SOFA support for the IAU 2006 precession model

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SOFA: Standards Of Fundamental Astronomy

- SOFA is an IAU Service which operates under Division 1 (Fundamental Astronomy) and reports through Commission 19 (Rotation of the Earth).
- It was created by the IAU at the 1994 General Assembly, to provide an authoritative and accessible set of fundamental-astronomy algorithms.
- It is run by the SOFA Review Board, comprising 9 members from China, France, Netherlands, Russia, UK, USA.
- The SOFA products are made available through a website that is at present run by HMNAO.
- The products are designed to be easy to use, either directly in applications or merely as a source of test results and algorithms.

SOFA website: www.iau-sofa.rl.ac.uk

Standards Of Fundamental Astronomy

The IAU created the SOFA initiative to establish and maintain an accessible and authoritative set of constants, algorithms and procedures that implement standard models used in fundamental astronomy.

Background

SOFA operates under the auspices of the International Astronomical Union to provide a collection of approved algorithms for use in astronomical computing. The initiative is managed by an international panel, the SOFA Reviewing Board, appointed through IAU Division 1. The Board obtains the latest IAU-approved models and theories from the fundamental-astronomy community, implements them as computer code and checks them for accuracy. The product pages contain indexes of the currently-available software, instructions of how to obtain it, and instructions for its use. Some reports and papers from the IAU SOFA initiative can be found here.

In the future, as well as acquiring newer algorithms as they become available, the SOFA libraries will expand and diversify, and variants in other programming languages will eventually appear.

SOFA Products

The SOFA products may be found on the index page:

SOFA Product Index

The SOFA product distribution service is coordinated from the Space Science and Technology Department, Rutherford Appleton Laboratory, Chilton, DIDCOT, Oxfordshire OX11 0QX, United Kingdom.

Acknowledgements

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Created: 2001-03-01
Updated: 2007-08-13
IAU SOFA Center, sofa@star.rl.ac.uk

SOFA routines

- 108 astronomy routines:
  - calendars (7)
  - time scales and Earth rotation (17)
  - ephemerides (16)
    - Earth $p,\dot{p}$ (Bretagnon 2000, abridged to give 10km, 5mm/s)
    - major planets (Simon et al. 1994)
    - fundamental arguments (same as IERS Conventions 2003)
  - precession-nutation and polar motion (60)
  - star data transformations (8)
- 52 vector-matrix routines:
  - 3-vectors and rotation matrices (30)
  - $p,\dot{p}$ vectors (17)
  - angles (5)
- All Fortran 77 at present, but work is under way on a C version.

SOFA tools for Earth orientation

- SOFA supports:
  - new (CIO-based) and classical (equinox-based) methods
  - angles-based and X,Y-series-based CIP predictions
  - IAU 1980 and IAU 2000A and IAU 2000B nutation
  - IAU 1976 and IAU 2000 and IAU 2006 precession
- All these choices mean that Earth orientation/rotation accounts for well over half of the 160 SOFA routines.
- The routines range from building blocks, e.g. compute nutation ($\Delta\psi$,$\Delta\varepsilon$) or CIP (X,Y) to high-level ensembles, e.g. compute celestial-to-terrestrial matrix, enabling different simplicity/efficiency trade-offs.

Four generations of precession

- IAU 1976 - Lieske precession, in z,ζ,θ form.
- IAU 2000 - Pole offsets and precession rate corrections absorbed into new nutation model.
- SOFA/IERS 2003 - Bias extended to include $\Delta\alpha$ and kept separate from nutation. Precession uses the Lieske $\varepsilon_0,\psi_A,\omega_A,\chi_A$ angles with rate adjustments applied to ψ_A, and ω_A. IERS offers CIP X,Y directly through series, SOFA through the classical NPB matrix.
- IAU 2006 – New precession model, consistent with dynamical theory but otherwise matching IAU 2000. Includes small adjustments to IAU 2000A nutation because of change to ε_0 etc. New SOFA routines, mainly using angles (now $\bar{\gamma},\bar{\varphi},\bar{\psi},\varepsilon_A$) but also providing direct X,Y series. To come in service at the start of 2009.

Precession-nutation algorithms: ingredients

- The goal is to generate these matrices:
  - ${}_{GCRS}\mathbf{R}_{CIRS}$ (or ${}_{GCRS}\mathbf{R}_{true}$)
  - ${}_{GCRS}\mathbf{R}_{TIRS}$
- This can be done if we know these vectors:
  - $\mathbf{v}_{CIP}$
  - $\mathbf{v}_{CIO}$ (or $\mathbf{v}_{equinox}$)
- ...and this angle:
  - ERA (or GST)
- The *equation of the origins* links classical and new:
  - EO = ERA−GST = distance from CIO to equinox

How the GCRS-to-TIRS matrix is built

$$
{}_{GCRS}\mathbf{R}_{TIRS} = R_3(\mathrm{ERA}) \times \begin{pmatrix} \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \end{pmatrix} = R_3(\mathrm{GST}) \times \begin{pmatrix} \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \end{pmatrix}
$$

$$
= \mathrm{ERA} - \mathrm{EO}
$$

Algorithm choices

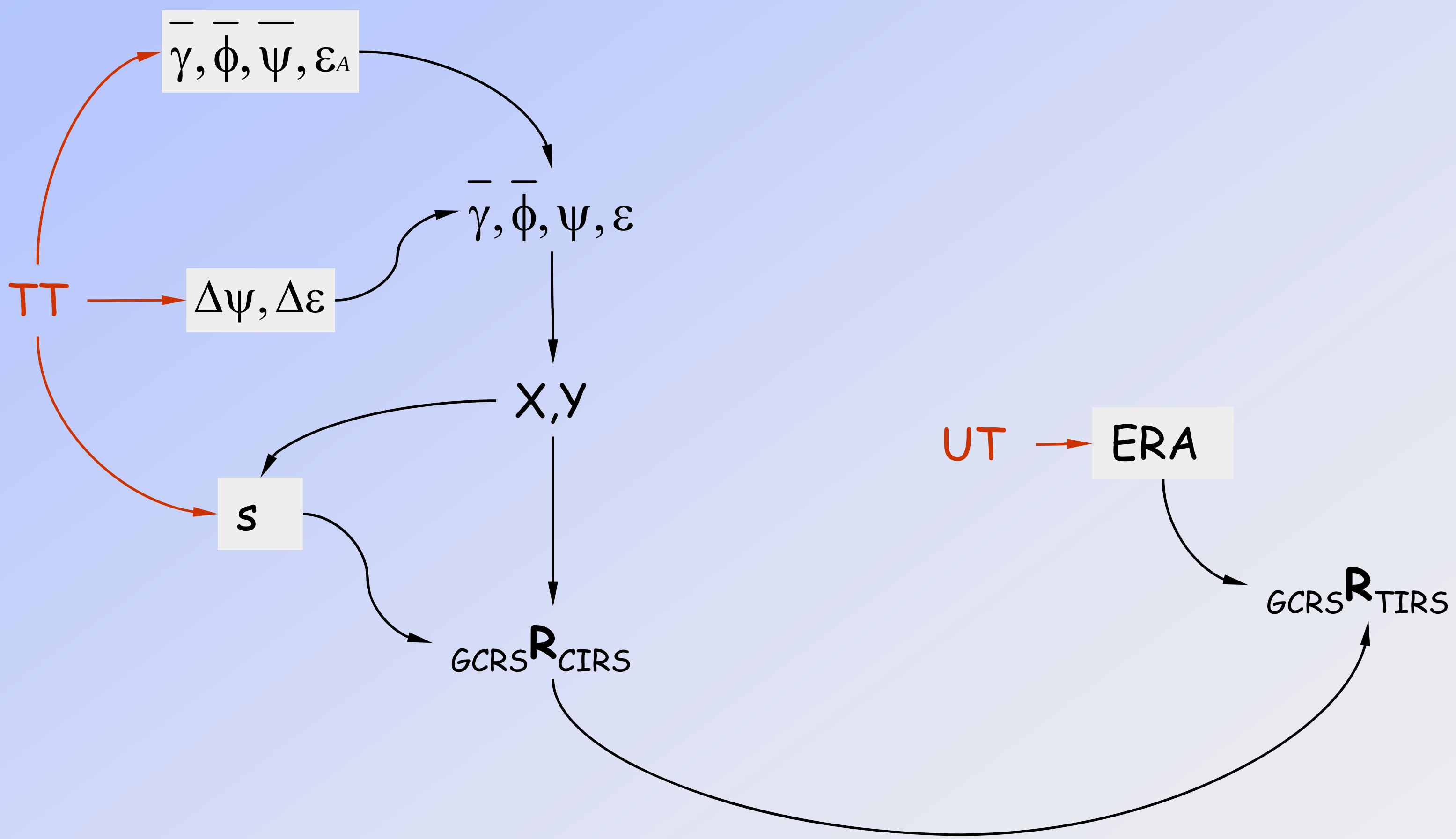
- IAU 2006 Resolution B1 leaves the choice of parameterization up to the user.

- Furthermore, Capitaine & Wallace (2006) describes:
  - 6 ways of forming ${}_{GCRS}\mathbf{R}_{CIRS}$
  - 3 ways of obtaining $\mathbf{v}_{CIP}$
  - 8 ways of obtaining $\mathbf{v}_{CIO}$

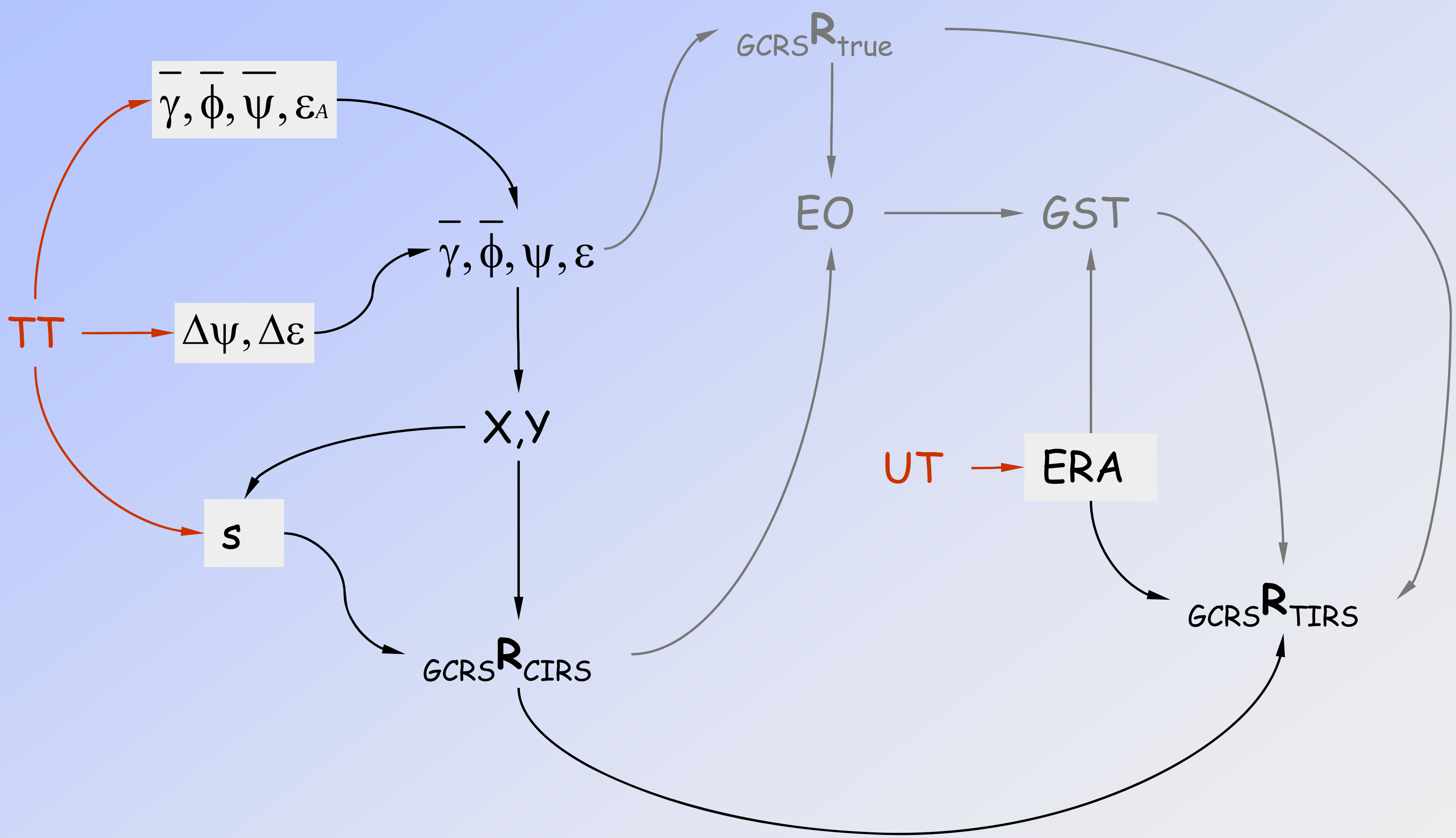
- Different applications have different priorities. For example:
  - A general-purpose toolkit such as SOFA must
    - have a clear provenance
    - be versatile, concise and efficient
    - be highly self-consistent
  - IERS Conventions addresses a demanding but highly focused application:
    - it must be straightforward to use
    - it must perform efficiently and accurately
  - In many applications accuracy can be traded off against size and speed.

- So there are many choices to make. Wallace & Capitaine (2006):
  - concentrates on two specific choices
  - provides detailed numerical examples

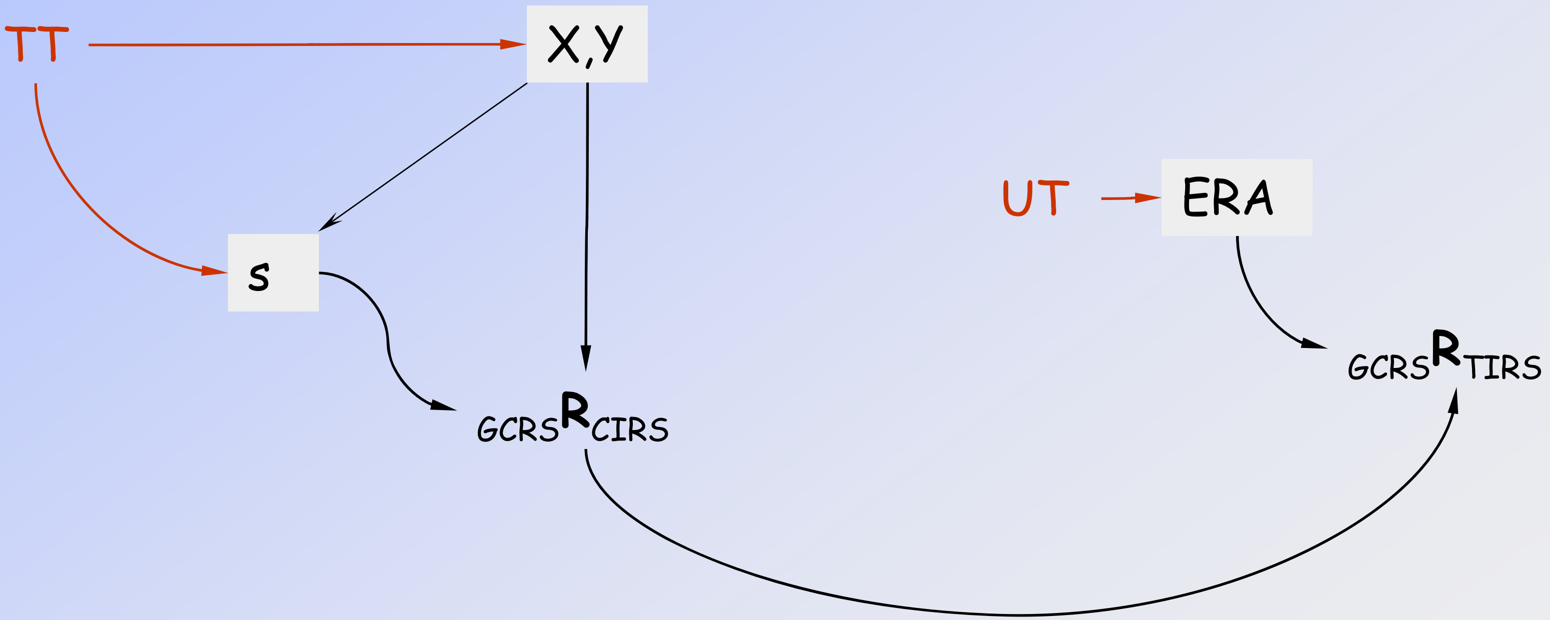
"Angles" method



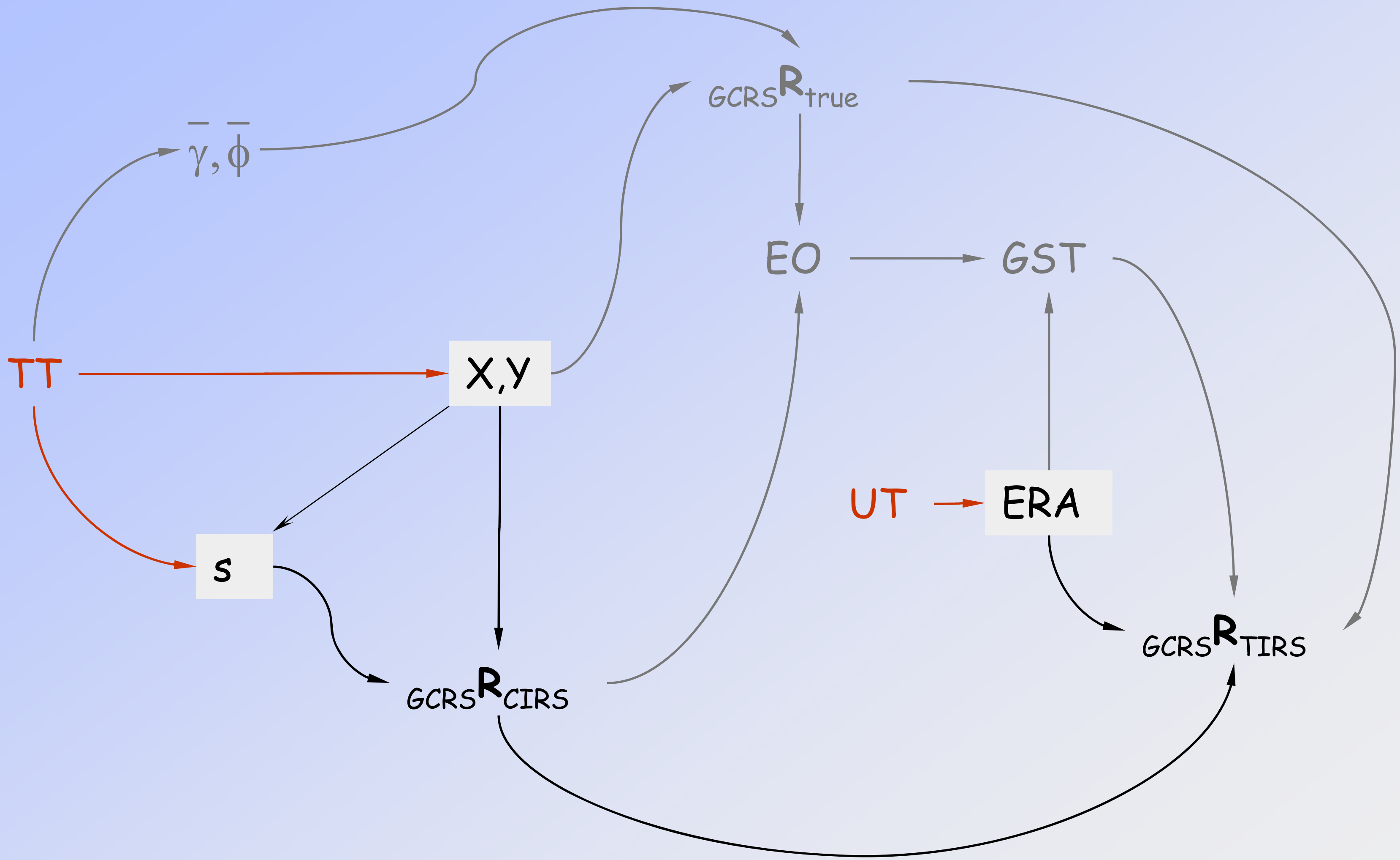
"Angles" method - equinox option



"X,Y series" method - the IERS recommendation



"X,Y series" method - equinox option

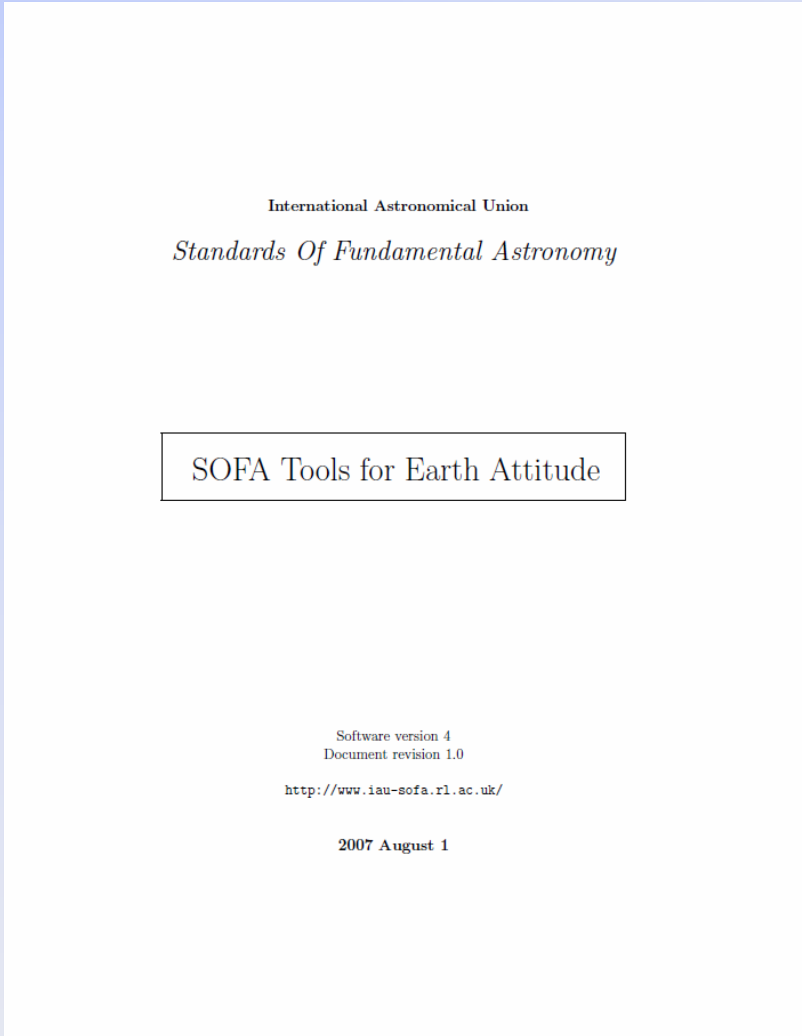


Methods supported by SOFA

- SOFA supports both the angles method and the X,Y series method, the latter through the new subroutine iau_XY06.
- Disadvantage: the user might mix the two canonical bases and so generate heterogenous results (at the 1μas level). However, the X,Y-series facilities are somewhat separate and self-contained, and the danger is documented.
- Advantage #1: the X,Y-series method is the preferred IERS method, and means that the post-2009 IERS software can use the SOFA subroutine directly. (The existing IERS Conventions 2003 subroutine XYS2000A which, like iau_XY06, is based on series, had no SOFA counterpart and was specially written for IERS use.)
- Advantage #2: the X,Y-series method is a better basis for "concise models".

The SOFA "cookbook"

- Introduction to celestial coordinates.
- SOFA models.
- The IAU 2006/2000A models in detail.
- Examples:
  - IAU 1976/80/82/94, equinox
  - IAU 2000A, CIO, angles
  - IAU 2000A, equinox, angles
  - IAU 2006/2000A, CIO, angles
  - IAU 2006/2000A, CIO, X,Y series



International Astronomical Union

Standards Of Fundamental Astronomy

SOFA Tools for Earth Attitude

Software version 4
Document revision 1.0

http://www.iau-sofa.rl.ac.uk/

2007 August 1

Example: IAU 2006/2000A, CIO based, using X,Y series

```
*  ===========================================
*  IAU 2006/2000A, CIO based, using X,Y series
*  ===========================================

*  CIP and CIO, IAU 2006/2000A.
      CALL iau_XY06 ( DJMJD0, TT, X, Y )
      S = iau_S06 ( DJMJD0, TT, X, Y )

*  Add CIP corrections.
      X = X + DX06
      Y = Y + DY06

*  GCRS to CIRS matrix.
      CALL iau_C2IXYS ( X, Y, S, RC2I )

*  Earth rotation angle.
      ERA = iau_ERA00 ( DJMJD0+DATE, TUT )

*  Form celestial-terrestrial matrix (no polar motion yet).
      CALL iau_CR ( RC2I, RC2TI )
      CALL iau_RZ ( ERA, RC2TI )

*  Polar motion matrix (TIRS->ITRS, IERS 2003).
      CALL iau_POM00 ( XP, YP, iau_SP00(DJMJD0,TT), RPOM )

*  Form celestial-terrestrial matrix (including polar motion).
      CALL iau_RXR ( RPOM, RC2TI, RC2IT )
```

$$
\begin{aligned}
X &= +0.000712264729525\\
Y &= +0.000044385248875\\
s &= -0.''002200475
\end{aligned}
$$

NPB matrix, CIO based

$$
\begin{pmatrix}
+0.999999746339445 & -0.000000005138822 & -0.000712264729999\\
-0.000000026475226 & +0.999999999014975 & -0.000044385241276\\
+0.000712264729525 & +0.000044385248875 & +0.999999745354420
\end{pmatrix}
$$

$$
\begin{aligned}
\mathrm{ERA} &= 13.^\circ 318492966097\\
&= 00^{h}\,53^{m}\,16.^{s}438311863
\end{aligned}
$$

celestial to terrestrial matrix (no polar motion)

$$
\begin{pmatrix}
+0.973104317573127 & +0.230363826247709 & -0.000703332818416\\
-0.230363798804181 & +0.973104570735574 & +0.000120888551078\\
+0.000712264729525 & +0.000044385248875 & +0.999999745354420
\end{pmatrix}
$$

celestial to terrestrial matrix

$$
\begin{pmatrix}
+0.973104317697536 & +0.230363826239128 & -0.000703163481769\\
-0.230363800456036 & +0.973104570632801 & +0.000118545368117\\
+0.000711560162594 & +0.000046626402444 & +0.999999745754024
\end{pmatrix}
$$

SOFA and IERS

- There is no direct formal connection between SOFA and IERS under the present arrangements.
- However, the SOFA software and the IAU 2000 precession-nutation software on the IERS Conventions web pages were developed in parallel and are consistent at microarcsecond level.
- It is intended that SOFA routines will, in the future, support IERS Conventions Chapter 5 directly, with no need for special versions or supplements.

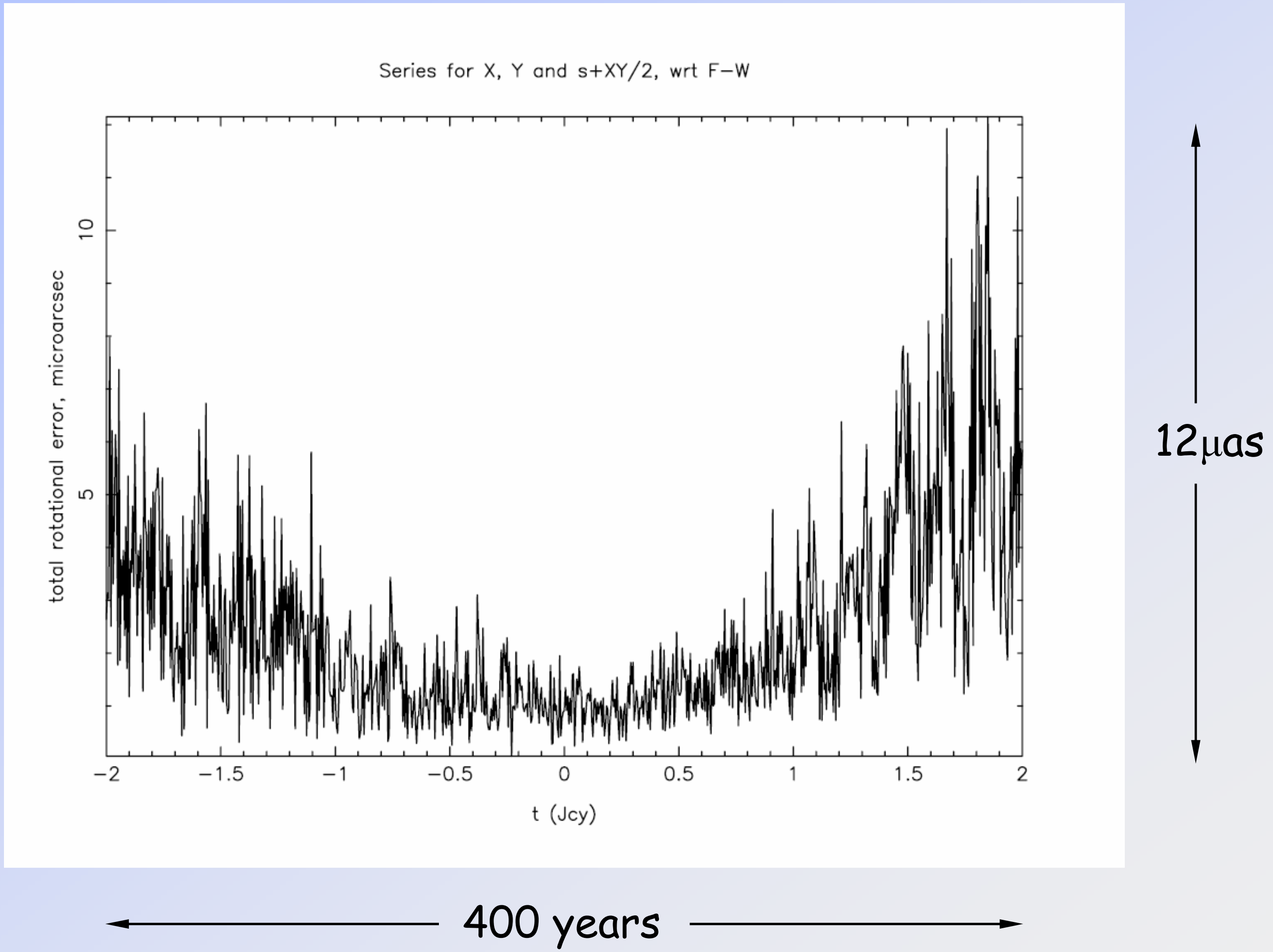
References

- Capitaine N., Wallace P.T., 2006, *High precision methods for locating the celestial intermediate pole and origin*, Astronomy & Astrophysics, **450**, 855-872
- Wallace P.T., Capitaine N., 2006, *Precession-nutation procedures consistent with IAU 2006 resolutions*, Astronomy & Astrophysics, **459**, 981-985

END

Additional material

How well do the "angles" and "X,Y series" methods agree?



The series compared

		number of terms					
		1	*t*	t^2	t^3	t^4	t^5
angles method	$\bar{\gamma}$	1	1	1	1	1	1
	$\bar{\phi}$	1	1	1	1	1	1
	ψ	1321	38	1	1	1	1
	ε_A	1038	20	1	1	1	1
X,Y method	X	1307	254	37	5	2	1
	Y	963	278	31	6	2	1
both	$s + XY/2$	34	4	26	5	2	1