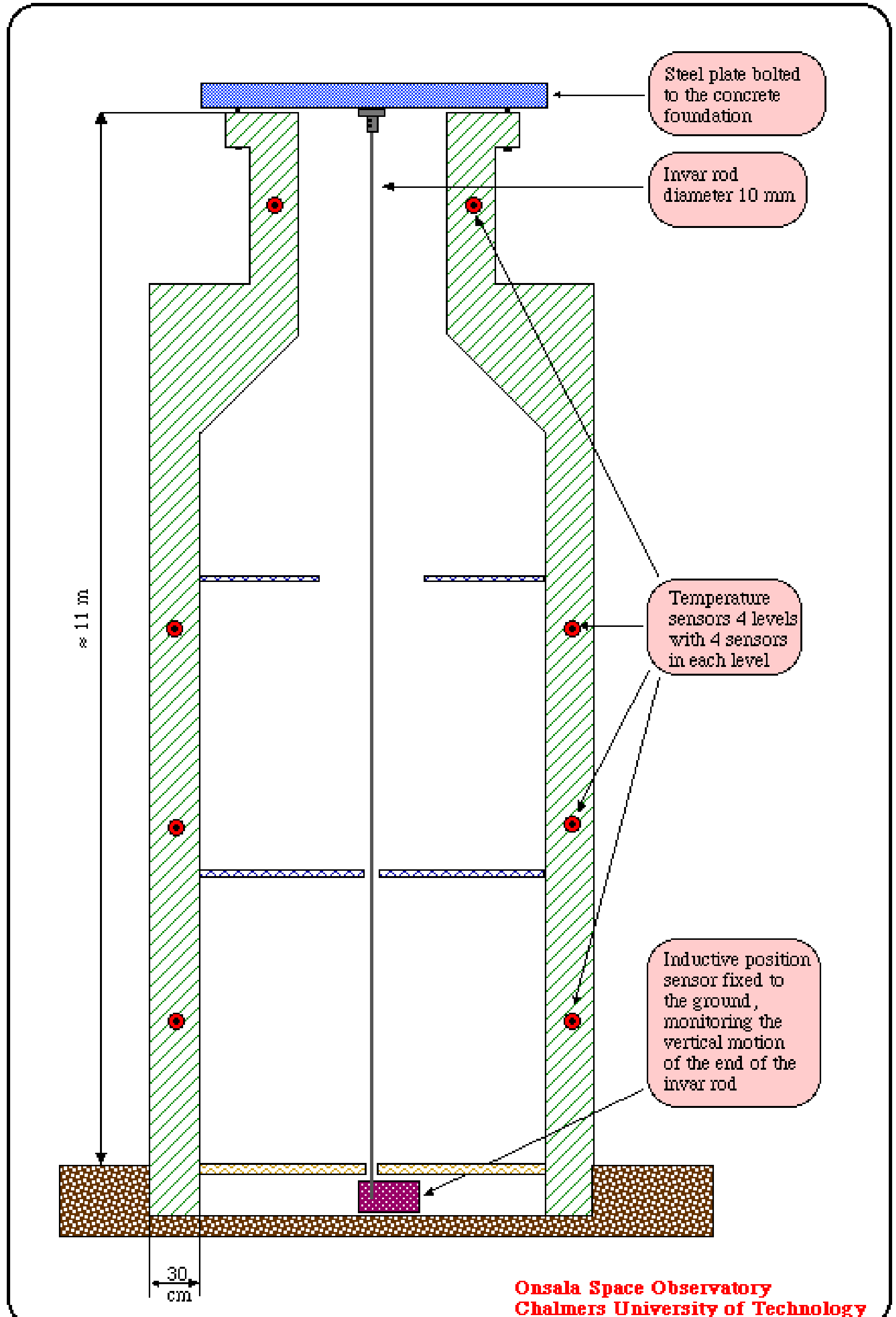
Thermal deformation of radio telescopes:

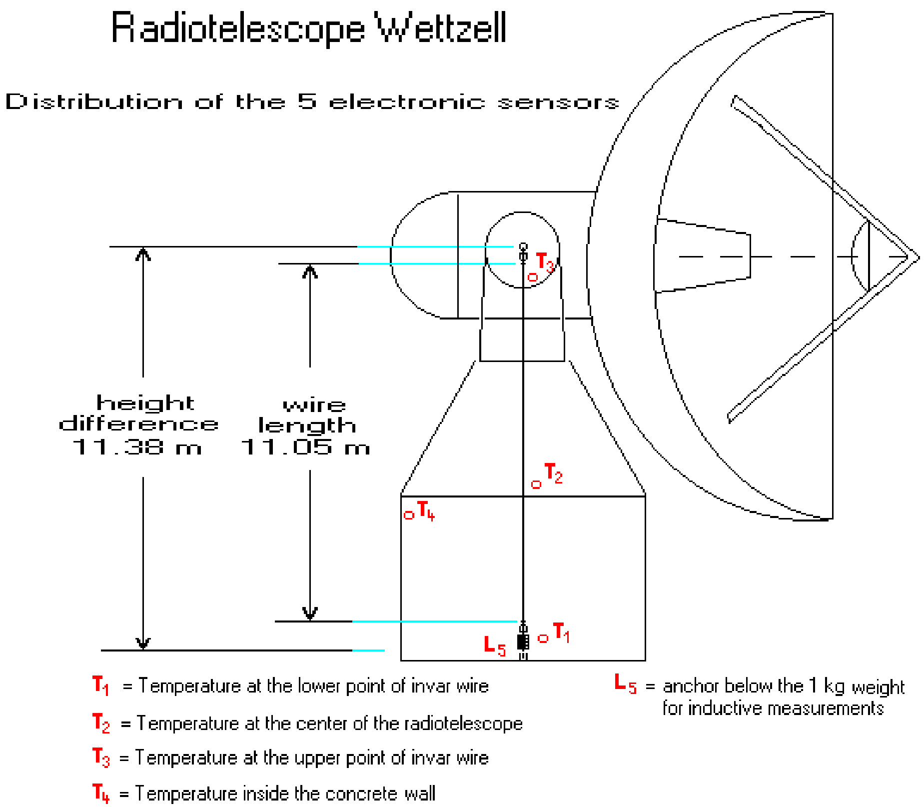
- Telescope structures expands as a function of temperature
- Reference point moves and shape of the reflector changes
- Effect on the VLBI signal path in the telescope

- Conventional model by Nothnagel (2009)
- Temperature propagation model by Wresnik et al. (2007)

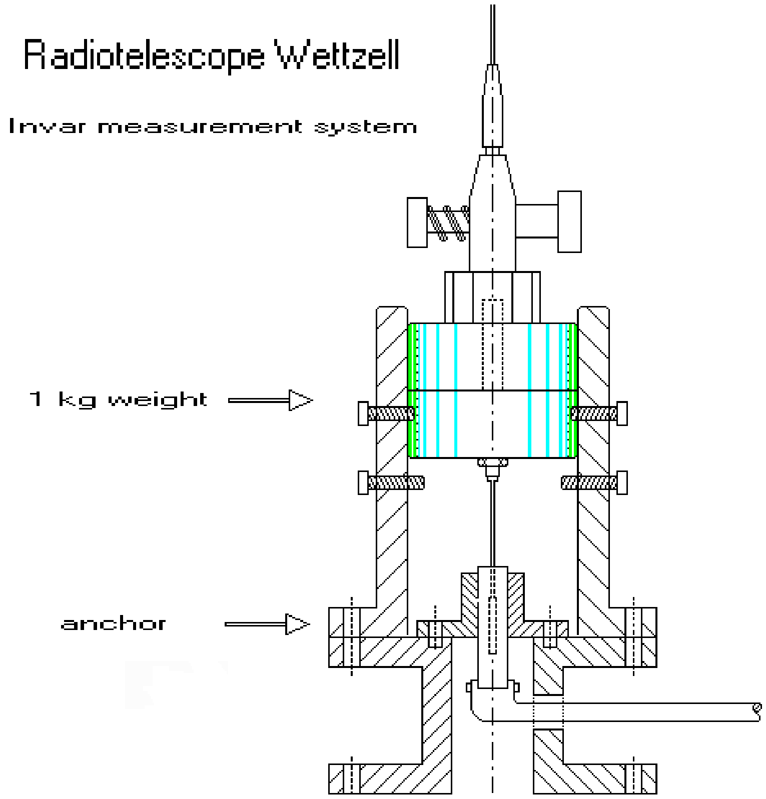
The invar-measurement system at Onsala:

- Invar rod
- Temperature sensors



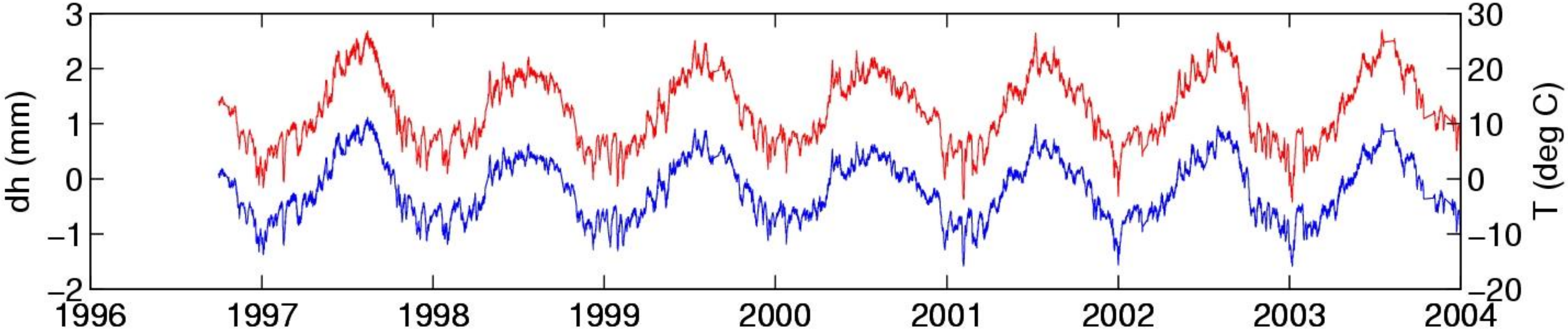


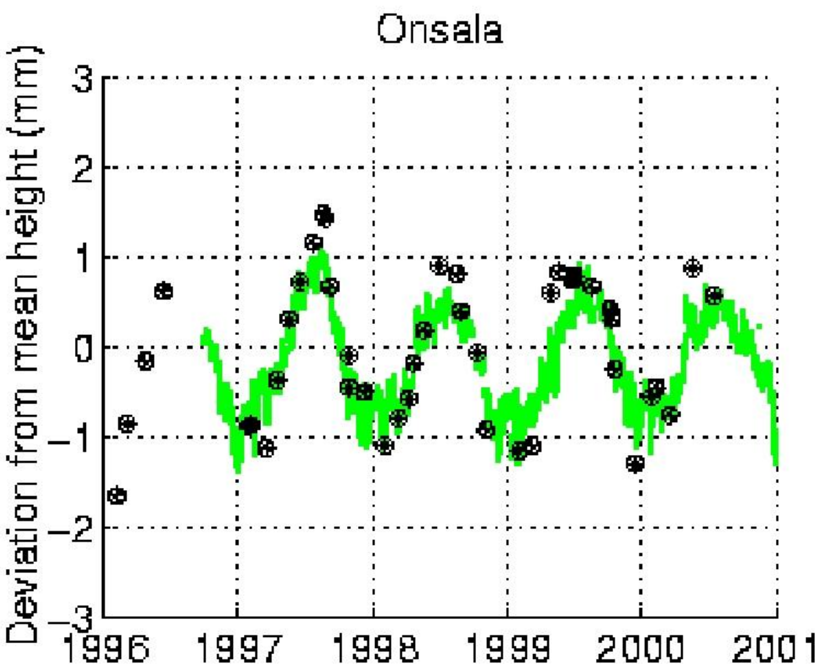
From: Zernecke (1999)

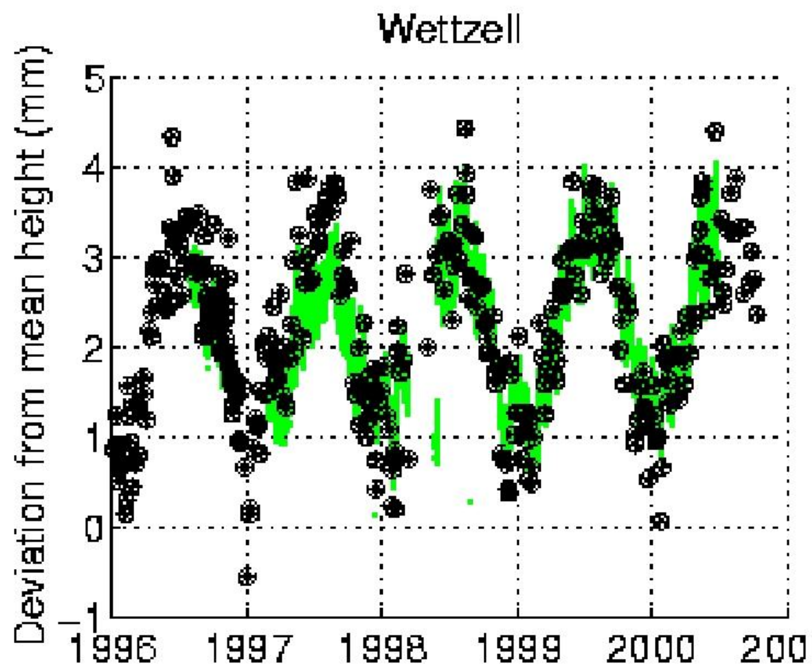


From: Zernecke (1999)

Vertical height change and mean concrete temperature at Onsala







Nothnagel, JGeod 2009

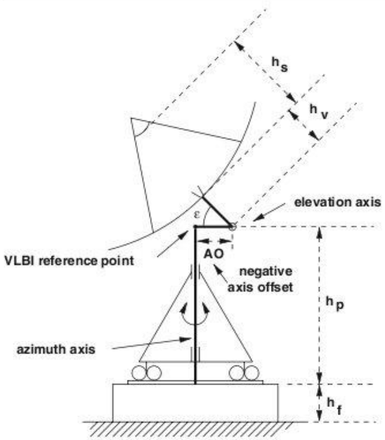


Fig. 2 Alt-azimuth telescope mount with negative axis offset

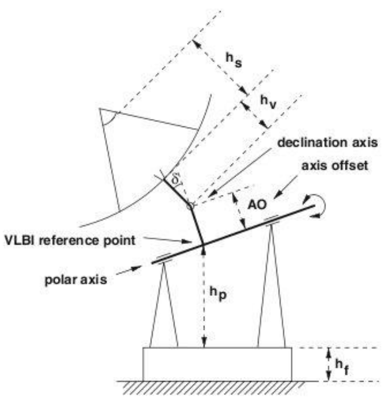


Fig. 3 Polar telescope mount

mount telescopes as well as X/Y mount antennas always have secondary axes which lie above the primary axes and, thus, these axis offsets have to be considered with a positive sign. In contrast to this, the secondary axis of an alt-azimuth telescope may lie behind the azimuth axis (Fig. 2) and in such a case the sign of the axis offset is negative.

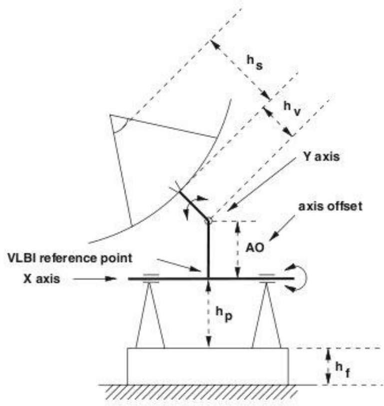


Fig. 4 X/Y telescope mount

At present, one exception exists for the general assumption of perpendicular axes which is the telescope of GARS O'Higgins, Antarctica, with the secondary axis tilted by 45°. However, since the primary and secondary axes intersect with zero axis offset, it can be modelled as a standard azimuth-elevation antenna.

Figures 1, 2, 3, and 4 serve as a reference for the description of the construction elements. The height of the concrete foundation is denoted by h_f, the height of the antenna pillar by h_p, the height of the vertex by h_v, the height of the subreflector by h_s and the axis offset between the two axes by AO. A list of antenna construction elements and other necessary parameters is maintained under http://vlbi.geod.uni-bonn.de/IVS-AC/Conventions

2.3 Delay corrections

The time delay caused by thermal expansion of the telescope's construction elements is modelled according to the following formulas which are partly collected from Nothnagel et al. (1995), Haas et al. (1999) and Skurikhina (2001). The latter contained a general sign error and a duplicate application of the axis offset for polar mounts which are corrected here.

In the equations below, c is the speed of light (m/s), γ_f and γ_a are the expansion coefficients (1/°C) for the foundation and for the antenna construction elements (mostly steel), respectively, and h_f, h_p, h_v, and h_s are the dimensions of the telescopes (m) while AO (m) is the axis offset between primary and secondary axes. δ is the declination of the radio

source being observed while azimuth and elevation of the observed radio source are denoted by α and ε. The elevation may be corrected for refractivity but the effect on the thermal expansion modelling is negligible. F_a is the antenna focus factor which reflects the effective signal path between the main reflector and the feed horn. For prime focus antennas F_a is 0.9 and for secondary focus antennas F_a is 1.8 (Otoshi and Young 1982).

The temperatures of the telescope structure elements are normally not available unless a dense set of temperature sensors is mounted. Therefore, the current model for most antennas has to be based on the surrounding air temperature denoted by T and the respective time lags for the expansion taking effect (e.g. 2 h for steel and 6h for concrete). T_0 is the reference (air) temperature with t being the observation epoch and Δt_a (antenna) and Δt_f (foundation) denoting the time lags.

Except for the one case where the elevation axis of an alt-azimuth antenna lies behind the azimuth axis (Fig. 2), the antenna axis offset brings the receiver closer to the incoming wavefront. A wavefront, thus, arrives earlier at the receiver by the axis offset contribution $\Delta\tau_{\text{axis}}$ which depends on the unit vector in source direction **s** and the unit vector in the direction of the fixed axis **f** (Skurikhina 2001, corrected):

$$\Delta\tau_{\text{axis}} = \frac{1}{c} AO \cdot \sqrt{1-(\mathbf{s}\cdot\mathbf{f})^2}. \tag{1}$$

This term changes for the different antenna mounts. With the axis offset term expressed w.r.t. azimuth, elevation or declination, the contribution of the thermal expansion effects on the time of arrival of a wavefront relative to the VLBI reference point is as follows:

(a) For alt-azimuth mounts

$$\begin{aligned}\Delta\tau_{\text{therm},i} = \frac{1}{c}\cdot\big[&\gamma_f\cdot(T(t-\Delta t_f)-T_0)\cdot(h_f\cdot\sin\varepsilon)\\ &+\gamma_a\cdot(T(t-\Delta t_a)-T_0)\cdot(h_p\cdot\sin\varepsilon\\ &+AO\cdot\cos\varepsilon+h_v-F_a\cdot h_s)\big].\end{aligned} \tag{2}$$

(b) For polar mounts:

$$\begin{aligned}\Delta\tau_{\text{therm},i} = \frac{1}{c}\cdot\big[&\gamma_f\cdot(T(t-\Delta t_f)-T_0)\cdot(h_f\cdot\sin\varepsilon)\\ &+\gamma_a\cdot(T(t-\Delta t_a)-T_0)\cdot(h_p\cdot\sin\varepsilon\\ &+AO\cdot\cos\delta+h_v-F_a\cdot h_s)\big].\end{aligned} \tag{3}$$

(c) For polar mounts with axis displaced (Skurikhina 2001):

$$\begin{aligned}\Delta\tau_{\text{therm},i} = \frac{1}{c}\cdot\big[&\gamma_f\cdot(T(t-\Delta t_f)-T_0)\cdot(h_f\cdot\sin\varepsilon)+\gamma_a\cdot(T(t-\Delta t_a)-T_0)\cdot(h_p\cdot\sin\varepsilon\\ &+AO\cdot\sqrt{1-[\sin\varepsilon\cdot\sin\phi_0+\cos\varepsilon\cdot\cos\phi_0\cdot(\cos\alpha\cdot\cos\Delta\lambda+\sin\alpha\cdot\sin\Delta\lambda)]^2}+h_v-F_a\cdot h_s)\big].\end{aligned} \tag{4}$$

with $\Delta\lambda$ being the error of the fixed axis if it is not oriented due north and ϕ_0 being the inclination of the fixed axis w.r.t. the local horizon. In the case of Richmond these angles are −0.12° and 39.06°, respectively.

(d) For X/Y mounts, primary axis north–south:

$$\begin{aligned}\Delta\tau_{\text{therm},i} = \frac{1}{c}\cdot\big[&\gamma_f\cdot(T(t-\Delta t_f)-T_0)\cdot(h_f\cdot\sin\varepsilon)\\ &+\gamma_a\cdot(T(t-\Delta t_a)-T_0)\cdot(h_p\cdot\sin\varepsilon\\ &+AO\cdot\sqrt{1-(\cos\varepsilon\cdot\cos\alpha)^2}+h_v\\ &-F_a\cdot h_s)\big].\end{aligned} \tag{5}$$

(e) For X/Y mounts, primary axis east–west

$$\begin{aligned}\Delta\tau_{\text{therm},i} = \frac{1}{c}\cdot\big[&\gamma_f\cdot(T(t-\Delta t_f)-T_0)\cdot(h_f\cdot\sin\varepsilon)\\ &+\gamma_a\cdot(T(t-\Delta t_a)-T_0)\cdot(h_p\cdot\sin\varepsilon\\ &+AO\cdot\sqrt{1-(\cos\varepsilon\cdot\sin\alpha)^2}+h_v\\ &-F_a\cdot h_s)\big].\end{aligned} \tag{6}$$

It should be noted that $\Delta\tau_{\text{therm},i}$ is the total contribution of the thermal expansion effect at one telescope. With a negative sign, it can be used as a correction for the observed time delays.

3 Consequences

The contributions of thermal expansion have to be computed independently for each VLBI delay observation and for each of the two telescopes forming a baseline. With the VLBI time delay being defined as

$$\tau = \mathcal{T}_2 - \mathcal{T}_1. \tag{7}$$

with $\mathcal{T}_1$ and $\mathcal{T}_2$ being the generalized arrival times of the signals at sites 1 and 2, respectively, the observed delay has to be corrected by subtracting $\Delta\tau_{\text{therm.baseline}}$ with

$$\Delta\tau_{\text{therm.baseline}} = \Delta\tau_{\text{therm},1} - \Delta\tau_{\text{therm},2}. \tag{8}$$

The magnitude of annual or daily variations in the topocentric height components depends primarily on the height of the fixed structures (h_f and h_p) and the amplitude of the temperature variations. The 100 m telescope of the Max Planck Institute for Radio Astronomy at Effelsberg (EFLSBERG)

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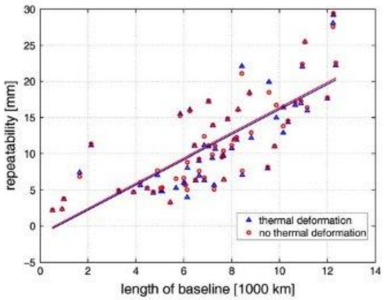

Fig. 8 Repeatabilities of the baseline lengths determined from all IVS-R1 and IVS-R4 experiments between January 2002 and August 2004. The triangles (Δ) show the repeatability with the thermal deformations of the antennas taken into account. Reference temperatures from Malkin (2002) were used for the thermal deformation model. The *circles* (o) show the repeatability without correction due to thermal deformations

5 A new model for temperature penetration into the telescope structure

The temperature T used in Eqs. (1) and (2) is the temperature of the telescope structure (Nothnagel et al. 1995). The optimum way of handling thermal deformation would be to equip all telescopes with temperature sensors in their structures to allow the direct multiplication of the temperature of the material by the expansion coefficient. Unfortunately, most VLBI stations are not equipped with such sensors, but the air temperature is recorded at all sites.

However, calculating the correction for thermal deformation from the surrounding air temperature yields imprecise corrections of the vertical displacement, because the measured air temperature does not directly correspond to the structure's temperatures of VLBI antennas. Nevertheless, as shown in the previous section, already the simple model for thermal deformation that uses the surrounding air temperature results in a slight improvement in baseline length repeatability. A further improvement can be expected when the actual temperature of the telescope structure is used instead.

Therefore, the idea to be applied in future is to use a model for temperature penetration of the air temperature into the telescope structure. The digital filter model described in Eq. (5) shown later uses the surrounding air temperature to calculate the structure temperature. Such filters are commonly applied in digital signal processing (e.g., Robinson 1980), and a first example for the thermal deformation of radio telescopes has been presented in Haas and Scherneck (2003).

The model temperature T_{m_i} at epoch i is calculated as weighted sum of the recent air temperatures T_{A}, with temperatures from earlier time-epochs, getting lower weights than temperatures close to epoch i. The weighting is performed exponentially and the weighting factor q is affected by the parameter z, which represents the memory effect of the temperature influence and the sampling rate d of the records of T_{A}.

To correspond to the different time-scales of temperature variations, this procedure is applied for short and medium terms, with the short term defined by a temperature interval of 12 h and the medium term by an interval of 3 days before epoch i. The weighting factors q_{short} and q_{med} are adjusted empirically and fixed to the values that correspond to the best-calculated standard deviation of model temperature T_{m} and the measured temperature of the antenna structure. The short and medium periodic terms are weighted differently by the factors f_{short} and f_{med}. These factors and the *offset* are calculated by least-squares adjustment (Wresnik 2005).

$$T_{\mathrm{m}\_i} = f_{\mathrm{short}} \cdot \frac{\sum_{j=i-z_{\mathrm{short}}}^{i} T_{\mathrm{A}j} \cdot q_{\mathrm{short}}^{(z_{\mathrm{short}}-(i-j)\cdot d)}}{\sum_{j=i-z_{\mathrm{short}}}^{i} q_{\mathrm{short}}^{(z_{\mathrm{short}}-(i-j)\cdot d)}} + f_{\mathrm{med}} \cdot \frac{\sum_{j=i-z_{\mathrm{med}}}^{i} T_{\mathrm{A}j} \cdot q_{\mathrm{med}}^{(z_{\mathrm{med}}-(i-j)\cdot d)}}{\sum_{j=i-z_{\mathrm{med}}}^{i} q_{\mathrm{med}}^{(z_{\mathrm{med}}-(i-j)\cdot d)}} + \text{offset} \qquad (5)$$

By applying Eq. (5) to the Onsala antenna, it was possible to model the tower temperature with an RMS of 0.82°C and the vertical deformation with an RMS of 0.07 mm. The same strategy was used for the Wettzell antenna, and the results for this antenna during the years 2001–2003 give an RMS of 1.42°C for the structure temperature and an RMS of 0.13 mm for the vertical deformation.

The difference between using the surrounding air temperature and the modeled structure temperature can be seen in Fig. 9 for the antennas in Onsala and Wettzell. The correction of vertical antenna deformation by using the structure temperature modeled according to Eq. (5) corresponds very well to the measured vertical deformation. For Onsala, the RMS agreement is 0.07 mm, and for Wettzell the RMS agreement is 0.13 mm.

Using the measured air temperature only, which is stored in the VLBI log-files, the calculated vertical deformation agrees less well with the measured deformation; the RMS agreement for Onsala is 0.42 mm, and for Wettzell the RMS agreement is 0.48 mm.

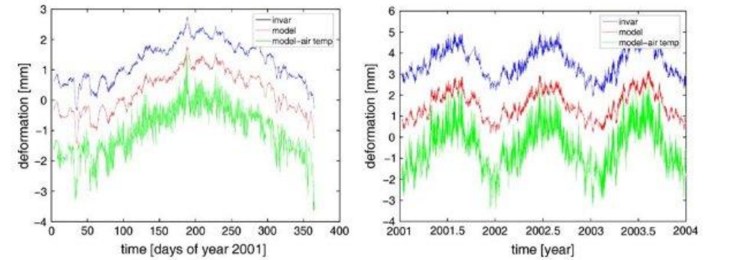

Fig. 9 Measured vertical deformation (*upper curve*), calculated antenna deformation using the air temperature to model the structure temperature according to Eq. (5) (*central curve*), and the thermal deformation using the measured air temperature directly to derive the deformation (*left* for the Onsala antenna and *right* for the Wettzell antenna). The curves are plotted with an offset of 1 mm for Onsala and 2 mm for Wettzell for better illustration

6 Conclusions

At present, the accuracy of geodetic VLBI is at the sub-centimeter level. For further improvement, the modeling of thermal deformation of the VLBI radio telescopes (which can reach a few millimeters) has to be improved. Thermal deformation can be calculated with the simple model (Nothnagel et al. 1995; Haas et al. 1999) and/or the extended version by Skurikhina (2001), but so far only some VLBI analysis groups routinely apply this model.

The improvement from using this model in VLBI analysis is of the order of 3.5% for baseline length repeatability. However, a definition of a reference temperature is necessary, preferably in the IERS Conventions (McCarthy and Petit 2004). The GPT model (Boehm et al. 2006) could be used to get homogeneous reference temperatures for old and new sites.

The ideal case would be to use the temperature of the antenna structure for the modeling of thermal deformation. However, for most of the VLBI stations, the temperature of the antenna construction is unknown, but records of nearby measured air temperature are available. An installation of temperature sensors at all antenna structures is highly desirable, but might take time and does not allow the treatment of historic VLBI data.

Therefore, a temperature penetration model was developed in order to determine the antenna structure temperature as a function of surrounding air temperature. The model was tested for two telescopes that are equipped with thermal deformation monitoring systems, the Onsala antenna located in a radome and the open-air Wettzell antenna. This new temperature penetration model leads to significant improvements in RMS agreement between modeled and measured thermal deformation and provides RMS agreements of the order of 0.10 mm.

It has to be investigated whether this model (Eq. 5) can be applied to other stations to model the temperatures of the telescope structures. The measured air temperatures of all stations must be available and updated frequently. Then, time-series of structure temperatures for each station can be determined and used in the VLBI data analysis. A preferable alternative would be to install temperature sensors at all antenna structures, which would provide directly the structure temperature of the antennas and ensure the best possible thermal deformation correction.

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	Onsala	Wettzell
Using outside temperature	RMS: 0.42 mm	RMS: 0.48 mm
Using advanced temp. model	RMS: 0.82 C, 0.07 mm	RMS: 1.42 C, 0.13 mm

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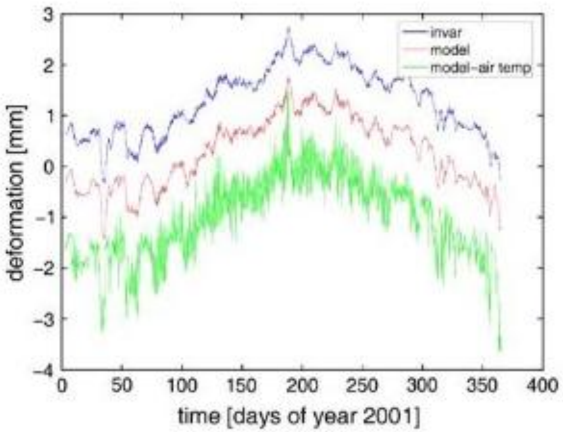

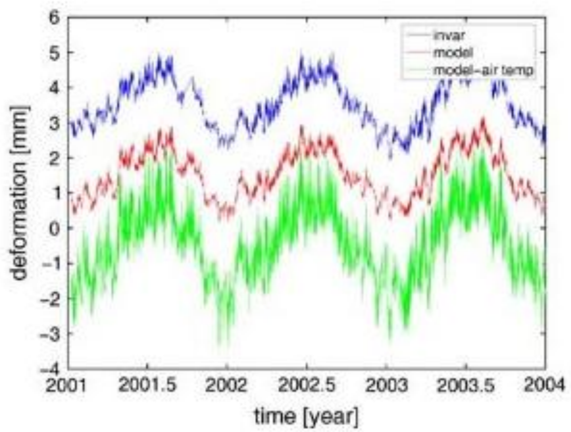

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Thermal deformation of radio telescopes:

- Thermal deformation model exists and is used in VLBI analysis
- Only few telescopes with invar and temperature monitoring devices exist (Onsala, Wettzell)
- Question: What temperatures to use ?
  - Logfiles usually give outside air temperature, RMS-agreement < 0.5 mm can be achieved compared to invar measurements
  - Wresnik-model for temperature propagation from outside temperature to temperature of telescope structure, gives RMS agreement ~0.1 mm (Adapt model to further sites?)
  - Reference temperature GPT (Böhm et al., 2007)