# A Unified Approach to Modeling the Effects of Earthquakes on the Three Pillars of Geodesy

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# Forcing Mechanisms

- External
  - Tidal potential
    - Body, ocean, atmosphere
- Surficial
  - Atmospheric winds and surface pressure
  - Oceanic currents and bottom pressure
  - Water stored on land (liquid, snow, ice)
- Internal
  - Earthquakes and tectonic motions
  - Glacial Isostatic Adjustment
  - Mantle convection
  - Outer core dynamics / coupling with mantle
  - Inner core dynamics / coupling with outer core and mantle

# Geodetic Measures of Earth's Response

- Rotation
  - Angular momentum exchange (surface fluids, outer core)
  - Torques (external, surficial, internal)
  - Changes in inertia tensor (earthquakes, GIA)
- Gravity and geocenter
  - Mass distribution (static field)
  - Mass redistribution (time varying field)
  - Surface displacements (ground-based)
- Shape
  - Tidal displacements
  - Surface loading and unloading
  - Internal deformation
    - Earthquakes, core pressure => mantle deformation

# Introduction

- The three pillars of geodesy
  - Geometric shape, rotation, and gravity
    - Static and time varying
- Are related by common observing systems
  - Examples: SLR, GNSS, DORIS, VLBI (shape and rotation)
- Are related by common sources of excitation
  - Examples: atmosphere, oceans, hydrology, earthquakes, GIA
- But are sometimes modeled separately
  - Example: flat Earth models for earthquake displacements, spherical Earth models for rotation
- A unified modeling approach
  - Allows excitation process to be studied using all geodetic data
  - Example: earthquakes

# Geodetic Effects of Earthquakes

- Flat Earth models
  - Commonly used to model site displacements
  - May or may not include effects of layering
  - Do not include effects of sphericity
    - Important for great earthquakes having rupture lengths of 1000 km (10°) or more
- Spherical Earth models
  - Commonly used for rotation effects
  - Often used for gravity effects
- A unified mode sum approach
  - Based on a realistic Earth model (PREM)
  - Automatically accounts for effects of sphericity, layering, self-gravitation

# Site Displacements

- Equation of motion

$$\nabla \cdot \tau + \mathbf{f_g} + \mathbf{f_s} = \rho(\mathbf{r}) \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

- Solve by expanding displacement field

$$\mathbf{u}(\mathbf{r},t) = \sum_k a_k(t)\, \mathbf{u}_k^*(\mathbf{r})$$

  - Normal mode eigenfunctions

$$\mathbf{u}_k(\mathbf{r}) = {}_nU_l(r)\, Y_{lm}(\theta,\lambda)\, \hat{\mathbf{r}} + {}_nV_l(r)\, \frac{\partial Y_{lm}}{\partial \theta}\, \hat{\boldsymbol{\theta}} + {}_nV_l(r)\, \frac{1}{\sin\theta}\, \frac{\partial Y_{lm}}{\partial \lambda}\, \hat{\boldsymbol{\lambda}}$$

  - Expansion coefficients (static limit)

$$a_k(\infty) = \frac{M_o}{\omega_k^2}\, \hat{\mathbf{M}} : \boldsymbol{\varepsilon}_k(\mathbf{r_s})$$

- Normal mode eigenfrequencies and eigenfunctions
  - Computed using MINOS program provided by Guy Masters

# Earth Rotation

- Conservation of angular momentum
  - Earthquakes are internal to the Earth (no load Love numbers)
  - Earthquakes occur suddenly (motion effects neglected)
- Length of day

$$\frac{\Delta\Lambda(t)}{\Lambda(t)} = \frac{\Delta I_{zz}(t)}{I_{zz}^{(m)}}$$

- Polar motion excitation

$$\Delta\chi_x(t) + i\,\Delta\chi_y(t) = \frac{1.61}{I_{zz} - I_{xx}}\left[\Delta I_{xz}(t) + i\,\Delta I_{yz}(t)\right]$$

- Inertia tensor

$$\mathbf{I} = \int \rho(\mathbf{r})\left[(\mathbf{r}\cdot\mathbf{r})\,\mathbf{I} - \mathbf{r}\,\mathbf{r}\right]dV$$

- Perturbed inertia tensor $(\mathbf{r} \Rightarrow \mathbf{r} + \mathbf{u})$

$$\Delta\mathbf{I} = \int \rho_o(\mathbf{r})\left[2\,(\mathbf{r}\cdot\mathbf{u})\,\mathbf{I} - (\mathbf{u}\,\mathbf{r} + \mathbf{r}\,\mathbf{u})\right]dV$$

# Gravitational Field

- Gravitational potential of Earth

$$U(\mathbf{r}) = \frac{GM}{r} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l} \left[C_{lm} \cos m\lambda + S_{lm} \sin m\lambda\right] \tilde{P}_{lm}(\cos\theta)$$

- Stokes coefficients

$$C_{lm} + i\, S_{lm} = \frac{N_{lm}}{Ma^{l}} \int_{V_o} r^{l}\, Y_{lm}(\theta,\lambda)\, \rho(\mathbf{r})\, dV$$

- Perturbed Stokes coefficients $(\mathbf{r} \;=>\; \mathbf{r} + \mathbf{u})$

$$\Delta C_{lm} + i\, \Delta S_{lm} = \frac{N_{lm}}{Ma^{l}} \int_{V_o} r^{l-1}\, \mathbf{u} \cdot (\hat{\mathbf{r}}\, l + \nabla_h) Y_{lm}(\theta,\lambda)\, \rho(\mathbf{r})\, dV$$

# Earth Rotation & Gravitational Field

- Related via the inertia tensor
  - Elements of inertia tensor are related to the degree-2 Stokes coefficients

$$I_{xz} = -\sqrt{5/3}\, M a^2\, C_{21}$$

$$I_{yz} = -\sqrt{5/3}\, M a^2\, S_{21}$$

$$I_{zz} = \frac{1}{3}\left[T - \sqrt{20}\, M a^2\, C_{20}\right]$$

- Trace of inertia tensor

$$T = 2\int \rho(\mathbf{r})\, r^2\, dV$$

- Perturbed trace of inertia tensor $(\mathbf{r} \;\Rightarrow\; \mathbf{r} + \mathbf{u})$

$$\Delta T = 4\int \rho_o(\mathbf{r})\, \mathbf{r}\cdot\mathbf{u}\, dV$$

# 2004 Sumatran Earthquake

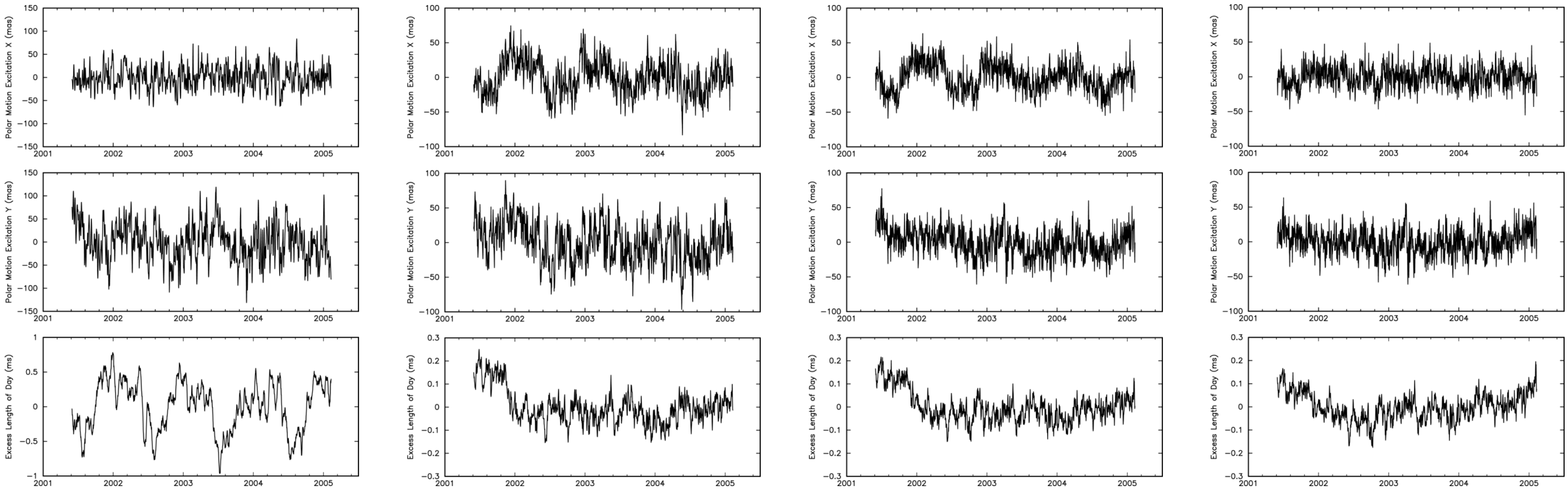
- Earth model
  - Preliminary Reference Earth Model (PREM)
    - Spherically symmetric, non-rotating, anelastic, transversely isotropic
- Source properties
  - Standard Harvard CMT solution (surface wave data)
    - Point source located at 3°N, dipping 8° towards 59°E
    - Moment magnitude $M_w = 9.0$ (scalar moment $M_o = 3.95 \times 10^{29}$ dyne-cm)
  - Free oscillation data (Stein & Okal, *Nature*, 2005)
    - Moment magnitude $M_w = 9.3$ (scalar moment $M_o = 10.0 \times 10^{29}$ dyne-cm)
    - Centroid of source located at 7°N
  - Multiple source Harvard CMT solution (Tsai *et al.*, *GRL*, 2005)
    - 5 sub-events distributed in space (3°N – 13°N) and time
    - Total moment magnitude $M_w = 9.3$ (scalar moment $M_o = 11.66 \times 10^{29}$ dyne-cm)

# Modeled Change in Earth Rotation

| | $\Delta lod$ ($\mu$sec) | $\Delta\chi_x$ (mas) | $\Delta\chi_y$ (mas) |
|---|---|---|---|
| *2004 Sumatran earthquake* | | | |
| $M_w$ = 9.0, 3°N (standard CMT solution) | –2.70 | –0.67 | 0.50 |
| $M_w$ = 9.3, 3°N (larger moment) | –6.84 | –1.70 | 1.27 |
| $M_w$ = 9.3, 7°N (larger moment, centroid to N) | –6.59 | –1.80 | 2.20 |
| $M_w$ = 9.3, 3°N – 13°N (Tsai *et al.*, 2005) | –6.77 | –1.41 | 1.85 |
| *Approximate measurement uncertainty (1$\sigma$)* | 20 | 5 | 5 |
| *Other great earthquakes* | | | |
| 1960 Chilean (Chao & Gross, 1987) | –8.40 | –9.53 | 20.45 |
| 1964 Alaskan (Chao & Gross, 1987) | 6.79 | –7.11 | –2.31 |

# Observed Residual Excitation
## (2001.4 – 2005.1)

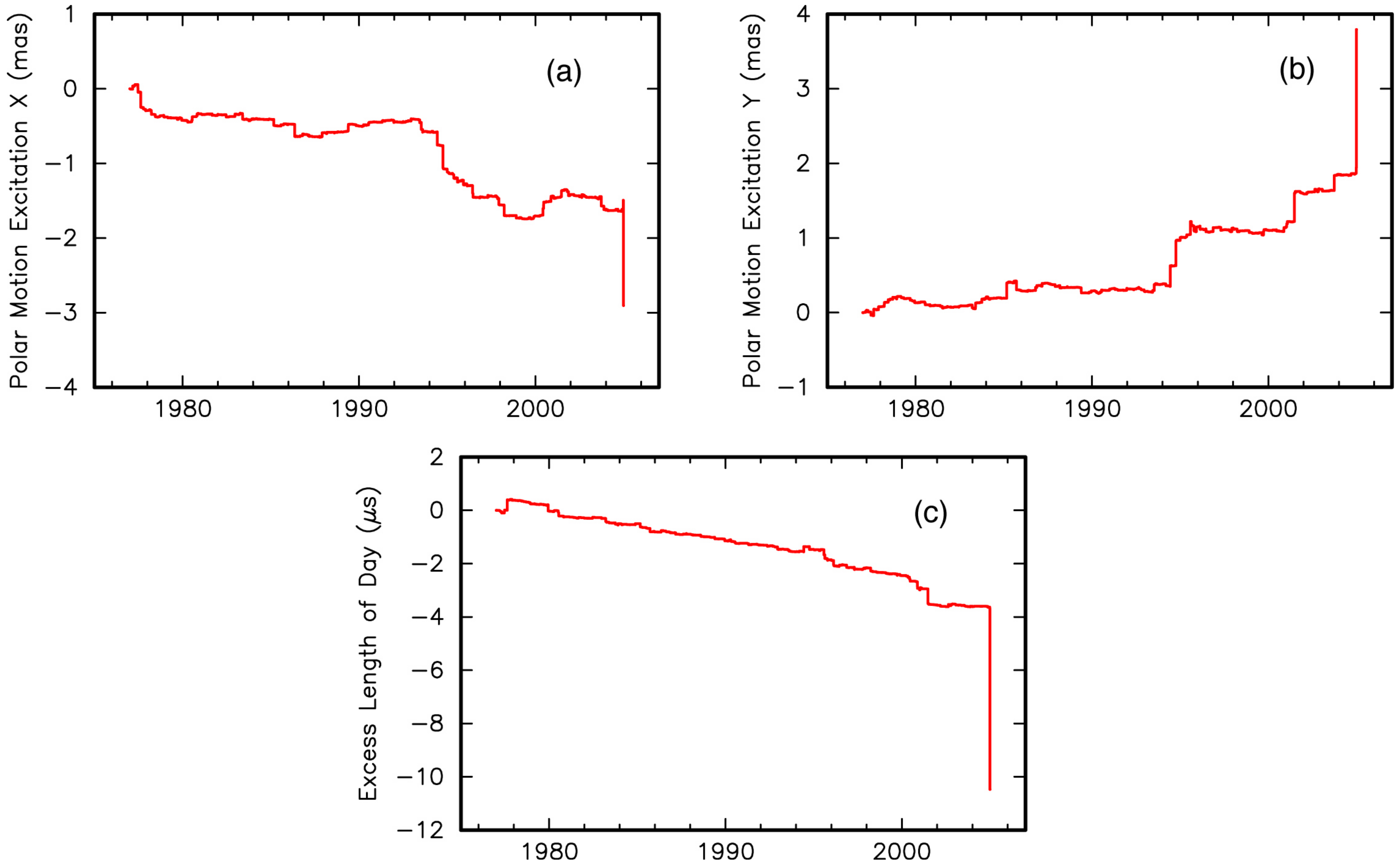
| observed | obs-atm | obs-atm-ocn | nonseasonal obs-atm-ocn |
|---|---|---|---|




## Variance (percent reduction)

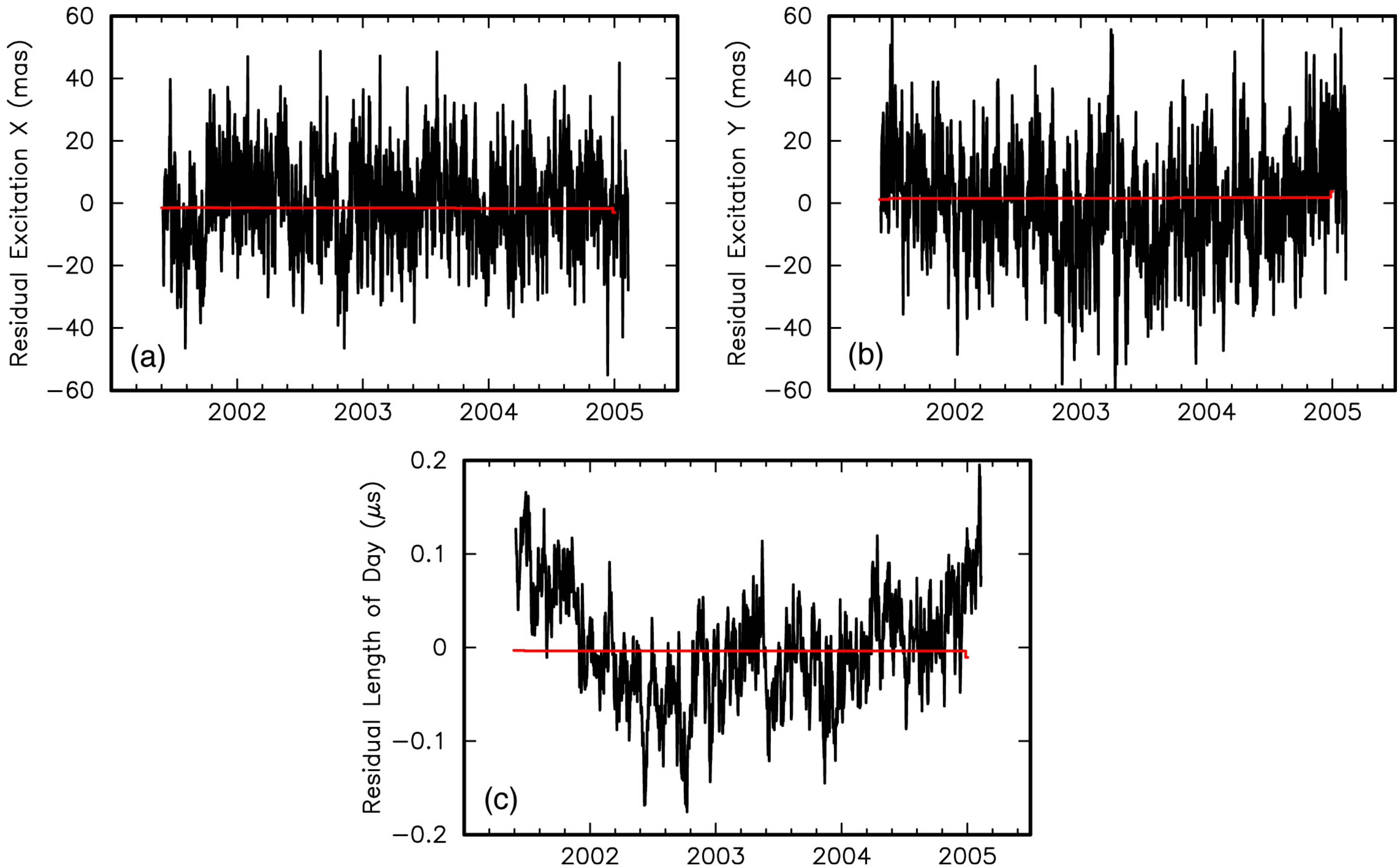
| | observed | obs-atm | obs-atm-ocn | nonseasonal obs-atm-ocn |
|---|---|---|---|---|
| chi-x: | 554 $mas^2$ | 570 $mas^2$ ( -3%) | 405 $mas^2$ (27%) | 232 $mas^2$ (58%) |
| chi-y: | 1611 $mas^2$ | 863 $mas^2$ (46%) | 412 $mas^2$ (74%) | 340 $mas^2$ (79%) |
| lod: | 0.127 $ms^2$ | 0.0055 $ms^2$ (95.7%) | 0.0045 $ms^2$ (96.5%) | 0.0033 $ms^2$ (97.4%) |

# Modeled Change in Earth Rotation




(Gross & Chao, 2006)

# Observed and Modeled Change in Rotation




(Gross & Chao, 2006)

# Change in Gravity Observed by GRACE

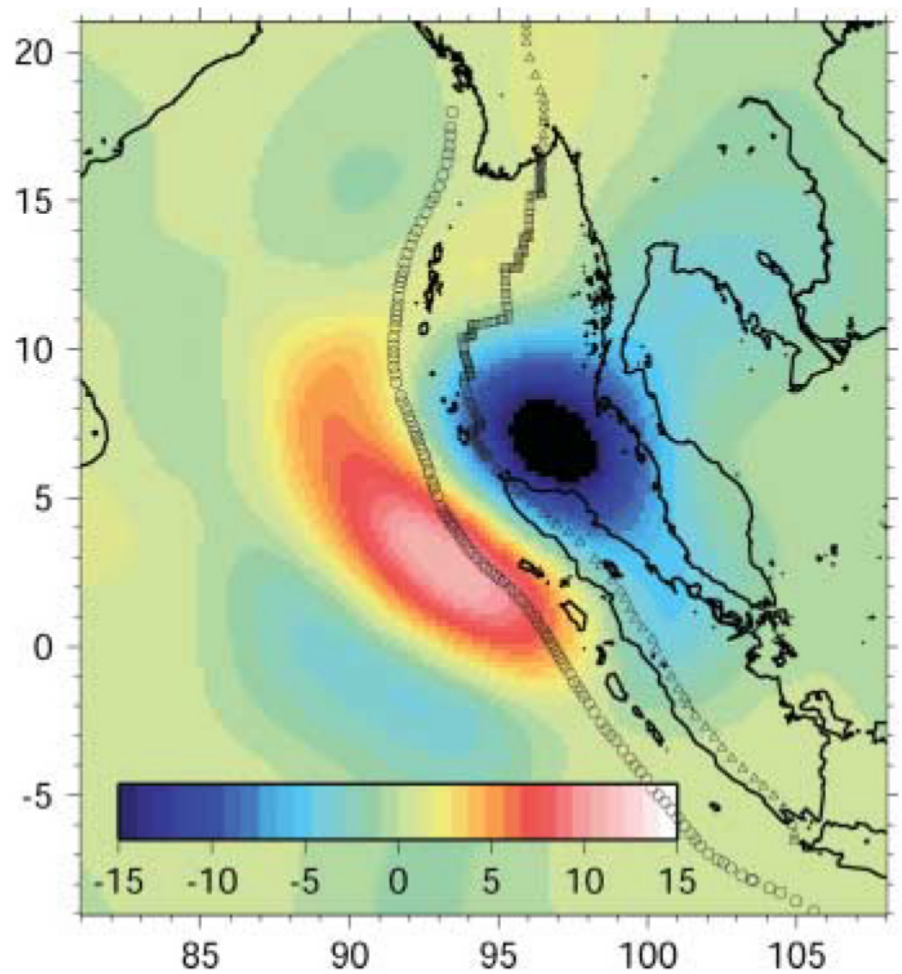



**Fig. 2.** Gravity changes (in μGal) after the Sumatra-Andaman earthquake, computed from averaging and filtering the two gravity changes between two different time periods (Fig. 1, E and F).

(Han et al., 2006)

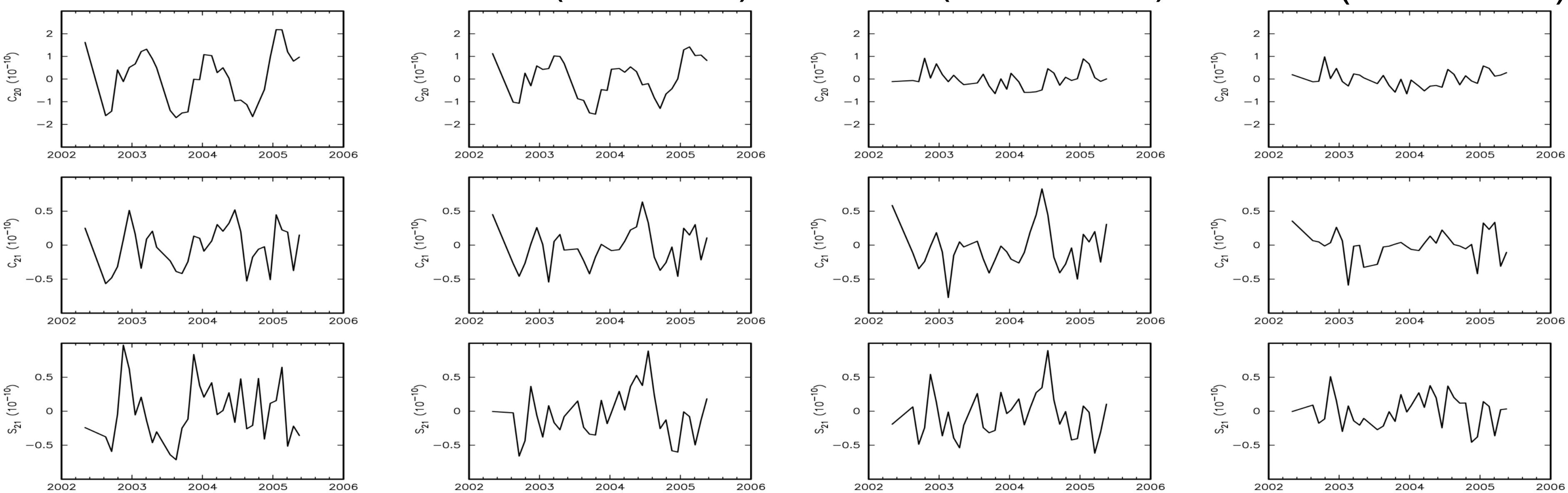
# Modeled Change in Gravity

| | $\Delta J_2$ ($10^{-11}$) | $\Delta J_3$ ($10^{-11}$) | $\Delta J_4$ ($10^{-11}$) | $\Delta J_5$ ($10^{-11}$) |
|---|---|---|---|---|
| *2004 Sumatran earthquake* | | | | |
| $M_w$ = 9.0, 3°N (standard CMT solution) | –0.91 | –0.21 | 0.09 | 0.14 |
| $M_w$ = 9.3, 3°N (larger moment) | –2.29 | –0.52 | 0.23 | 0.35 |
| $M_w$ = 9.3, 7°N (larger moment, centroid to N) | –2.17 | –0.75 | 0.10 | 0.28 |
| $M_w$ = 9.3, 3°N – 13°N (Tsai *et al.*, 2005) | –2.37 | –0.63 | 0.25 | 0.31 |
| *Approximate SLR measurement uncertainty (1$\sigma$)* | 1.3 | 1.6 | 4.9 | |
| *Other great earthquakes* | | | | |
| 1960 Chilean (Chao & Gross, 1987) | –0.83 | 3.29 | –1.89 | 3.64 |
| 1964 Alaskan (Chao & Gross, 1987) | 5.25 | 2.35 | 1.40 | 1.62 |

$$\Delta J_l = -\sqrt{2l+1}\,\Delta C_{l0}$$

# Observed Residual Gravity Coefficients from Satellite Laser Ranging
## (2002.3 – 2005.4)

observed | obs−(atm+ocn) | obs−(all surface) | nonseasonal obs−(all surface)

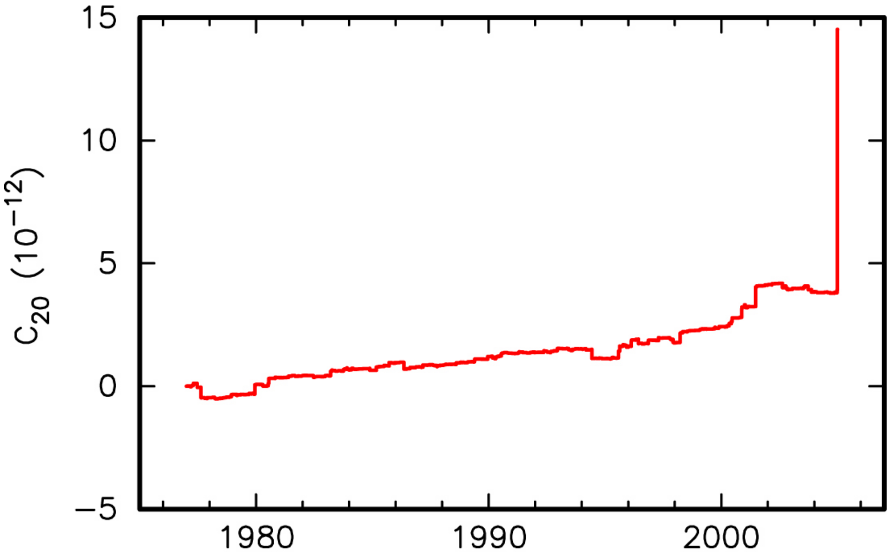


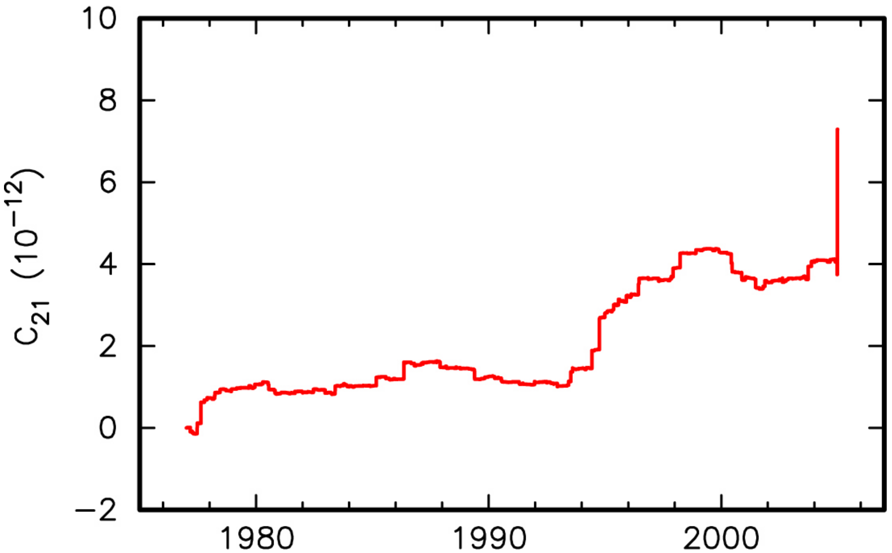

## Variance (percent reduction)

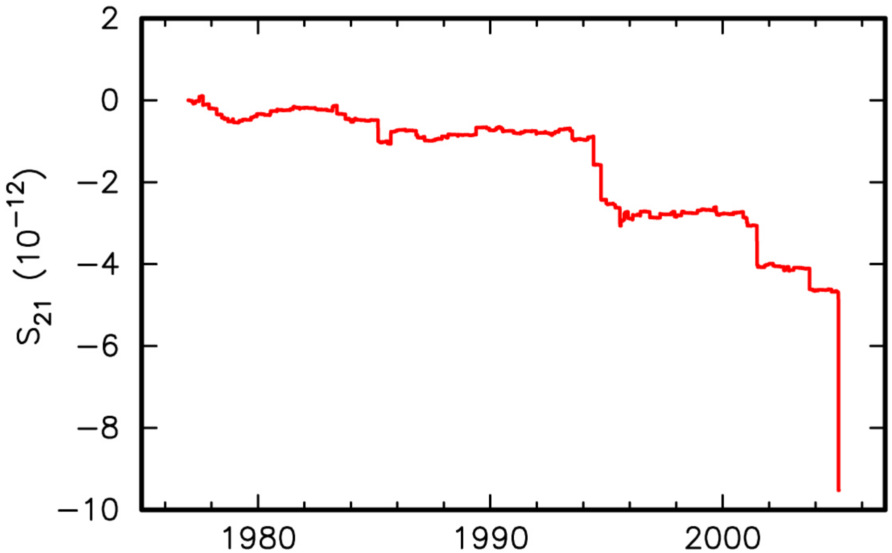
| | observed | obs−(atm+ocn) | obs−(all surface) | nonseasonal obs−(all surface) |
|---|---|---|---|---|
| $C_{20}$: | 134.55 | 72.94 (46%) | 15.74 (88%) | 12.32 (91%) |
| $C_{21}$: | 9.73 | 7.64 (21%) | 10.30 (–6%) | 4.22 (57%) |
| $S_{21}$: | 18.30 | 11.70 (36%) | 10.55 (42%) | 5.28 (71%) |

units of variance: $10^{-22}$

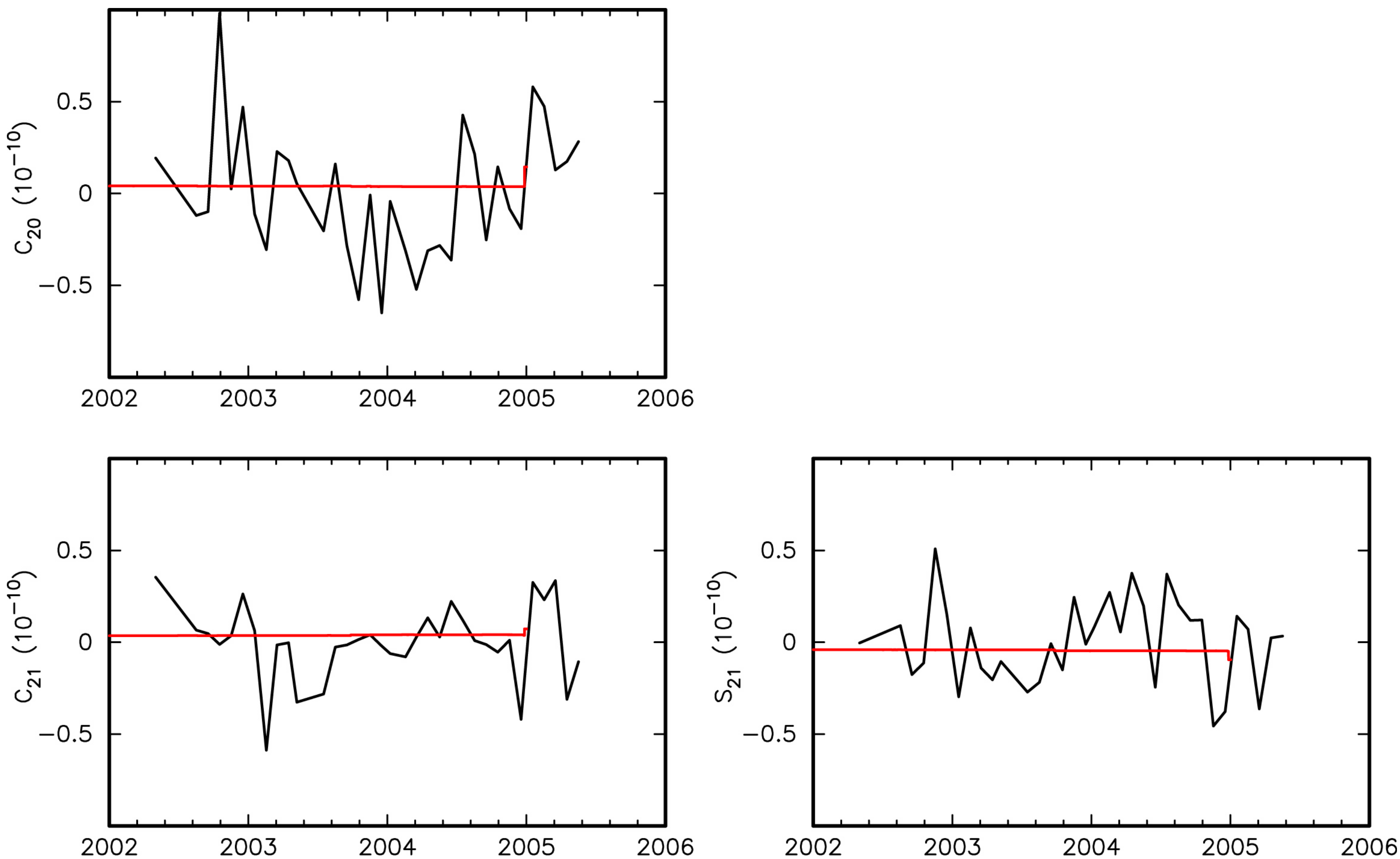
# Modeled Change in Gravity



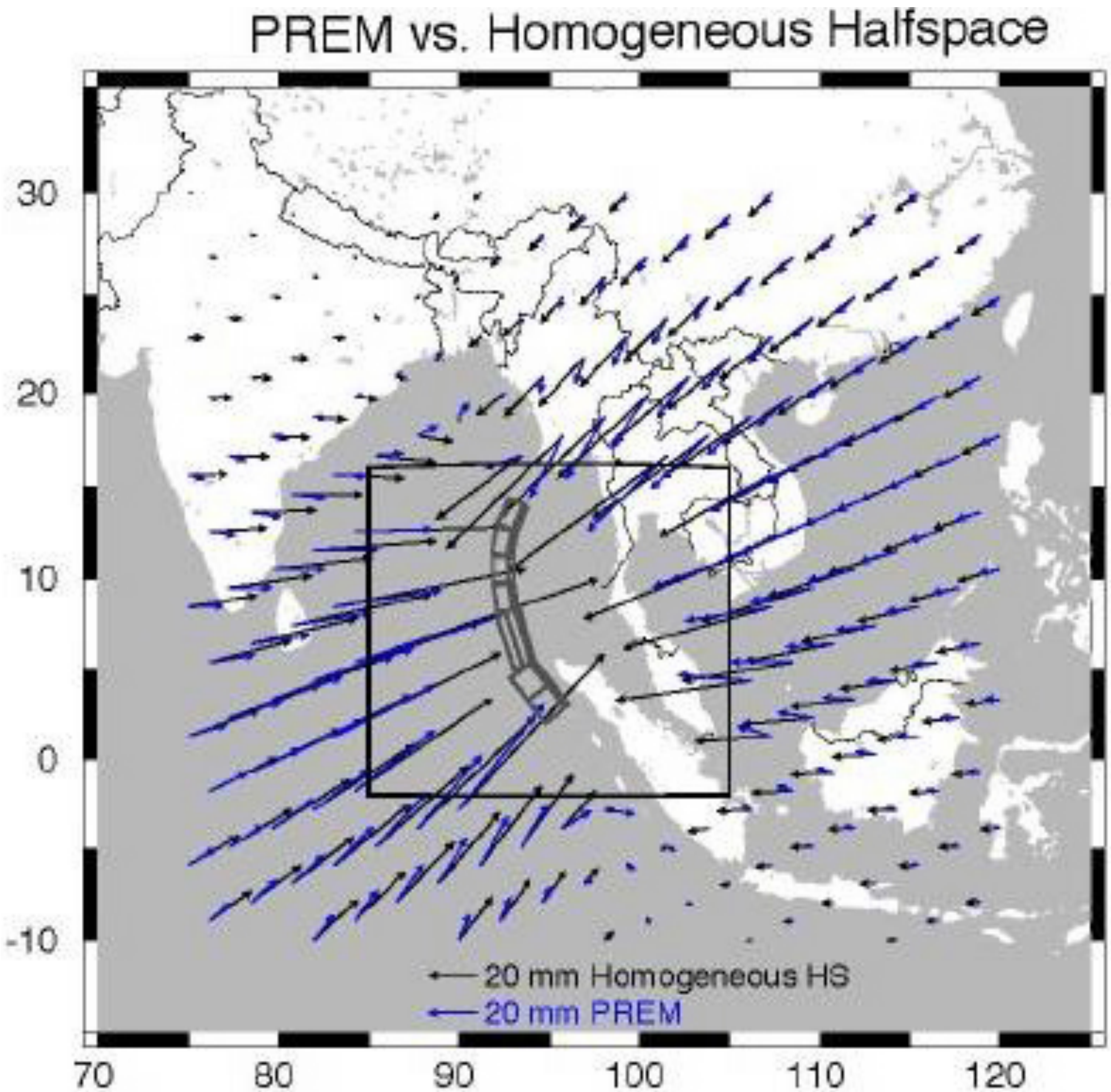



# Observed and Modeled Change in Gravity



# Modeled Change in Shape




(Banerjee et al., 2007)

# Summary

- Changes in Earth's shape, rotation, and gravity
  - Are measured by the same observing system
    - SLR, GNSS, DORIS, VLBI (shape and rotation)
  - Often have a common cause
    - Atmosphere, oceans, hydrology, earthquakes, global isostatic adjustment
  - But are sometimes modeled separately
    - Flat Earth models for earthquake displacements, spherical for rotation
- Unified model
  - Allows common shape, rotation, and gravity observations to be used to determine model parameters
  - Allows consistent geodetic modeling of
    - Surface change
    - Mass transport and exchange
    - Angular momentum exchange

# Unified Observations

- Eliminates inconsistencies
  - Between observations taken by different techniques
    - VLBI, SLR, GNSS, DORIS
  - Between EOPs and reference frames
    - IERS Working Group on Combination at the Observation Level
    - Gravity (?)
  - Between measurements taken in different reference frames
    - Example: satellite altimetric measurements of sea surface height
- Strengthens solution
  - Better global distribution of stations
    - VLBI + SLR + GNSS + DORIS
  - More measurements
- Requires common standards

# Unified Models

- Changes in observed shape, rotation, and gravity
  - Often have a common cause
    - Examples: atmosphere, oceans, hydrology, earthquakes, GIA
  - But are often modeled separately
    - Example: flat Earth models for earthquake displacements; spherical for rotation
- Develop common models
  - From common shape, rotation, and gravity observations
    - Surface change
    - Mass transport and exchange
    - Angular momentum exchange
- Dynamic Earth theme

# Foundation for a Scientific Vision

- Unify observations
  - Example: IERS WG on Combination at the Observation Level
- Unify models
  - Same model used to predict all geodetic observations
- Unify observations with models
  - Assimilate geodetic observations into models
- Dynamic Earth
  - Surface change (natural hazards)
    - Shape and deformation
  - Mass transport and exchange (climate change)
    - Gravity and geoid
  - Angular momentum exchange
    - Earth rotation