A. Brumberg

Nicole Capitaine

Agnès Fienga

Toshio Fukushima

Bernard Guinot

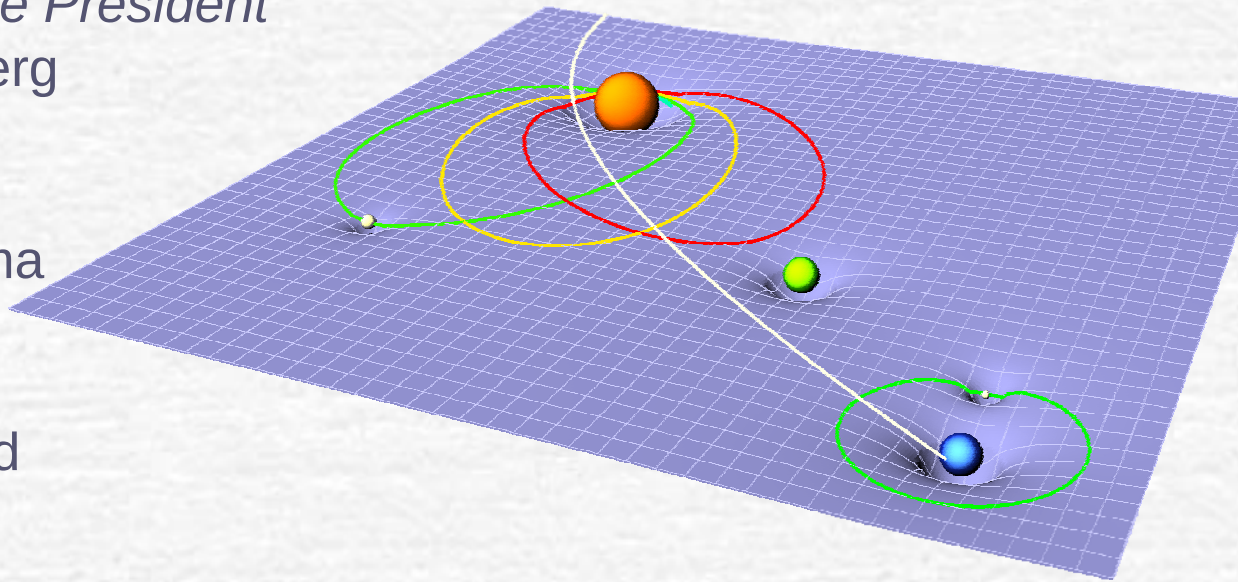
Cheng Huang

François Mignard

Ken Seidelmann

Michael Soffel

Patrick Wallace

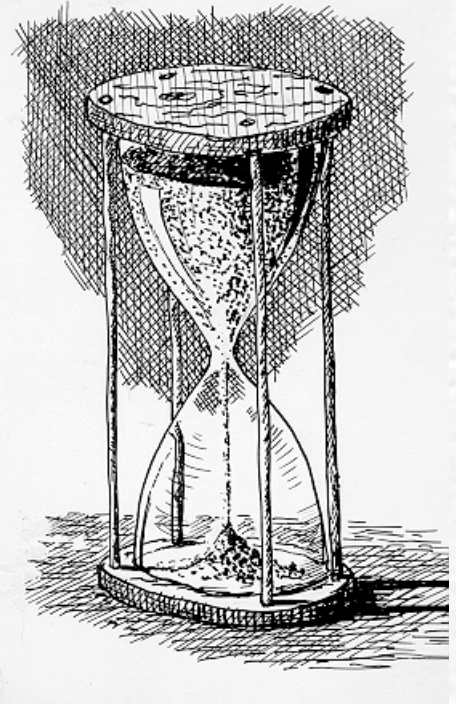


IAU Commission 52

“Relativity in Fundamental Astronomy”

- is thought to stimulate the exchange on relativistic issues
- Current projects:
 - Compile a list of unsolved problems
 - Frequently asked questions
 - Relativistic glossary for astronomers
 - “Task teams”, an ad hoc discussion group for very well posed issues:
 - One task team is currently activated: “**TDB units**”

Question: “What are the units of TDB?”



IAU Commission 52

“Relativity in Fundamental Astronomy”

- All interested members of the IAU are kindly invited to join:

<http://astro.geo.tu-dresden.de/RIFA>

RIFA
IAU Commission 52
Relativity in Fundamental Astronomy

Manifesto and invitation to join

Dear Colleagues,

On behalf of the Organizing Committee I would like to invite you to become a member of the new IAU Commission 52 "Relativity in Fundamental Astronomy". The web page of the Commission can be found at

<http://astro.geo.tu-dresden.de/RIFA>

Please find below a manifesto setting out the long-term and short-term goals of the Commission. We hope that the subject of the new Commission is of interest for you and that you will choose to join it. In order to join please send an e-mail to

Sergei.Klioner@tu-dresden.de

Sincerely Yours,

Sergei Klioner
President of the IAU Commission 52

IAU Commission 52 "Relativity in Fundamental Astronomy"

Relativity has been becoming increasingly important for modelling and interpretation of high accuracy astronomical observations during at least the last 20 years. Relativistic questions have been playing an important role in the work of several IAU Commissions and Working Groups for an extended period of time. For these reasons the 26th General Assembly of the IAU, held in Prague in August 2006, has decided to establish IAU Commission 52 "Relativity in Fundamental Astronomy" (RIFA). The general scientific goals of the new Commission are:

clarify geometrical and dynamical concepts of Fundamental Astronomy within a relativistic framework

Table of Contents:

Manifesto and invitation to join
Scientific objectives
Projects
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List of members
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Last modified on:
11 August 2007

Section 1.2 Numerical Standards: AU

Table 1.1 IERS Numerical Standards

$\tau_A^\dagger$	499.0047838061s	0.000000002s	[3]	Astronomical unit in seconds
$c\tau_A^\dagger$	149597870691m	6m	[3]	Astronomical unit in meters

† The values for τ_A , $c\tau_A$, and ψ_1 are given in “TDB” units (see discussion above).

(not clear why to have both quantities: $c\tau_A$ is sufficient)

Suggestion:

Just “astronomical unit in meters”, no reference to TDB

Reasoning:

- With its current definition “astronomical unit” should have the same value with both TDB and TCB (better choice of the convention among infinite possible)
- 2. If some time later, the AU in meters becomes a defining constant, nothing will have to be changed

Do we need astronomical units?

- The reason to introduce astronomical units was that angular measurements were many orders of magnitude more accurate than distance measurements.

- BUT

- The situation has changed crucially since that time!
- Solar mass is time-dependent just below current accuracy of ephemerides

$$\dot{M}_{Sun} / M_{Sun} \approx 10^{-13} \text{ yr}^{-1}$$

- Very confusing situation with astronomical units in relativistic framework
- Why not to define AU conventionally as fixed number of meters?
- Do you see any good reasons for astronomical units in their current form?

Section 1.2 Numerical Standards: AU

Table 1.1 IERS Numerical Standards

$GM_{\odot}$	$1.32712442076 \times 10^{20} m^3 s^{-2}$	$5 \times 10^{10} m^3 s^{-2}$	[from 3]	Heliocentric gravitational constant
$\tau_A^{\dagger}$	$499.0047838061 s$	$0.00000002 s$	[3]	Astronomical unit in seconds
$c\tau_A^{\dagger}$	$149597870691 m$	$6 m$	[3]	Astronomical unit in meters

$\dagger$ The values for τ_A , $c\tau_A$, and ψ_1 are given in “TDB” units (see discussion above).

- Any reasons to have “astronomical units” outside of the software for developing solar system ephemerides?
- As a result of the current AU definition we have errors in both AU and GM_{Sun} expressed in metric units...

Section 1.2 Numerical Standards: TDB

Table 1.1 IERS Numerical Standards

				TDB
L_B	$1.55051976772 \times 10^{-8}$	2×10^{-17}	[4]	Average value of $1-d(\text{TT})/d(\text{TCB})$
L_C	$1.48082686741 \times 10^{-8}$	2×10^{-17}	[4]	Average value of $1-d(\text{TCG})/d(\text{TCB})$
L_G	$6.969290134 \times 10^{-10}$	Defining	[4]	$1-d(\text{TT})/d(\text{TCG})$

- Where L_C is used? Is it needed for any practical calculations?

- The new TDB (IAU 2006)? If it is adopted by the IERS:

$$L_B = 1.550519768 \times 10^{-8} \quad \text{Defining} \quad [?] \quad 1-d(\text{TDB})/d(\text{TCB})$$

$$L_G = 6.969290134 \times 10^{-10} \quad \text{Defining} \quad [?] \quad 1-d(\text{TT})/d(\text{TCG})$$

- L_C is no longer used to compute L_B
- Average value of $1-d(\text{TCG})/d(\text{TCB})$ for a particular ephemeris is no longer $1-(1-L_B)/(1-L_G)$

- If the new TDB is adopted, Section 10.1 should be also modified

Section 11.2 Laser Ranging: L_C

- Where L_C is used? Is it needed for any practical calculations?

In Section 11.2 Laser Ranging we have

$$\vec{r}_{TDB} = \vec{r}_{TT} \left(1 - \frac{U}{c^2} - L_C \right) - \frac{1}{2} \left(\frac{\vec{V} \cdot \vec{r}_{TT}}{c^2} \right) \vec{V}, \quad (19)$$

- Suggestion:

$$\vec{r}_{TDB} = \frac{(1 - L_B)}{(1 - L_G)} \vec{r}_{TT} - \frac{U}{c^2} \frac{\vec{V} \cdot \vec{r}_{TT}}{c^2} \vec{V}$$

Section 1.2 Numerical Standards: GM_Earth

Table 1.1 IERS Numerical Standards

$GM_{\oplus}$	$3.986004418 \times 10^{14} m^3 s^{-2}$	$8 \times 10^5 m^3 s^{-2}$	[1]	Geocentric gravitational constant (EGM96 value)
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Suggestion:

TCG-compatible value of ...

Section 1.2 Numerical Standards: GM_Earth

Table 1.1 IERS Numerical Standards

$GM_{\oplus}$	$3.986004418 \times 10^{14} m^3 s^{-2}$	$8 \times 10^5 m^3 s^{-2}$	[1]	Geocentric gravitational constant (EGM96 value)
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- This number contradicts the DE405 value which can be computed from constants given in Tables 3.1 and 3.2

$$\begin{aligned}
 & (149597870691)^3 (86400)^{-2} k^2 \\
 & \times 328900.561400^{-1} (1 + 1/81.30056)^{-1} \\
 & \times (1 - L_B)^{-1} \\
 & = 3.986004391 \times 10^{14}
 \end{aligned}$$

a deviation of 3.4σ

it would be better to avoid such inconsistencies...

Section 1.2 Numerical Standards: ψ

Table 1.1 IERS Numerical Standards

$\psi_1^\dagger$	$5038.47875''/c$	$0.00040''/c$	[6]	IAU(1976) value of precession of the equator at J2000.0 corrected by $-0.29965''$. See Chapter 5.
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† The values for τ_A , $c\tau_A$, and ψ_1 are given in “TDB” units (see discussion above).

- why “TDB” units?

- Suggestion:

TT-compatible value of precession constant

Section 11.2 Laser Ranging

$$t_2 - t_1 = \frac{|\vec{x}_2(t_2) - \vec{x}_1(t_1)|}{c} + \sum_J \frac{2GM_J}{c^3} \ln \left(\frac{r_{J1} + r_{J2} + \rho}{r_{J1} + r_{J2} - \rho} \right), \quad (17)$$

- What is the accuracy requirements for this formula?
- Why “2” and not “1+ γ ” here?
- Two possible issues:
 1. the post-post-Newtonian time delay due to the Sun (not very relevant for IERS?)
 2. motion of the body “J” during light propagation:

simple way to take the main term into account is to change

$$r_{J1} = |\vec{x}_1 - \vec{x}_J|, \quad r_{J2} = |\vec{x}_2 - \vec{x}_J| \quad \longrightarrow \quad r_{J1} = |\vec{x}_1 - \vec{x}_{J1}|, \quad r_{J2} = |\vec{x}_2 - \vec{x}_{J2}|$$

$$\text{with } \vec{x}_{J1} = \vec{x}_J(t_1), \quad \vec{x}_{J2} = \vec{x}_J(t_2)$$

Post-post-Newtonian light propagation

Full post-post-Newtonian expression for the Shapiro time delay with PPN parameters (Klioner, Zschocke, 2007):

$$\begin{aligned} c \tau = & R + (1 + \gamma) m \log \frac{x + x_0 + R}{x + x_0 - R} \\ & + \frac{1}{8} \alpha \epsilon \frac{m^2}{R} \left(\frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\ & + \frac{1}{4} \alpha (8(1 + \gamma) - 4\beta + 3\epsilon) m^2 \frac{R}{|\mathbf{x} \times \mathbf{x}_0|} \arctan \frac{x^2 - x_0^2 + R^2}{2|\mathbf{x} \times \mathbf{x}_0|} \\ & - \frac{1}{4} \alpha (8(1 + \gamma) - 4\beta + 3\epsilon) m^2 \frac{R}{|\mathbf{x} \times \mathbf{x}_0|} \arctan \frac{x^2 - x_0^2 - R^2}{2|\mathbf{x} \times \mathbf{x}_0|} \\ & + \frac{1}{2} (1 + \gamma)^2 m^2 \frac{R}{|\mathbf{x} \times \mathbf{x}_0|^2} (x - x_0 - R) (x - x_0 + R) . \end{aligned}$$

$$m = \frac{GM}{c^2}$$

The higher-order terms give up to 10 meters. Are all these terms relevant?

Post-post-Newtonian light propagation

NO!

The only numerically relevant term can be written as

$$c \tau = R + (1 + \gamma) m \log \frac{x + x_0 + R + (1 + \gamma) m}{x + x_0 - R + (1 + \gamma) m}$$

This has already been derived by Moyer (2003) in a different way.

All other terms can be estimated as

$$c \delta \tau \leq \frac{m^2}{d} \left(\frac{3}{4} + \frac{15}{4} \pi \right)$$

This gives maximally 4 cm for Sun-grazing ray,
and much less in typical cases...

Possible future developments

1. The details of relativistic models for different kinds of observations are not the same:

the VLBI model is given in full detail

the LLR one is just sketched

Do we need the same level of detail?

2. Required/claimed accuracies should be stated (clearly stated for VLBI, but not, say, for LLR)
3. General Relativity or PPN formalism?

IAU 2000 framework is formulated in General Relativity

IERS Conventions 2003: a mixture of PPN formalism
and General Relativity



What could be interesting in principle

1. (More) detailed description of the LLR model
2. Relativity in GPS model:

adding, e.g., the influence of the quadrupole gravitational field of the Earth on the clocks on the satellites

GPS models used in Bernese have never been seriously discussed by scientific community

Can we risk inconsistencies between the GPS and VLBI models?

(some of the statements in the Ashby's description of the GPS model are not compatible with the IAU framework)