Local tie adjustment: from principle to implementation

Patrick Sillard, Pierguido Sarti
& Luca Vittuari

Introduction

Physical def. of Ref. point

Local survey

Geometrical constraints

SINEX

- What to measure?
- How to measure it?
- How to parametrize the problem?
- How to compute?
- What is the final product?

Reference point definition

- Completely linked with the way it is modeled in SG softwares
- Goal= to realize from terrestrial measurement, the physical definition
- Trying to reproduce in the sequence of terrestrial measurements what is done in SG Analysis softwares

Types of instruments

- “Big” instruments (VLBI & SLR) which have degrees of freedom and cannot be removed to observe a physically linked ground marker
- “Small” instruments which can be removed and which are potentially physically linked to a ground marker

  - Recovering the information reproducing the physical definition
  - Direct measurement of the ground marker

Example

Medicina 2001 and 2002 surveys

VLBI

+

GPS

Basic observations

- Direct measurement of VLBI antenna targets
- Direct measurements of GPS antenna

CLASSICAL GEODESY OBSERVATIONS

Scheme

$$\Sigma^{-1}\xi = \Sigma^{-1}\widetilde{\xi}$$

$$F(\xi,\pi) = 0$$

- Normal equation coming from classical geodesy observations with respect to target coordinates (ξ)
- Additional constraints which link previous coordinates (ξ) to Ref. Points coordinates (π)

Optimisation problem

$$\min_{\xi,\pi} (\xi - \tilde{\xi})^T \Sigma^{-1} (\xi - \tilde{\xi})$$

$$u.c.\ F(\xi,\pi) = 0$$

$$F(\xi,\pi) = 0 \Leftrightarrow \left| \begin{array}{ll} g(\xi,\pi) = 0 & (\lambda) \\ q(\pi) = 0 & (\mu) \\ h(\xi,\pi) = 0 & (\eta) \end{array} \right.$$

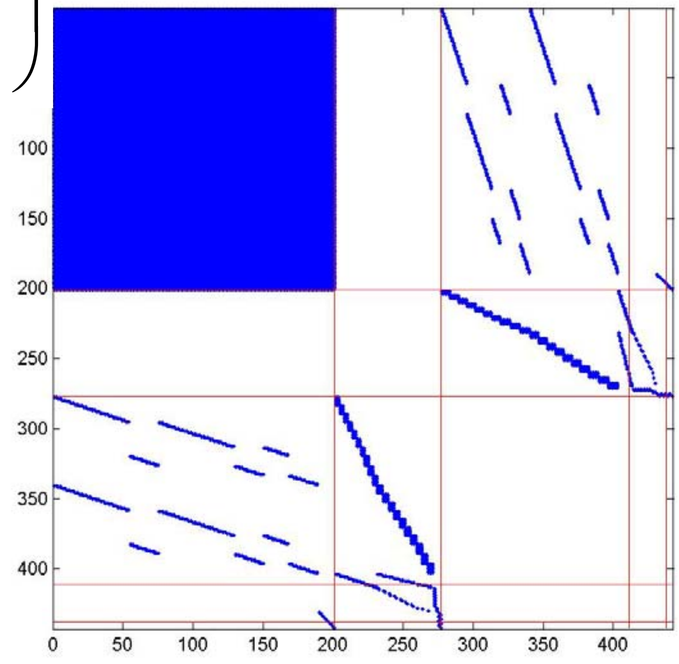
Constraints

$\pi = \left| \begin{array}{l} \text{Planes par.} \\ \text{Circle centers} \\ \text{VLBI Ref. Point} \\ \text{GPS Ref. Point} \end{array} \right.$

- While the VLBI antenna is moving around one axis, a target belongs to a circle
- The centers are aligned and are connected to the antenna Ref. Point
- Link between GPS antenna observations and GPS Ref. Point

System

$$
\left(\begin{array}{c|c|c|c|c}
\Sigma^{-1} & 0 & G_1^T & 0 & H_1^T \\ \hline
0 & 0 & G_2^T & Q^T & H_2^T \\ \hline
G_1 & G_2 & 0 & 0 & 0 \\ \hline
0 & Q & 0 & 0 & 0 \\ \hline
H_1 & H_2 & 0 & 0 & 0
\end{array}\right)
\begin{pmatrix} \xi \\ \pi \\ \lambda \\ \mu \\ \eta \end{pmatrix}
=
\begin{pmatrix} \Sigma^{-1}\tilde{\xi} \\ 0 \\ K \\ L \\ J \end{pmatrix}
$$



Solution

Inversion of

$$\begin{pmatrix} \Sigma^{-1} & 0 & G_1^T & 0 & H_1^T \\ 0 & 0 & G_2^T & Q^T & H_2^T \\ G_1 & G_2 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 & 0 \\ H_1 & H_2 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \pi \\ \lambda \\ \mu \\ \eta \end{pmatrix} = \begin{pmatrix} \Sigma^{-1}\underline{\tilde{\xi}} \\ 0 \\ K \\ L \\ J \end{pmatrix}$$

Gives estimates $\hat{\xi}, \hat{\pi}$

And residuals $\hat{V} = \tilde{\xi} - \hat{\xi}$

Statistics

$$\left(\begin{array}{c|c|c|c|c} \Sigma^{-1} & 0 & G_1^T & 0 & H_1^T \\ \hline 0 & 0 & G_2^T & Q^T & H_2^T \\ \hline G_1 & G_2 & 0 & 0 & 0 \\ \hline 0 & Q & 0 & 0 & 0 \\ \hline H_1 & H_2 & 0 & 0 & 0 \end{array}\right)^{-1}$$

Covariance of π (must be multiplied by a variance factor σ²)

$$\hat{\sigma}^2 = \frac{\hat{V}^T \Sigma^{-1} \hat{V}}{n_{obs} + n_{cons} - n_{unkn}}$$

Example: Medicina 2001 and 2002 surveys

Local tie / Number of	2001	2002
stations	110	105
azimuth angles	297	308
zenith angles	289	339
distances	272	327
unknowns	324	312

$$\hat{\sigma}^2 \approx 3.1$$

Uncertainties

Coordi-nate	E_{vlbi}	N_{vlbi}	U_{vlbi}	E_{gps}	N_{gps}	U_{gps}
σ(mm) 2001	0.30	0.40	0.82	1.02	1.07	0.85
σ (mm) 2002	0.22	0.29	0.56	0.82	0.56	0.53

$$\rho_{2002} = \begin{pmatrix} 1 & 0.0595 & -0.0004 & 0.0035 & -0.0131 & -0.0005 \\ & 1 & -0.0054 & -0.0149 & 0.1773 & -0.0002 \\ & & 1 & 0.0003 & -0.0009 & 0.3985 \\ & & & 1 & -0.0587 & -0.0169 \\ & & & & 1 & -0.0122 \\ & & & & & 1 \end{pmatrix}$$

Transformation into the ITRF

- The underlying reference frame is arbitrary
- So, an arbitrary transformation can be applied
- Anyway, a set a local tie should be assumed to be expressed in a fuzzy reference frame
- In our case, application of a mean rotation and a mean translation without introducing associated uncertainty

SINEX

```
%=SNX 1.00
*-------------------------------------------------------------------------------
+FILE/COMMENT
* The DOMES are correct
* Variance factor equal to 1.00
-FILE/COMMENT
*-------------------------------------------------------------------------------
+SITE/ID
*CODE PT __DOMES__ T _STATION DESCRIPTION__ APPROX_LON_ APPROX_LAT_ _APP_H_
 VLBR  A 12711S001
 GPSR  A 12711M003
-SITE/ID
*-------------------------------------------------------------------------------
+SOLUTION/ESTIMATE
*INDEX TYPE__ CODE PT SOLN _REF_EPOCH__ UNIT S __ESTIMATED VALUE____ _STD_DEV___
     1 STAX   VLBR  A    1  0:000:00000 m    2 4.461369978747157E+06 2.67134E-004
     2 STAY   VLBR  A    1  0:000:00000 m    2 9.195968255608416E+05 5.44588E-004
     3 STAZ   VLBR  A    1  0:000:00000 m    2 4.449559200474912E+06 2.80537E-004
     4 STAX   GPSR  A    1  0:000:00000 m    2 4.461400895152843E+06 6.81391E-004
     5 STAY   GPSR  A    1  0:000:00000 m    2 9.195934231391583E+05 5.43265E-004
     6 STAZ   GPSR  A    1  0:000:00000 m    2 4.449504680825088E+06 7.14008E-004
-SOLUTION/ESTIMATE
*-------------------------------------------------------------------------------
+SOLUTION/MATRIX_ESTIMATE L COVA
*PARA1 PARA2 ____PARA2+0__________ ____PARA2+1__________ ____PARA2+2__________
     1     1 7.136055600058919E-08
     2     1 2.715051926328614E-09 2.965758690842930E-07
     3     1 -1.957946925859 22E-08 6.082195561979286E-08 7.870072882320828E-08
     4     1 1.258598565862185E-08 2.544143821923472E-09 -1.306511842288 35E-08
     4     4 4.642941241946729E-07
     5     1 3.092006786654302E-09 1.113665739364520E-07 2.422511344148857E-08
     5     4 -5.223528824292 44E-08 2.951370324747009E-07
     6     1 -1.501342154514 20E-08 2.450421396973857E-08 2.347633522844270E-08
     6     4 1.762122104338922E-07 -6.116258732701 11E-08 5.098075311688250E-07
-SOLUTION/MATRIX_ESTIMATE L COVA
%ENDSNX
```

Conclusion

- Definition of the Reference Point
- Design of local survey
- Implementation
- documentation