

Tropospheric ties

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Outline

Radio wavelength techniques, such as VLBI, GPS, and DORIS can provide

the same type of tropospheric parameters:

- tropospheric zenith delays**
- N-S and E-W gradients**

**Are there systematic differences?
How to combine?**

Homogeneously processed data (e.g. GGOS-D) provide the best basis for the assessment of such systematics.

Tropospheric model

Tropospheric delay in line of sight $L=L(\text{elev},a)$:

$$L = \textcolor{red}{mf}_h \textcolor{violet}{ZHD} + \textcolor{blue}{mf}_w \textcolor{blue}{ZWD} + \textcolor{green}{mf}_g [\textcolor{green}{G}_n \cos(a) + \textcolor{green}{G}_e \sin(a)]$$

„hydrostatic“

„wet“

„azimuthal asymmetry“

1) Mapping functions

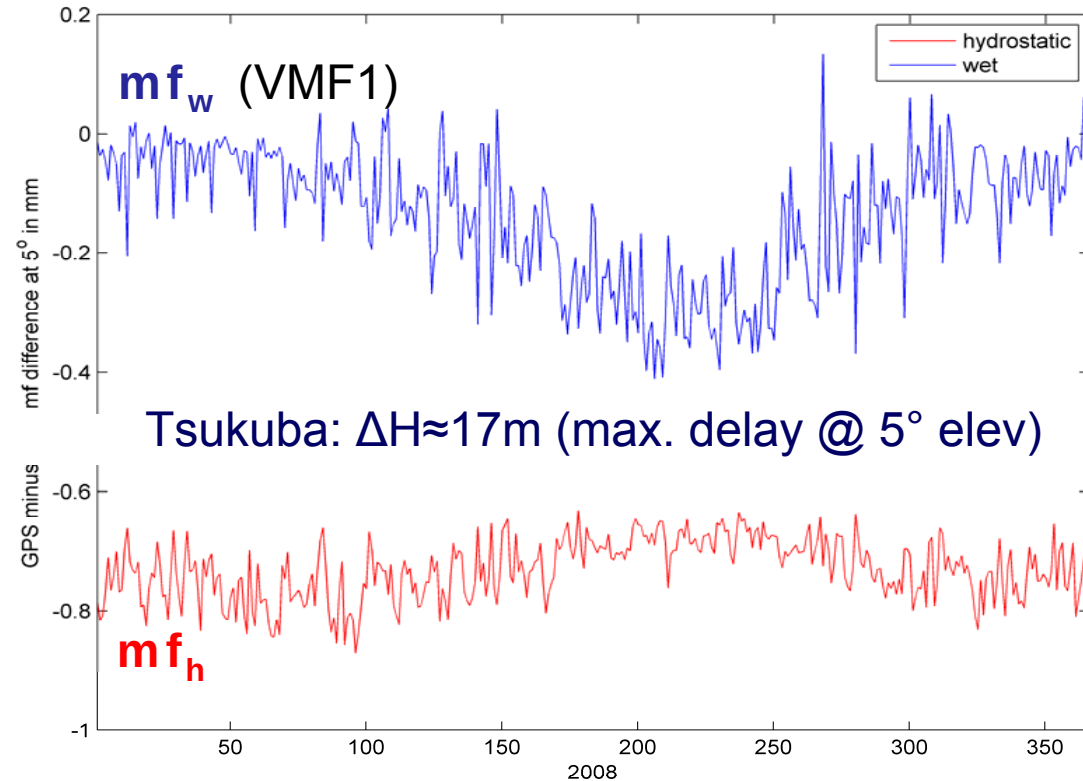
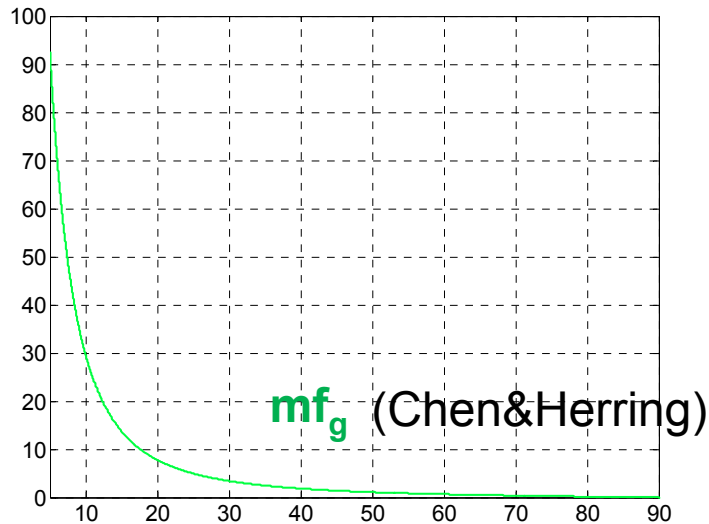
- Hydrostatic $\textcolor{red}{mf}_h$: $\Delta H \Rightarrow \textcolor{red}{mf}_h$ identical?
- Wet $\textcolor{blue}{mf}_w$: $\Delta H \Rightarrow \textcolor{blue}{mf}_w$ identical?
- Gradient $\textcolor{green}{mf}_g$: $\Delta H \Rightarrow \textcolor{green}{mf}_g$ identical?

2) Delays

- Hydrostatic $\textcolor{violet}{ZHD}$: $ZHD = ZHD(H)?$
- Wet $\textcolor{blue}{ZWD}$: $ZWD = ZWD(H)?$
- Gradients $\textcolor{green}{G}_N$ and $\textcolor{green}{G}_E$: $G=G(H)?$

Mapping functions: $mf = mf(H)$?

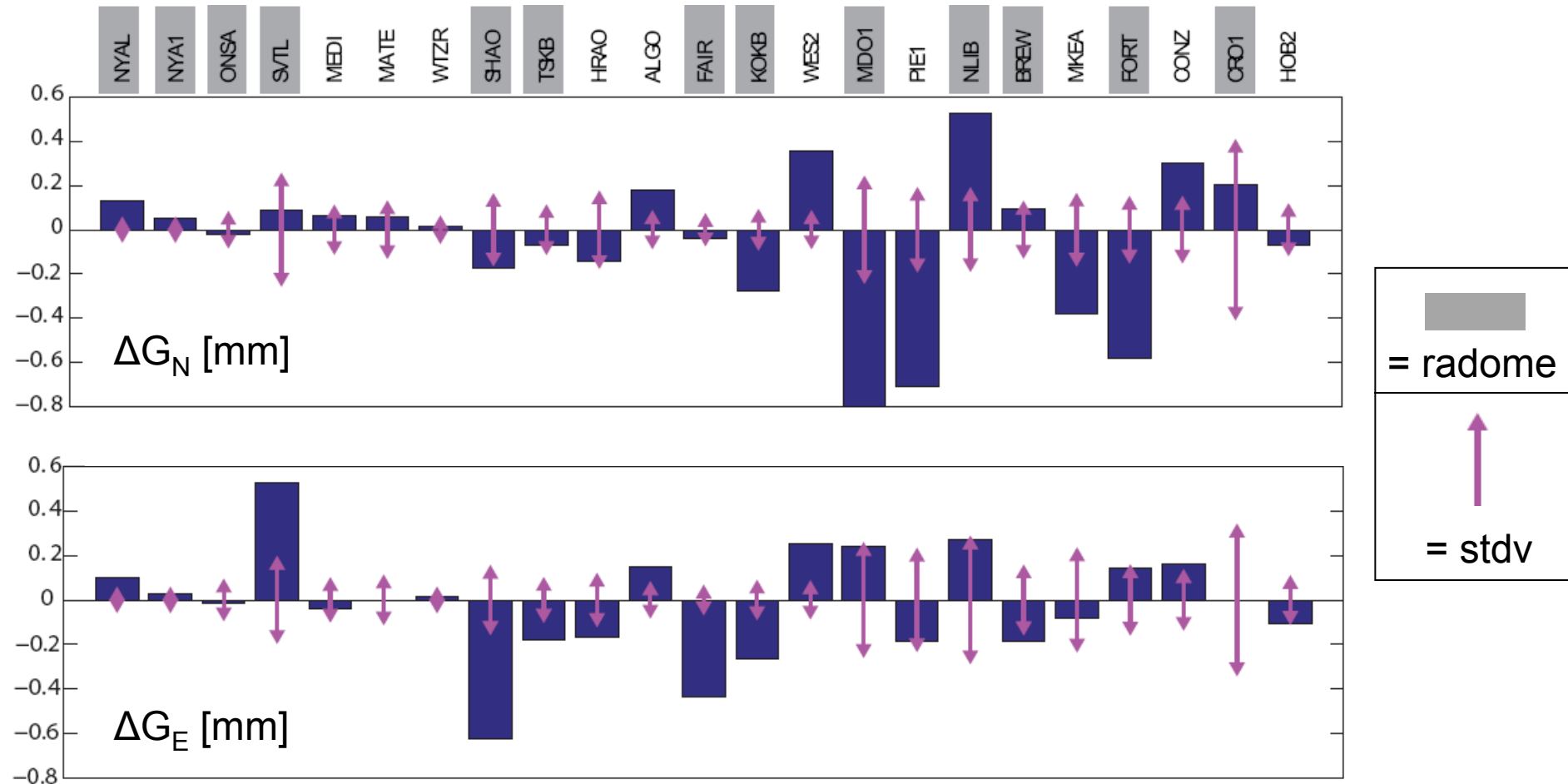
1) Mapping functions



Additional delays due to $mf(H)$

- stay small, if $\Delta H < \sim 100m$
- are repeatable $\Rightarrow$ can be corrected, if necessary

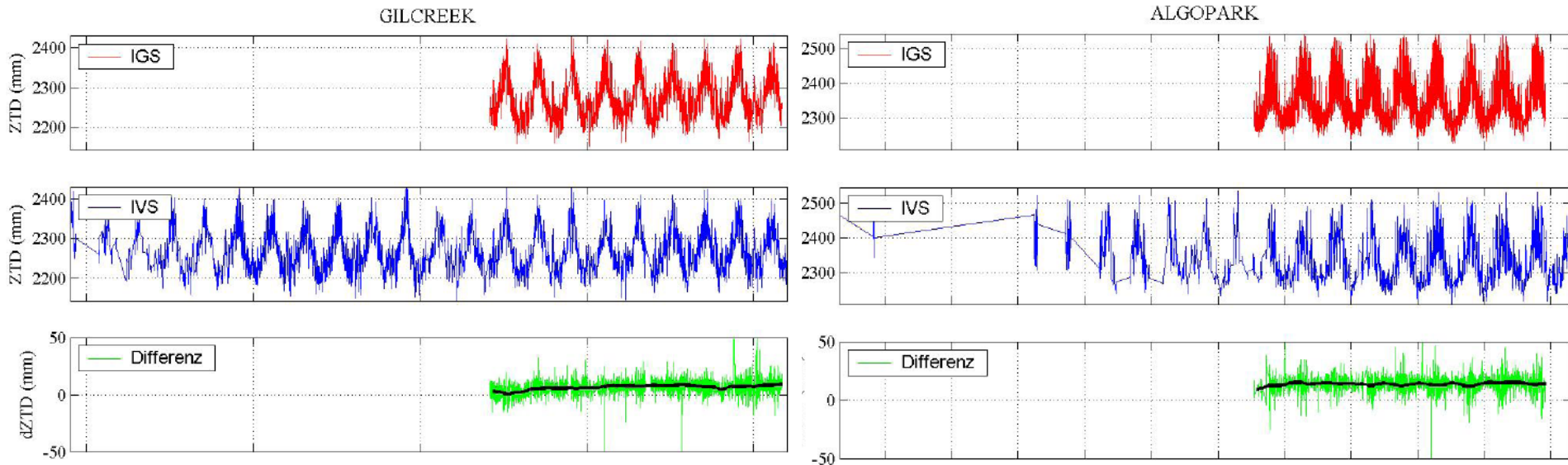
Tropospheric gradients



- GPS and VLBI gradients agree within 0.8 mm

Zenith total delay ZTD

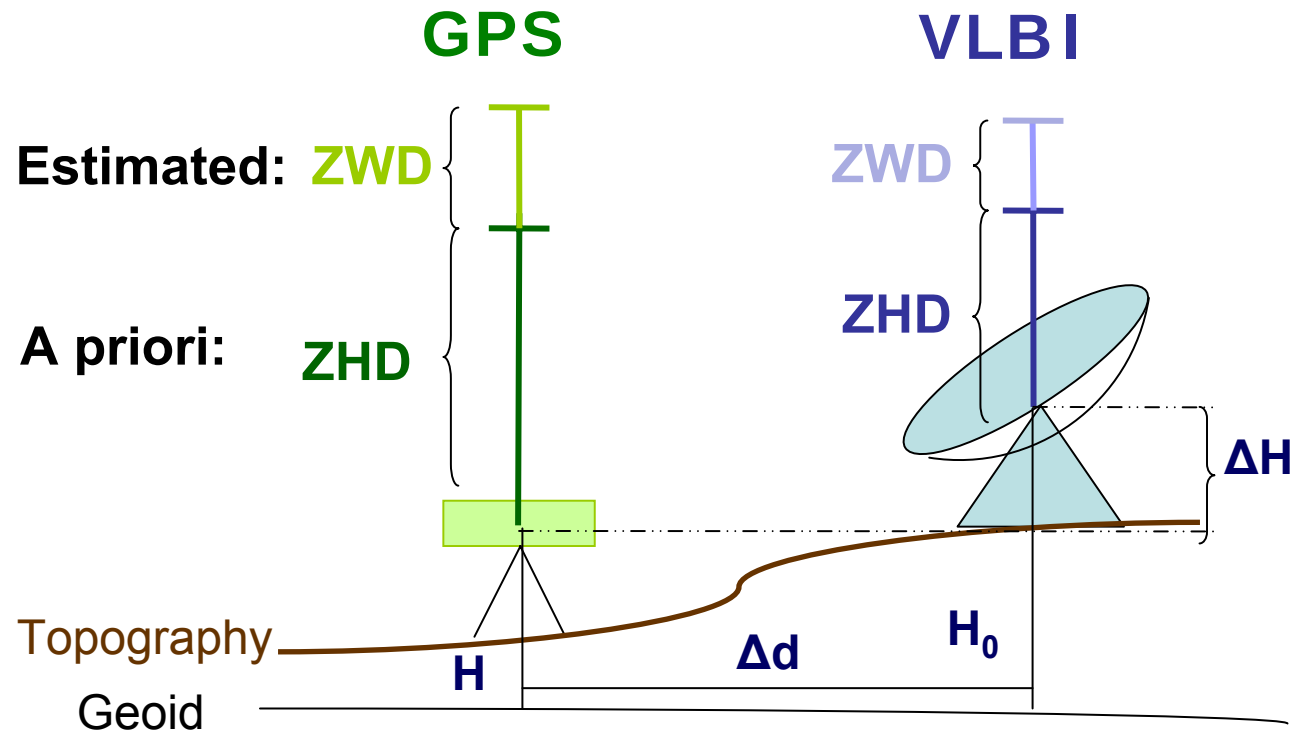
Zenith total delays ($ZTD = ZHD + ZWD$)



- GPS and VLBI ZTD show a very good agreement
- but also significant offsets
- offsets are not constant over time

Tropospheric ties

Example: colocated GPS & VLBI



Vertical distance

$$\Delta H = H_0 - H$$

= >

$$\text{ZHD} > \text{ZHD}$$

$$\text{ZWD} > \text{ZWD}$$

Horizontal distance

$$\Delta d = > ?$$

Height difference (VLBI-GPS)

Site	GPS	VLBI	ΔH (m)	Site	GPS	VLBI	ΔH (m)
Ny Alesund	NYAL	7331	8.83	Algonquin P.	ALGO	7282	23.11
	NYA1	7331	3.10	Fairbanks	FAIR	7225	13.09
Onsala	ONSA	7213	13.71	Kokee	KOKB	7298	9.23
Svetloe	SVTL	7380	9.36	Westford	WES2	7209	1.75
Medicina	MEDI	7230	17.15	Fort Davis	MDO1	7613	-398.08
Noto	NOT1	7547	16.89	Pietown	PIE1	7234	16.95
Matera	MATE	7243	7.72	North Liberty	NLIB	7612	15.22
Wettzell	WTZR	7224	3.10	Brewster	BREW	7614	11.88
Urumqi	URUM	7330	1174.3	Fortaleza	FORT	7297	3.63
Tsukuba	TSKB	7345	17.37	Concepcion	CONZ	7640	-9.76
Hartebeesth.	HRAO	7232	1.54	Hobart	HOB2	7242	24.03

still comparable?

Horizontal distance (VLBI-GPS)

Site	GPS	VLBI	Δd (m)	Site	GPS	VLBI	Δd (m)
Ny Alesund	NYAL	7331		Algonquin P.	ALGO	7282	89.8
	NYA1	7331	97.5	Fairbanks	FAIR	7225	
Onsala	ONSA	7213	65.9	Kokee	KOKB	7298	
Svetloe	SVTL	7380		Westford	WES2	7209	
Medicina	MEDI	7230	45.6	Fort Davis	MDO1	7613	8020
Noto	NOT1	7547	54.8	Pietown	PIE1	7234	18
Matera	MATE	7243	60.9	North Liberty	NLIB	7612	
Wettzell	WTZR	7224	136.3	Brewster	BREW	7614	
Urumqi	URUM	7330		Fortaleza	FORT	7297	24.5
Tsukuba	TSKB	7345	285.7	Conception	CONZ	7640	
Hartebeesth.	HRAO	7232	162.3	Hobart	HOB2	7242	

still comparable?

Tie1, ZHD

- Davis et al. (1985)

$$\mathbf{ZHD = ZHD(p)}$$

(based on Saastamoinen)

$$ZHD = \frac{(0.0022768 \pm 0.00000005) \cdot p}{1 - 0.00266 \cdot \cos(2\phi) - 0.00028 \cdot H}$$

p = p(H) significant

$\downarrow$ $\phi \neq \phi(H)$
 $\downarrow$ H [km] negligible

$$p = p_0 \cdot \left(1 - \frac{\gamma(H - H_0)}{T_0} \right)^{\frac{g}{R_L \gamma}}$$

$\gamma = -0.0065 \text{ K} \cdot \text{m}^{-1}$ average lapse rate
 $R_L \approx 287.058 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ specific gas constant

Tie 1: **ΔZHD due to air pressure difference** by analytical equation :

$$\Delta ZHD = ZHD(p_0) - ZHD(p) \quad \checkmark$$

Tie1, ZWD

$$\mathbf{ZWD = ZWD(e,T)}$$

Tie 1: **$\Delta\mathbf{ZWD}$** through water vapor pressure and temperature difference, e.g. :

- Saastamoinen (1972, 1973):
$$\mathbf{ZWD = \frac{0.0022768 \cdot e \cdot (1255/T + 0.05)}{1 - 0.00266 \cdot \cos(2\phi) - 0.00028 \cdot H}}$$
$$\mathbf{\Delta ZWD = ZWD(e_0, T_0) - ZWD(e, T)}$$

Tie 1: **$\Delta\mathbf{ZWD}$** through height difference (direct), e.g. :

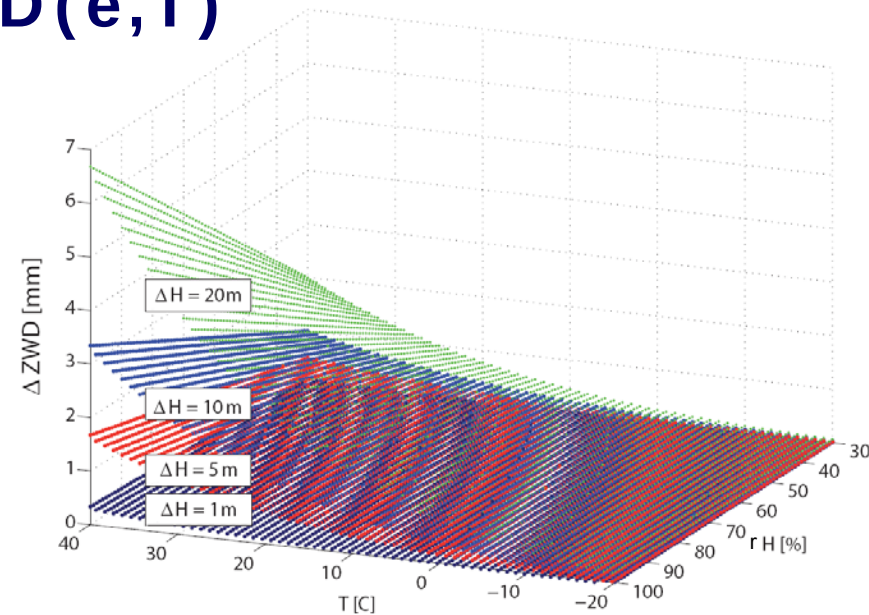
- Brunner & Rüeger (1992):
(based on Saastamoinen)
$$\mathbf{\Delta ZWD = \frac{-2.789 \cdot e}{T^2} \left(\frac{5383}{T} - 0.7803 \right) \gamma (H_0 - H)}$$

Tie1, ZWD

$$\mathbf{ZWD} = \mathbf{ZWD}(e,T)$$

- Brunner & Rüeger (1992):

$$\Delta\text{ZWD} = \frac{-2.789 \cdot e}{T^2} \left(\frac{5383}{T} - 0.7803 \right) \gamma (H_0 - H)$$

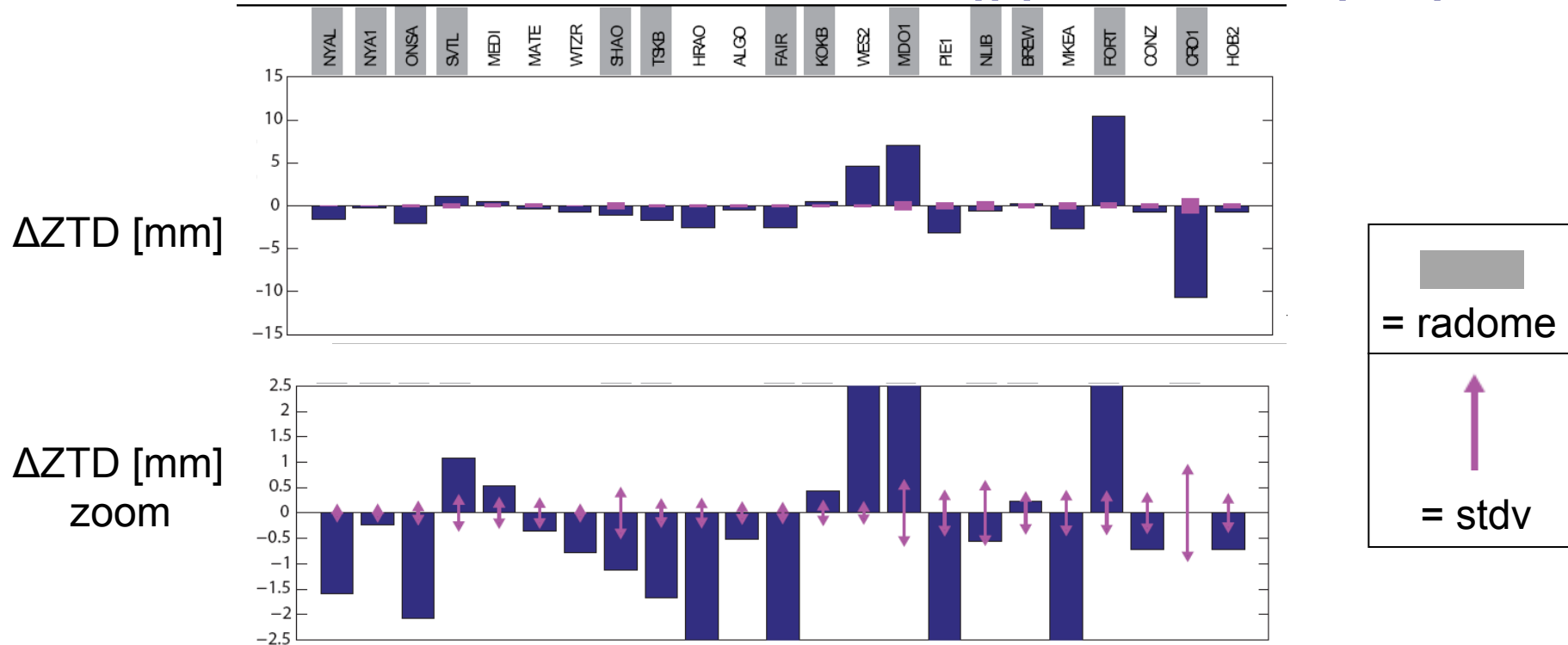


Tsukuba: $\Delta H \approx 17\text{m}$ (using in situ meteorology)



Remaining systematics

- ZTD offsets after correction of $\Delta ZHD(p)$ and $\Delta ZWD(e,T)$!



- Further unconsidered systematics present:
horizontal distance, gradients, radomes, mapping functions,
different spatio-temporal sampling

Tie2

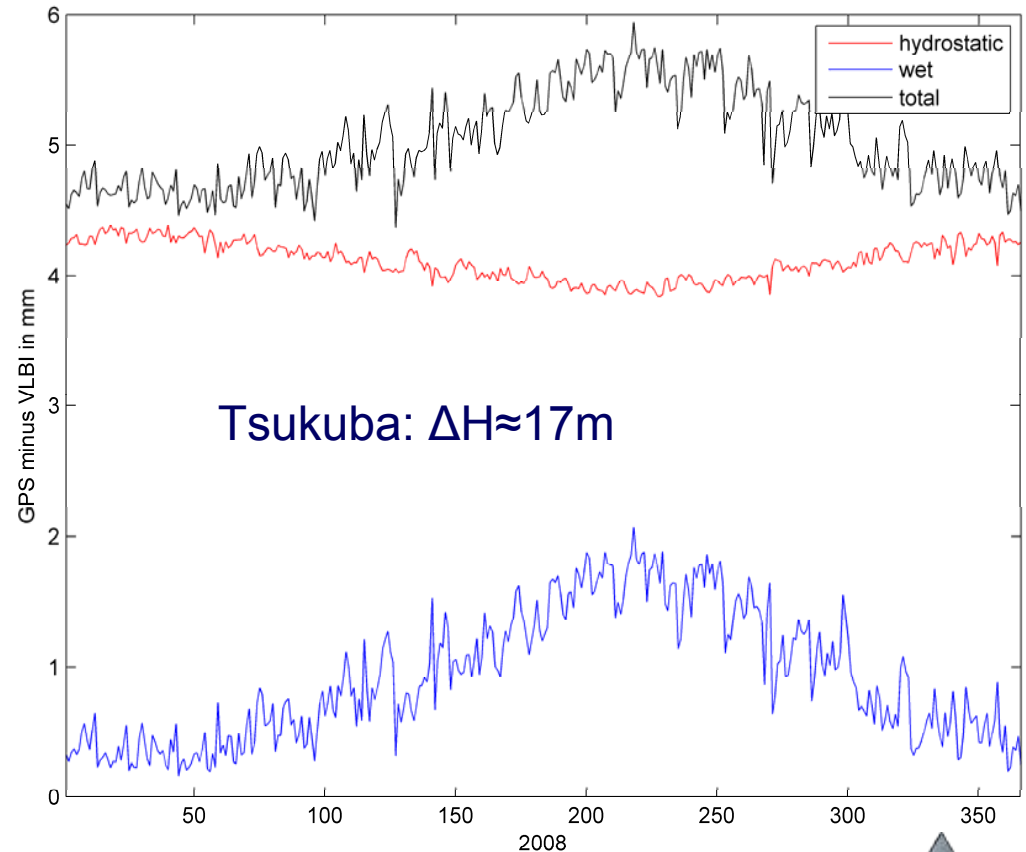
Tie 2: **Numerical integration** through a weather model, e.g. ECMWF:

$$\Delta ZTD = \Delta ZHD + \Delta ZWD$$

$$\Delta ZHD = 10^{-6} \sum_{h=H}^{H_0} \left[k_1 \frac{p(h)}{T(h)} \right] \cdot Z_d^{-1} \delta h$$

$$\begin{array}{c} \downarrow \\ H_0 = H(\text{VLBI}) \\ H = H(\text{GPS}) \\ \uparrow \end{array}$$

$$\Delta ZWD = 10^{-6} \sum_{h=H}^{H_0} \left[k'_2 \frac{e(h)}{T(h)} + k_3 \frac{e(h)}{T^2(h)} \right] \cdot Z_w^{-1} \delta h$$



Summary and conclusion

- **very good agreement** in temporal behaviour of tropospheric parameters estimated from GPS and VLBI
- **tropospheric delays and gradients** still show some small offsets, which have to be explained
- **equations** (Saastamoinen, Brunner) and **numerical integration** (ECMWF) have to be tested more extensively
- **$m_f(\Delta H)$** and their temporal behaviour have to be studied in order to provide consistent tropospheric ties
- **remaining inter-technique systematics** (radomes, etc.) have to be identified and studied

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